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Abstract. In this paper, we discuss a viewpoint to regard individuals in a society as cognitive agents having internal dynamics, in order to study the dynamic nature of social structures. Internal dynamics is the autonomous changes of an agent's internal states that govern his/her behavior. We first discuss the benefit of introducing internal dynamics into a model of humans and the dynamics of society. Then we propose a simple recurrent network with self-influential connection (SRN-SIC) as a model of an agent with internal dynamics. We report the results of our simulation in which the agents play a minority game. In the simulation, we observe the dynamics of the game as a macro structure itinerating among various dynamical states such as fixed points and periodic motions via aperiodic motions. This itinerant change of the macro structures is shown to be induced by the internal dynamics of the agents.

# 1 Introduction

The spontaneous social structures in a society, such as institutions, classes, and markets, usually cannot be separated from the individuals in the society, since individuals both shape and are influenced by such structures. The key notion when considering spontaneous structures is *the micro-macro loop*[14][15]. However, we think that this notion alone cannot explain some changes in social structure seen in an actual society. In this paper, we introduce *internal dynamics* in addition to the micro-macro loop and illustrate with a multi-agent simulation where both the social structure at a macro level and the individuals' behavior at a micro level keep changing.

In traditional economics, individuals are often assumed to be isolated from each other, with independent utilities and preferences. On the other hand, Egashira and Hashimoto[3] propose the notion of *socially developmental individuals* whose cognitive frameworks, including utilities and preferences, are shaped through their interaction among themselves. They show the emergence of an institution as a pattern of cognitive frameworks common to the individuals[7]. However, once organized, the institution in their model never changes. In general, if influences from the macro structure to the micro level have a self-enforcement function to regulate the behavior of individuals, it is thought that an institution

can emerge and be maintained[1]. But, spontaneous changes of the social structures are not seen in such a case. In reality, social structures change dynamically. Changes in the macro structures are often thought to be caused by changes coming from outside the micro-macro loop, but a mechanism of endogenous change is not explained.

In addition to the idea of socially developmental individuals, we introduce the notion of *internal dynamics*, representing the basic nature of cognitive individuals in a society, in order to understand the endogenous change of social structures. Internal dynamics refers to autonomous changes of the individuals' internal states. Recent cognitive science has developed into clarifying the dynamic nature of cognitive systems. Gelder, for example, advocates that humans are regarded as a kind of dynamical systems, since the complex behavior of dynamical systems can well express cognitive phenomena [17] [18]. Varela manifests the importance of structural coupling, which appropriately connects the internal states of a cognitive system to its environment through the interaction between them [19]. These studies place importance on the dynamic change of cognitive systems. In the present study, we also focus on internal dynamics, which has received attention in the field of cognitive science<sup>1</sup>. In the next section, we discuss how internal dynamics is important in considerations of human behavior. Further, we propose a model of an agent with internal dynamics which can be used in multi-agent simulations.

The purpose of this study is to illustrate the importance of viewing individuals in a society as cognitive agents having internal dynamics. In this paper, we perform the following. 1) We propose a simple model of an agent having internal dynamics. This model is expressed by a kind of recurrent neural network. 2) We construct a dynamic social simulation by a multi-agent system which is composed of the agents proposed in 1). Here, a simulation showing the dynamics of social structure is referred to as a dynamic social simulation. We adopt the minority game as a social interaction among the agents. 3) We use this dynamic social simulation to study what causes the macro level dynamics.

The rest of the paper is organized as follows. In section 2, we discuss the internal dynamics and propose a model of agent having internal dynamics. In section 3, a multi-agent system for dynamic social simulation is introduced. Results of the simulations are depicted in section 4. We present a discussion of the results in section 5 and deliver our conclusions in section 6.

<sup>&</sup>lt;sup>1</sup> An example of a cognitive phenomenon which supports the effect of internal dynamics on cognition is the experiment using a reversible figure. In this experiment, although the figure never changes objectively, the subjective vision of the figure changes with time. This result suggests that cognitive processing is evoked by autonomous changes in the internal states. Moreover, there are some studies about a perception of ambiguous patterns by using chaotic neural network, namely, a network with internal dynamics[11][10].

