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Chaotic Itinerancy: A Multi-Disciplinary Perspective

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Abstract

Chaotic itinerancy is characterized here as a general mechanism of high-dimensional systems. The paper focuses on the interplay between the low-dimensional behavior in the vicinity of attractors and the transients of the embedding larger system. Using this interplay we chart the relationship of chaotic itinerancy to some other dynamical concepts and applied notions. From the dynamic computational point of view, chaotic itinerancy is shown to extend naturally to the concept of 'computation with trajectories', which involves the use of low-dimensional systems based entirely on transients. In a more applied perspective, chaotic itinerancy is discussed in the context of emergent multi-level systems. As an example, molecular dynamics are argued to be conceptualizable in the form of a series of low-dimensional systems which emerge temporarily in a very large system that performs itinerant motion. Finally, as motivated by the properties of chaotic itinerancy, we discuss the phenomenology of high-dimensional systems and its relationship to the problem of universality of dynamics.

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Chaotic itinerancy has been studied for more than a decade. It was found in a great number of examples which include various natural systems and computer simulations. The principal issue, in most cases, was to understand nonstationary behavior observed in high-dimensional systems. Chaotic itinerancy is of more general interest, however. It is used as a theoretical tool in the study of encoding and cryptography, as well as of episodic memory and other biological models of cognition. In this paper the concept of chaotic itinerancy is characterized as a special case for a more general idea, that of computing with trajectories. We discuss both notions from a multi-disciplinary perspective that encompasses biology, chemistry and cognitive science. We draw a picture of chaotic itinerancy as a powerful tool for obtaining dynamical behaviors of a new, important kind of complexity, and we show how the concept of 'computing with trajectories' arises naturally in that context. Finally, in a series of remarks about the phenomenology of temporal behavior and the problem of universality of dyamics, we characterize high-dimensional systems from a new viewpoint, partially motivated by the properties of chaotic itinerancy.

I. Introduction

The concept of chaotic itinerancy was introduced in the works of Kaneko,[1],[2] Ikeda et al.[3], and Tsuda.[4] Although the notion is relatively new, there are various definitions of it. According to one formulation, forwarded by Tsuda[5], chaotic itinerancy can be described as chaotic transition dynamics resulting from a weak instability of Milnor-type attractors. Similar behaviors often arise when a slow system forces a fast one. As a result, nonstationary itinerant motion can emerge. The orbit visits many regions in the phase space, and the system maintains high dimensionality, while repeatedly entering and leaving consecutive domains of low-dimensional behavior typical of attractors. A simple example (and a good visualization) is a leaf tossed into a smoothly flowing river. As soon as the river flows around some big boulders or through rapids, the leaf will be carried into various eddies and backflows, each time making different turns. The leaf can follow a highly irregular path, characterized by oscillatory behaviors, intervals of turbulence, and domains of smooth motion interlinked by the visits of the same particle.

The mathematical notion underlying the involved attractor-like temporary structures is that of Milnor attractor[6],[7],[8]. Although (as shown by Kaneko[9] and others) Milnor attractors exist in dynamical systems with a few ($n = 7$) degrees of freedom, chaotic itinerancy is typically a phenomenon of the very high dimensions. A fully developed itinerant motion tends to visit several (quasi-) attractors, which is only possible if there are enough variables to host these attractors. As the possibility of exhaustive simulation dramatically breaks down with the increasing of the number of variables, the study of systems of high dimensionsality is strongly underdeveloped. In this situation, chaotic itinerancy can be conceived as a microscope that allows us to peek into the behaviors typical for these systems. In what follows we elaborate elements and consequences of this idea.

II. A Plea for Chaotic Itinerancy

Transitory behaviors similar to those generated by Milnor attractors are possible in a large number of more conventional systems. Examples are generalized Lotka – Volterra equations[10] which produce heteroclinic cycles[11]; input-output systems[12]; and complex dyamical systems equivalent to switching networks[13]. There are claims to the effect that chaotic itinerancy is not significantly different from any of these[14]. In this section we discuss why this is wrong and why there is reason to believe that chaotic itinerancy is an important conceptual tool for the study of complicated dynamic phenomena.