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# 2 Internal Dynamics

#### 2.1 Importance of Internal Dynamics

From a mechanistic viewpoint, humans can be regarded as a kind of state transition machine. They have internal states that change with external stimuli and return some responses that have one-to-one correspondences with the stimuli.

However, this viewpoint is not always appropriate, since the internal states of humans do not change only in response to external stimuli. It is difficult to explain such characteristics of human behavior as *diversity* and *consistency* by regarding humans as mere state transition machines. The term diversity means here that humans can and often do show various behaviors in the same situation. The sequence of behaviors is usually not random, but has a certain causality. We call this feature of human behavior consistency.

The internal states of humans change even in situations in which the same external stimuli are constantly given or when no external stimuli are given. We refer to this autonomous change of the internal states as *internal dynamics*. By taking the internal dynamics into consideration, we can account for some features of human behavior. Humans can behave variously, even if the same stimuli are given, since their internal states, on which their behavior depends, change autonomously. Accordingly, the human can form a *one-to-many relationship* among a stimulus and his/her responses by means of internal dynamics. The internal states change with actions as well as the external situation. Namely, various influences from the past actions, internal states, and external stimuli are stored in the current internal states. Thus, causality of the human behavior arises, since the actions depend on the internal states and correlate with a history of the past internal states.

#### 2.2 Architecture of Agent having Internal Dynamics

We conceptualize an architecture of an agent with internal dynamics. As we discussed above, the agent's internal states change autonomously. In addition, the internal states are affected by the agent's past action and the present external stimuli. These assumptions lead to a basic architecture of an agent having internal dynamics, as shown in Fig. 1. In the figure, the agent is regarded as a kind of dynamical system. Thus, we model the agent by means of a dynamical system.

A recurrent network, which is regarded as a kind of dynamical systems, agrees with Gelder's approach of treating humans as dynamical cognitive systems. The recurrent network is known to have various functions such as pattern recognition, motion control, and time series prediction. It is often used in the field of computational cognitive science [13][16][8]. Although recurrent networks can produce many behaviors, their computational cost is typically prohibitive for modeling a great number of agents necessary for a large-scale social simulation. On the other hand, the computational cost of a simple recurrent network (SRN) designed by

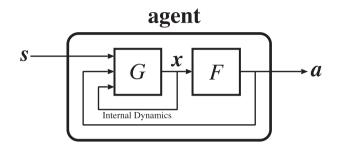


Fig. 1. The basic architecture of agent having internal dynamics. The symbols s, x and a are external stimuli, the agent's internal states and his/her actions, respectively. The boxes labeled by G and F are functions to change the internal states and to decide how the agent behaves when he/she has certain internal states, respectively. The arrows indicate the direction of interactions between the elements.

Elman[4] is comparatively less. Furthermore, the SRN is as effective in simulating dynamic phenomena, since it has a powerful ability to learn/predict a time series[4][5][6]. In spite of these advantages, the SRN is not often used in social simulations.

We propose a concrete agent model corresponding to the basic architecture illustrated in Fig. 1. The model is a modification of the SRN. We call this model a *SRN with self-influential connection* (*SRN-SIC*). Figure 2 shows the proposed architecture of the agent. The SRN has an input layer to accept external stimuli; an output layer to decide the output value based on received signals; and a hidden layer to process input values and to pass them to the output layer. Further, the SRN has a context layer in which each neuron has one-to-one connections with each neuron of the hidden layer, in order to copy a previous state of the hidden layer. Therefore, the state of the network at a certain time is decided by mixing current stimuli and a history of the past states. Moreover, the SRN-SIC has an additional recurrent connection between the output and the input layers so that the agent decides its own action based on his/her past action.

We show a mathematical form of the SRN-SIC. Each layer has its own index variable: l for recursive output nodes, k for output nodes, j for hidden nodes, i for input nodes, and h for context nodes. The output function of each neuron other than the input and the context neurons is the differentiable nonlinear function L whose range is between -1.0 and 1.0. The function L is defined by

$$L(net) = \tanh(\beta net) , \qquad (1)$$

where *net* is the sum of weighted input values, and  $\beta$  decides the nonlinearity of the function L. The output of SRN-SIC is determined by

$$o_k(t) = L(net_k(t)) \quad , \tag{2}$$

$$net_k(t) = \sum_{j=0} w_{kj} v_j(t) + \theta_k \quad , \tag{3}$$

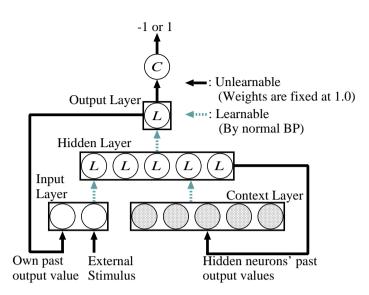


Fig. 2. The SRN-SIC as the proposed architecture of the agent. This is a particular Elman-type network with an additional recurrent connection between the output and the input layers. The symbol L represents a nonlinear function to output a real number between -1.0 and 1.0. The symbol C represents a step function which classifies an output value into two value -1 or 1 in order to be suited for the minority game. Not all connections are shown.

where  $o_k(t)$  is the k-th output neuron's value at time t,  $w_{kj}$  is the connection weight between the k-th output and the j-th hidden neurons,  $v_j(t)$  is the j-th hidden neuron's value at time t, and  $\theta_k$  is a bias of the k-th output neuron. The hidden neuron's activation is calculated by

$$v_j(t) = L(net_j(t)) \quad , \tag{4}$$

$$net_j(t) = \sum_{i=0} w_{ji} x_i(t) + \sum_{h=0} w_{jh} u_h(t) + \sum_{l=0} w_{jl} z_l(t) + \theta_j \quad , \tag{5}$$

where  $w_{ji}$  is the connection weight between the *j*-th hidden and the *i*-th input neurons,  $x_i(t)$  is the *i*-th input neuron's value at time *t*,  $w_{jh}$  is the connection weight between the *j*-th hidden and the *h*-th context neurons,  $u_h(t)$  is the *h*-th context neuron's value at time *t*,  $w_{jl}$  is the connection weight between the *j*-th hidden and the *l*-th recursive output neurons,  $z_l(t)$  is the *l*-th recursive output neuron's value at time *t*, and the  $\theta_j$  is a bias of the *j*-th hidden neuron. Each value of *u* and *z* can be replaced by the past hidden and the past output neuron's activation, respectively. Therefore, the equation (5) is rewritten as

$$net_j(t) = \sum_{i=0} w_{ji} x_i(t) + \sum_{h=0} w_{jh} v_h(t-1) + \sum_{l=0} w_{jl} o_l(t-1) + \theta_j \quad .$$
 (6)

When we consider the context layer as a type of input layer at each time step, the network can be regarded as a kind of feedforward type neural network. Therefore, as a learning method, we adopt the error Backpropagation learning. Each weight of all recurrent connections is fixed at 1.0 and is not adjusted by learning.

# 3 Multi-Agent System for Dynamic Social Simulation

We show a dynamic social simulation by using the multi-agents with internal dynamics proposed in the previous section. In this simulation, we adopt the minority game (MG) proposed by Challet and Zhang[2] as a social interaction among the agents. The game is characterized by the following two basic rules:

- 1. N (odd) players must choose one out of two alternatives (-1/1 meaning buy/sell, or etc) independently at each step<sup>2</sup>.
- 2. Those who are in the minority side win.

To consider a micro-macro loop in our system, we establish influence from the macro level to the micro level by the following two ways. One is that the previous move of minority side is given to all players as an external stimulus at each step. The other is that all players learn a time series of the past minority move.

This simulation is concretely carried out by the following procedure:

- 1. Each agent independently decides a move (-1 or 1) based on its own past action and the move of minority side at the last play.
- 2. A current move of minority side is determined from all players' moves.