Common to many simpler transitory systems we find some kind of 'designed' behavior. The Lotka-Volterra systems contain terms that ensure that orbits stay at hyperplanes characterized with a constant value of some variables, while showing characteristic oscillations in the others. The variables can change roles later. Switching networks are based on a somewehat similar principle. They contain dedicated nodes which stay dynamically inactive or 'dormant' for some time, just to be activated later, when some specific condition is met – for instance, when the remainder of the system approaches a fixed point. Then the dormant nodes jump into action and attempt to dislodge the rest from the attractor. The dynamics of the overall system is itinerant if this is successful. Although quite easy to design, such nodes exist in a small minority of systems, however.

By contrast, chaotic itinerancy appears to be a generic phenomenon of high-dimensional systems[15]. Also, as opposed to the orbits of switching networks, chaotic itinerancies lack a robust pre-determination. This makes their itinerant motion contingent upon many factors, and allows transitions to be stochastic. In other words, motiond generated from destabilized Milnor attractors can connect (quasi-) attractors in a complex and flexible way. Unlike other kinds of transient systems which are 'exotic' by brute force and are sensitive to parameters, systems with chaotic itinerancies show their astounding behavior as a consequence of a broad class of definitions. At the moment, chaotic itinerancy seems to be the only mechanism by which nonstationary dynamics can be produced in such a 'cheap and easy' way.

The above properties make chaotic itinerancy a useful tool in a new context, the hunt for more and more powerful dynamical systems. This non-mathematically motivated line of development[16] is of importance for a variety of applications such as brain theory, where plastic, reconfigurable behaviors are required in realistic, natural systems. Such a perspective on dynamics is in line with the so-called constructive[17],[18] use of nonlinear systems.

How far can we get? How can chaotic itinerancies be of help? In lack of a general theory of high-dimensional systems, the constructive framework requires – and permits – a good deal of imagination. In what follows, we outline one possible view of high-dimensional systems, and discuss the role of chaotic itinerancy in it.

III. Computing with Attractors *versus* Computing with Trajectories

In chaotic itinerancy the low dimensional systems visited by the orbits are attractors (most frequently in the form of limit cycles). As a consequence, the phenomenology of the behavior consists of a series of coupled or synchronous oscillations interrupted by chaotic transients.

The usual understanding is that a dynamical system is essentially equivalent with an attractor structure. This view is further amplified in concepts like 'computing with attractors'[19],[20],[21]. The popular metaphor obtains its power from a combination of two paradigms: the computational use of nonlinear systems (as in e.g. neural networks[22]) and the attractor-based concept of dynamics. Literally speaking, however, even when 'computing with attractors', the process of interest does not take place on a (classical) attractor, as is true by definition. In a long paper offering a comprehensive approach, Hirsch and Baird[23] write:

> We view a computational medium as a set of structurally stable input-output subsystems which can be coupled in various ways into a larger system. By 'structurally stable' we mean that the dynamical behavior of each subsystem is largely immune to small perturbations due to noise or parameter changes. We assume that the dynamics of each subsystems is organized into attractor basins; the attractors can be stationary, periodic or chaotic. As the overall system evolves in time, each subsystem passes through a sequence of attractors, some function of which is presented to the observer as the 'output' of the system. *These sequences of attractors are the 'computation' of the system.*

It is clear, therefore, that the attractor-based view is an approximation. But is it a useful approximation? If we are now looking for the possible most general behavioral (i.e. computational) structures offered by dynamical systems, then it is worth noting that (1) the classical concept of attractor breaks down here (2) the actual computational process must involve transients as an essential part. Chaotic itinerancy is an important concept here because it generalizes both properties in a natural way. In other words, if a computational process is embedded in a dynamical system, it should utilize features of dynamical systems which are usually considered as being separete, but which are now seen inherently combined in both dynamic computations and chaotic itinerancies.