We call this flow *one step*. All agents learn a time series of the minority moves for the past 100 steps per every 10000 steps. We refer to the 10000 steps between the learning processes as *one turn*.

# 4 Simulation Results

In this section, we report the results of the multi-agent simulation. In the simulation, the population size of the agents is 101. The SRN-SIC of each agent has one output neuron, five hidden neurons (i.e., there also are context neurons) and two input neurons, as illustrated in Fig. 2. At the beginning of the simulation, all the input values including the feedback input values from the output and the hidden neurons are set to be 0.0. The initial connection weights are set to be random real numbers between -0.5 and 0.5, but only recurrent connection weights are fixed at 1.0.

We distinguish several observable dynamics in the micro and the macro levels, as indicated in Table 1.

 $<sup>^2</sup>$  An output value from the SRN-SIC, which is a real number, is converted to either -1 or 1 in order to correspond to a move in the MG. In this conversion, we regard 0.0 as a border.

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Macro Level		Micro Level		
A time series of minority move	The number of agents who belong to the minority side	Moves of each agent	Internal dynamics of each agent	

Table 1. The range of the macro and the micro level.

### 4.1 Dynamics at Macro Level – Classification of Change Patterns and Itinerant Dynamics –

The time series of the minority move shows various patterns. We classify these into six different patterns, as illustrated in Fig. 3. In order to understand the dynamics at the macro level in detail, we examine in this figure the transition of minority move (-1 or 1) multiplied by the number of agents belonging to the minority side, namely, the winners.

We focus our attention on Fig. 3(b) and (c), in which the minority side never changes. Although all agents continuously receive the same external stimulus, the number of winners changes periodically in Fig. 3(b) and aperiodically in Fig. 3(c). These dynamics imply that the agents can autonomously alter the way they interpret the external information utilizing their internal dynamics, even if the same information is successively given to them. This resembles a human's vision of a reversible figure. Note that the periodic/aperiodic changes happen in one turn in which no learning was executed.

We observe various patterns in the dynamics of the minority move, even in one turn. Figure 4 depicts typical itinerant dynamics at the macro level in one turn<sup>3</sup>. As can be seen, the patterns in the time series of the game itinerate among various dynamical states. The transitions among fixed points and periodic changes are mediated by aperiodic dynamics<sup>4</sup>.

The dynamics illustrated in Fig. 4 is observed in one turn. That is to say, it is confirmed that very complex changes at the macro level are induced by the internal dynamics of each agent, though the internal structures of the agents are not modified by learning.

<sup>&</sup>lt;sup>3</sup> To draw the graphs in Fig. 4, we encode the time series of the minority move. At first, the minority moves, -1 and 1, are coordinated to 0 and 1 as binary digit, respectively. Next, a 20 steps series of the minority move is regarded as a binary fraction. Then, it is converted to a decimal fraction.

<sup>&</sup>lt;sup>4</sup> These changes resemble *the chaotic itinerancy* proposed by Kaneko and Tsuda[9]. We still could not clarify whether the dynamics observed in our system is precisely chaotic itinerancy.

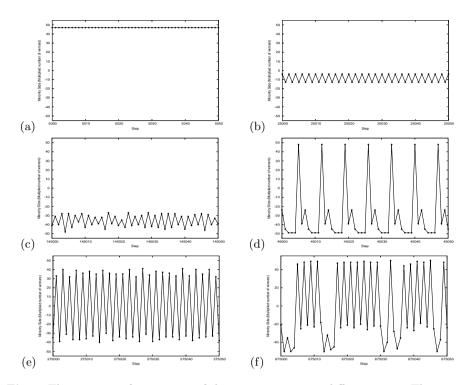
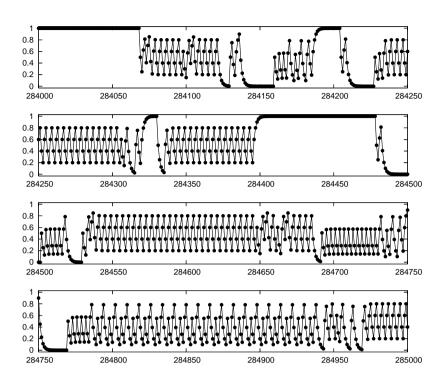