From the mathematical point of view, the above recognition speaks for the importance of orbits that can link attractors. But there is no reason to stop at this point. The attractors are, strictly speaking, never reached and must be unstable in certain directions, so it is equally justified to speak of orbits that link, or connect, other orbits. What is the role of attractors at all? A probable answer is that a high-dimensional system can only perform effective computations if it behaves like a lower dimensional system[24],[25]. Chaotic itinerancy achieves this by permitting the orbits to enter the vicinity of attractors, thereby significantly reducing the dimensions. However, generalizing the same idea, there is no reason why it should not be possible to obtain other kinds of 'piecewise low-dimensional systems' which are based entirely on transients, and correspond to computations with trajectories distant from attractors.

IV. Chaotic Itinerancies and Related Products in Various Fields

Let us pay a look at the applications. Chaotic itinerancies have been demonstrated in a number of systems including physiological neural nets (by Freeman[26], Kay et al.[27], Kozma and Freeman[28]), optical systems (by Otsuka[29]), artificial neural networks (by Tsuda[30]), globally coupled maps (by Kaneko and Tsuda[31]), and replicator equations (e.g. by Hashimoto and Ikegami[32]). Tsuda[33] has recently developed the concept into a new brain theory and a theory of coding[34]. Yet the relevance of chaotic itinerancies goes significantly beyond what is suggested by these examples. Here we mention a few extensions.

(1) Chaotic itinerancies for the study of emergent multi-level systems
When a system enters the domain of chaotic itinerancy, the behavior around the attractors could be rewritten using new variables that define the low dimensional process. That is to say, the dynamics is identical with that of a multi-level system where the higher (i.e. faster) aggregate levels emerge dynamically in time. In general, there is no reason to expect that the new, synthetic variables should follow the same equations as the ones that generated them. In other words, not just a reduction of the dimensions occur, but the emergence of entirely new systems is possible. This makes chaotic itinerancy unique for the experimental and theoretical study of emergent multi-level systems. Also, the new emergent level will eventually disappear, giving rise to another emergent level at the end of each chaotic transition. Itinerant systems can thus serve as a template for processes where a variety of temporary levels are produced and destroyed. The phenomenon is of interest in several fields including evolution and ecology, social science, or chemistry.

(2) Simple considerations – chemistry.
In one possible representation, a chemical system can be conceived as an extremely high dimensional dynamical system[35], defined over the individual degrees of freedom of the atoms constituting the individual molecules. Although the analysis or simulation of systems of such astronomic dimensionality is out ouf the question, small-scale toy examples can help the cartography of the general principles that may govern such systems. Chemical systems from a strictly dynamical viewpoint were recently put in the foreground of study by S. Kauffman[36], W. Fontana[37] and others, which makes the seemingly distanct question of these extraordinary systems of immediate interest.

Dynamical chemistry inherently involves the issue of levels. Especially in large nonequilibrium chemical systems[38] dynamical representation becomes counter-intuitive at the molecular level, because the variables are not permanent in the system. It can be more instructive, therefore, to conceptualize molecular dynamics in relation to a series of low-dimensional systems which represent molecules, and which emerge temporarily in some very large system that performs itinerant motion. Accordingly, a hypothetical research program can be based on chaotic itinerancy and orbit-to-orbit computations. Although this development is yet to be seen, there is a tangible tendency towards it, as exemplified by some recent works[39].

V. The Importance of Phenomenology in High-Dimensional Systems

Studies of dynamics frequently concentrate on behavior generated from a given sets of equations. Of equal interest is the inverse problem, where samples of behavior are given (often for only one variable) and the task is to reconstruct the system. Time series analysis and other identification methods exists that attempt to trace back complex behaviors to simple causes[40,41]. A central idea is to find the original variables of the system in some reconstruction space.[42,43] Now chaotic itinerancies and other nonstationary phenomena of multi-attractor systems suggest a new way to look at the issue.

If we are presented with records of an itinerant motion connecting several (quasi-) attractors, then not knowing *a priori* that the samples are generated by a single high-dimensional system, it is probably impossible to infer (or even suspect) that this is the case. As long as time series methods are applicable here at all, the reconstructed dynamics will inevitably fall into pieces, yielding a sequence of separate low-dimensional systems, each following a different set of equations. Inverse methods always operate at the risk of producing deceptive results, but this is a different situation now.