Fig. 3. The patterns of time series of the minority move in different turns. The x-axis is the steps. The y-axis is the minority move (-1 or 1) times the number of winners, i.e., agents in the minority side. The positive (negative) value in the y-axis signifies that the minority move is 1 (-1). (a) Both the minority side and the number of winners are fixed. (b) The minority side is fixed, and the number of winners periodically changes. (c) The minority side is fixed, and the change of the number of winners is aperiodic. (d) Both the minority side is periodic, and that of the number of winners is aperiodic. (f) Both the minority side and the number of winners aperiodic.

# 4.2 Dynamics at Micro Level – Emergence and Transition of Agent's Strategy –

In this section, we investigate the behavior of the agents. At first, we show that agents obtain particular strategies through learning and interaction in the minority game. A strategy is a way to determine how an agent reacts to external stimuli.

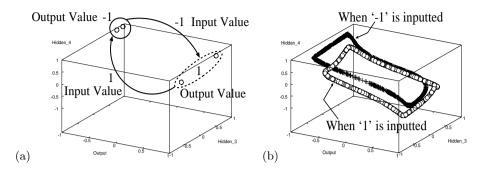
Figure 5 shows examples of two different agents' strategies, expressed as the relationship between the output value and the internal dynamics, namely, the changes of two hidden neurons' values. The agent exemplified in Fig. 5(a)acquires a simple strategy which can be denoted by a deterministic finite state transition machine with two states. He/She behaves regularly, depending on the



**Fig. 4.** An example of itinerant dynamics at the macro level in one turn. The *x*-axis and the *y*-axis of each figure are the steps and the minority moves converted to real numbers, respectively. The dynamical states of the game change frequently among fixed points and various periodic cycles via aperiodic motions.

input values. That is to say, there is a one-to-one correspondence between the external stimulus and the action. The other type of strategy, illustrated in Fig. 5(b), accurately uses two rules depending on two kinds of input value, -1 and 1. While the rules described by two closed curves seem simple at a glance, the agent's behavior is complex. The points on the closed curves are so dense that the output sequence of the agent is quasi-periodic. Further, since each closed curve ranges over almost the entire area of the output, the strategy creates a one-to-many relationship from an input to the agent's moves. We also found agents whose strategies are expressed by deterministic finite state transition machines with many states and complex forms like strange attractors.

We illustrate examples of the transitions of two different agents' strategies in several turns in Fig. 6. These strategies vary through the learning process. An interesting agent behavior is found in the 24th turn. Although the minority move in this turn is fixed on '-1', namely, the same stimulus is continuously given, the trajectory of the agent's internal dynamics drawn in Fig. 6(a) shows a chaotic motion. Time series analyses confirm that this motion is a low dimensional chaos



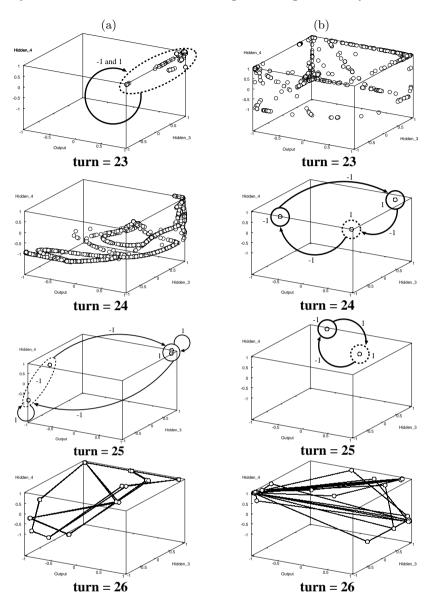
**Fig. 5.** Examples of two different agents' strategies. The x, y and z-axis of each figure are the values of the output neuron, the third hidden neuron and the fourth hidden neuron, respectively. (a) A strategy described by a simple deterministic finite state transition machine with two states is depicted in the phase space. The small circles show the actual outputs of network. The large circles and the arrows stand for the output values of the agent and the input value that he/she receives, respectively. The dotted circle is the initial state of the agent. The agent behaves periodically. (b) This is a strategy that has two closed curves corresponding to two input values. This means that the agent having the strategy can switch two output sequences according to the external stimuli.

with weak nonlinearity. In contrast, the other agent in the same turn depicted in Fig. 6(b) acquires a simple deterministic finite state transition machine with three states. In this turn, these agents alter their actions depending on only their past actions. In other words, they attain one-to-many relationships between an input and outputs.

#### 5 Discussion – Causes of Dynamics at Macro Level –

Time series of the minority move show definite features such as fixed points and periodic motions. This suggests that the agents have certain internal structures and form certain relationships with other agents, because the time series of the minority move is decided by the sequence of all agents' moves. Besides, since fixed points and periodic cycles can be described by some rules of dynamical systems, an agents' society showing such dynamics is considered as in some structuralized states with macro level rules. Accordingly, the feature of time series can reflect a macro structure in the agents' society. In our system, the feature of the time series changes with time, as shown in Fig 3 and 4. That is to say, the system realizes the dynamics of the macro structure in the agents' society.

It is thought that the dynamics of the macro structure is brought about by some instability in the system. If so, where is the instability? From the result showing different features of the time series for each turn (Fig. 3), instability must be caused by the learning between turns. Further, there seems to be another instability that is produced by the internal dynamics and interaction of the



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**Fig. 6.** Transitions of two different agents' strategies in turns  $23\sim26$ . All axes are the same as those in Fig. 5. The strategies of each agent vary through the learning process among various deterministic finite state transition machines and complex forms like strange attractors. The agents do not have the same strategy in one turn. For example, in the 24th turn the strategy in (a) forms a strange attractor and in (b) a deterministic finite state transition machines with three states. Although both of the strategies in (a) and (b) in turn 26 are deterministic finite state transition machines with 12 and 30 states, respectively, the number of the states is too many to illustrate. Therefore, we draw the trajectories instead of circles and arrows.

agents, since the system itinerate among various dynamical states in one turn (Fig. 4). In the following section, we discuss both of the instabilities.

#### 5.1 Instability between Turns –Effect of Learning–

In the learning process, each agent independently forms a prediction model from the sequence of minority moves of the past 100 steps, to estimate the transition of the game. The model is based on *a static expectation* that the past structure is preserved as is. All agents try to predict the macro structure in the future from a part of the past events. The structure is, however, constructed by all agents whose behavior has been modified by the learning process. Therefore, a static expectation model does not work well to predict the transition of the game.

This is structurally the same destabilization mechanism that is seen in Taiji and Ikegami's studies of the coupled dynamical recognizers [16][8]. There are two agents playing the iterated prisoners' dilemma game in their model. The agents try to make models of their opponents mutually through learning. For each agent, the opponent model used in the previous game is often different from the current opponent. Thus, the dynamics of the game becomes unstable, since the opponent model cannot predict the current opponent's move correctly. Our model can be thought of as an extension from the relationship between two persons in the model of Taiji and Ikegami [16][8] to one among many people. Even though only the moves of the minority side are input to each agent, the minority side is constructed from the moves of all agents. Accordingly, each agent relates to all agents indirectly.

The static expectation is an expression of the agent's bounded rationality. In actual societies, no one can make a complete prediction model that takes the consequences of behavioral changes of all people into consideration. Therefore, the cause of destabilization discussed here is inevitable when social structures are endogenously formed.

#### 5.2 Instability in One Turn –Effect of Chaotic Actions–

To know what feature at the micro level causes the itinerate dynamics at the macro level in one turn, we investigate the configuration of the agents' actions at the micro level. Table 2 shows the configuration corresponding to the classification in section 4.1. When the itinerant dynamics is shown at the macro level, the number of agents with aperiodic actions is much larger than the other cases. The aperiodic action may be chaotic dynamics as indicated in section 4.2. Chaotic dynamics has orbital instability, which expands small differences in the trajectories of agents' actions[12]. Therefore, even a small displacement at the micro level can induce a change in the macro level dynamics.