Here, the target of the reconstruction is not the dynamics itself, but the phenomenology generated by the dynamics. Perhaps a chemical analog similar to that of the previous section can help to illuminate the case. From the behavioral samples of a chemical system, we seldom think of chemical dyamics in any form other than reaction kinetics, which, however, necessarily neglect the composition and decomposition processes of the molecules, and take them as granted variables, ready for use in the system. The actual, underlying, dynamics disappears as a result of the temporary decoupling of these new variables from the rest. The chemical example is not very well understood beyond that point, and must stay at the level of analog at the moment, but chaotic itinerancy allows for a more precise treatment of the issue as such. From the phenomenological point of view, we can say that a high-dimensional system with Milnor attractors is not one single system, but a sequence of different systems.

Based on the mild conditions under which chaotic itinerancies were found, it can be conjectured that the above situation is typical. If that is true, the next question is, how many different low-dimensional systems can be embedded in a high-dimensional system? Apart from a few experimental studies[44], at the moment only the earlier mentioned analogy to heteroclinic chaos is at disposal. The analogy suggests that we can have as many new embedded systems as there are subsystems combinatorically possible, i.e. for n variables the number is *sum (n!)/k!(n-k)!*, which may be a distant upper limit, of course. A possible new research direction would be to search for systems that are actually capable of simulating any other system of lower dimension.

Still combinatorically speaking, there exist but a few low-dimensional systems and many high-dimensional systems. The above discussion suggests that the upper end (where the majority resides), is populated by systems for which the concept of attractor loses its significance.

VI. The Universality of Dynamics

After our remarks about the phenomenology of high-dimensional systems, we may also risk the hypothesis that the presented phenomenology undermines the usual concept of dynamics understood as simply flow.

The underlying question is, Are dynamical systems universal? It is generally assumed that the answer is positive. Dynamical systems are believed to be the most general tools for representing change. Some of the considerations presented in this paper may imply, however, that that can be otherwise. More precisely, we may want to reconsider the question: Universal with respect to what? Depending on the chosen representation, there appear to be are various answers to the problem of universality.

In the conventional representations, where dynamics are considered generated from given sets of rules, the class of dynamical systems is most probably universal, by the highest available standards, that of Turing universality (or possibly even beyond[45]). This is a strong but plausible statement, which is justified by the many known models of universal computation implemented in dynamical systems[46],[47],[48] – understanding, at the same time, that the various mathematical notions of 'computation' have so far been found synonymous with each other and with the intuitive notion of 'having a rule'.

The situation may be quite different if we consider a dynamical system as a temporal dependency relation that describes a given behavior. These dependency relations can be classified on the basis of the different roles variables play in them. Dynamical systems in the form of differential equations, maps, and the like – in general, dynamical systems generated from rules – can represent those dependency relations where the relevant dimensions of the problem do not change radically during dynamic behavior. Dependency relations of a different kind are constituted by systems in which several different sets of variables are needed in a sequence, in order to give account of the temporal behavior. Chaotic itinerancy occupies a middle ground where there is a liberty to switch freely between these two kinds of representations.

If we recall that chaotic itinerancies support emergent levels, we can reformulate the last remarks. Rule-based dynamic systems typically describe processes in terms of a single level of dependency. Chaotic itinerancy (and some other dynamic concepts) provide multiple dependency structures. Elesewhere[49] it was argued that similar multiple dependencies underlie causality. This may be the basis of an unexpected and yet untested relationship between exotic dynamical behaviors and natural complex systems that support them.

VII. Conclusions

Although a paper like this may not require a conclusion, we may, by way of summary, re-emphasize that chaotic itinerancy is a powerful concept, and we demonstrated that its usefulness goes far beyond the significance of Milnor attractors. We discussed mechanisms by which chaotic itinerancies may provide cues for further classes of complicated behavior. Among these, computation with trajectories, ultra-high dimensional systems, and systems with multiple dependency were discussed and suggested to require new attention.

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