Let us discuss the aperiodic action from the viewpoint of the relationship between inputs and outputs. A strategy with one-to-many relationships emits aperiodic actions. Periodic actions are, however, also derived from a strategy with one-to-many relationships, as shown in Fig.6(b), the 24th turn. The distinction of these strategies is that the one with aperiodic action, characterized

Table 2. The correspondence between changing patterns at the macro level and the configuration of agents at the micro level. The left-hand side of the table indicates the classification at the macro level dynamics (see section 4.1). The first and the second columns are the dynamical states of the minority side and that of the number of winners, respectively. Each number in the right-hand side of the table stands for the number of agents whose actions are in the specific dynamical states. In the case of itinerant macro dynamics (the bottom row), the number of agents showing aperiodic actions is much larger than in the other cases.

Macro Level		Micro Level		
Minority Side	The number of Winners	Fixed Point	Periodic Motion	Aperiodic Motion
Fixed Point	Fixed Point	101	0	0
Fixed Point	Periodic Motion	84	17	0
Fixed Point	Aperiodic Motion	50	46	5
Periodic Motion	Periodic Motion	63	38	0
Periodic Motion	Aperiodic Motion	20	77	4
Itinerant Motion	Aperiodic Motion	8	6	87

by strange attractors, forms a one-to-infinity relationship from an input to outputs. Accordingly, the condition for the dynamics of macro structure to appear may be that there exists a certain number of agents having a one-to-infinity relationship between an external stimulus and their actions. Internal dynamics is indispensable for obtaining such complex behavior.

### 5.3 Other Instabilities

In our system, there are also other causes of destabilization. One candidate is the nonlinearity of the SRN-SIC. It is a nonlinear dynamical system and has high dimensional nonlinearity if there are many neurons. Thus, we have to elucidate how this nonlinearity affects an agent's behavior and dynamics of the macro structure by explicating the mathematical structure of the SRN-SIC through experiments such as changing the number of neurons.

A feature of the MG may also be a cause destabilization. In the MG, there is a threshold at half of the players' population. When the number of agents in the minority side is around the threshold, the result of a game changes if only a few

players alter their behavior. The similar effect of such a threshold works when output value of the SRN-SIC is divided into '-1' or '1.' Thus, behavior of the SRN-SIC is easily changed by a small fluctuation in the input when the output value is around 0.0.

# 6 Conclusion

In this paper, we have discussed the effectiveness of viewing a human as a cognitive agent having internal dynamics when we account for the emergence and dynamical changes of social structures. We have proposed a model of a social agent having internal dynamics in terms of a simple recurrent network with self-influential connection (SRN-SIC) in order to illustrate the effectiveness concretely. Using a dynamic social simulation considered a micro-macro loop involving such agents, we have shown that complex dynamics emerged at both a micro and a macro levels.

The cause of the macro-level dynamics is conjectured as follows. First, each member of the society does not consider the behavioral change of all the other members to predict the future constructed by them; second, a one-to-infinity relationship between an external stimulus and the actions of each member leads to a chaotic behavior.

Our simulation results substantiate the significance of internal dynamics for forming and maintaining a dynamic social structure. Thus, we conclude that internal dynamics is necessary to form and maintain a dynamic social structure. We also argue that our proposed SRN-SIC is an efficient architecture of a social agent with internal dynamics to construct dynamic social simulations.

We have shown endogenous dynamics of social structures represented by itinerant dynamics, even though the agents' internal structures do not change by learning. It is not yet clear, however, how this dynamics emerged. In further studies, we will clarify the influence of the internal dynamics on agents' behavior and on the macro structure. By solving these problems, we will be able to better perform dynamic social simulations to address the essence of the dynamics in actual societies.

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