I113 オートマトンと形式言語
レポート2の解説
(I113 Automaton & Formal Languages Answer & Comments for Report 2)

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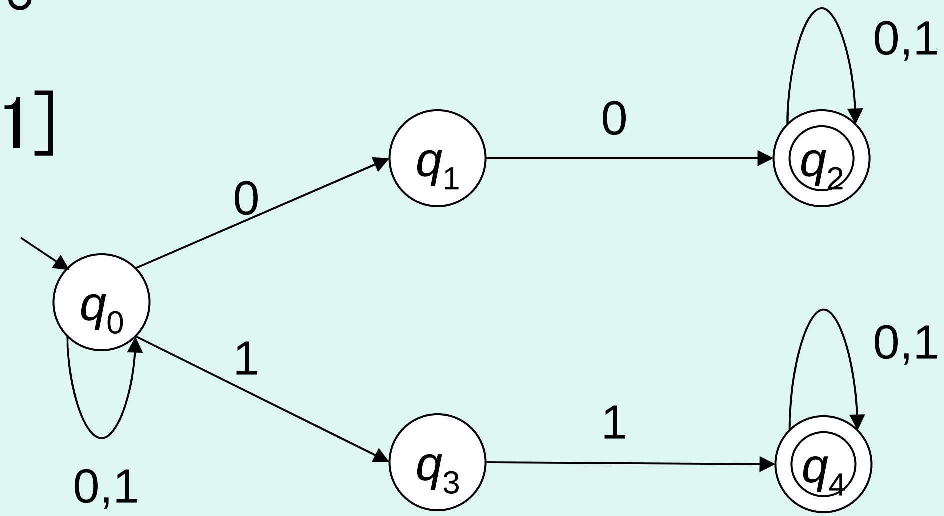
レポート (2)

[問題1] 図1で与えられるε-NFA $A=(\{q_0,q_1,q_2,q_3,q_4\}, \{0,1\}, \delta, q_0, \{q_2, q_4\})$が受理する言語と同じ言語を受理する正則表現を構成せよ。

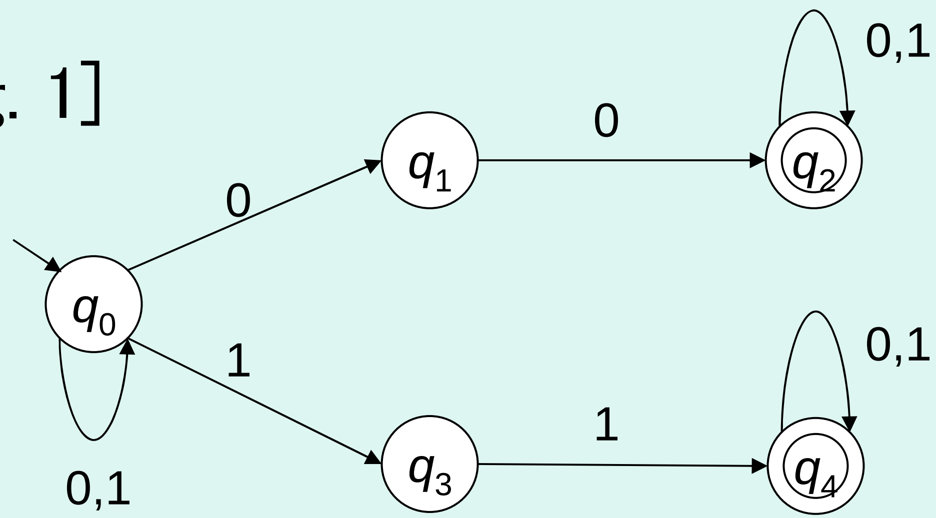
[図1]



Report (2)

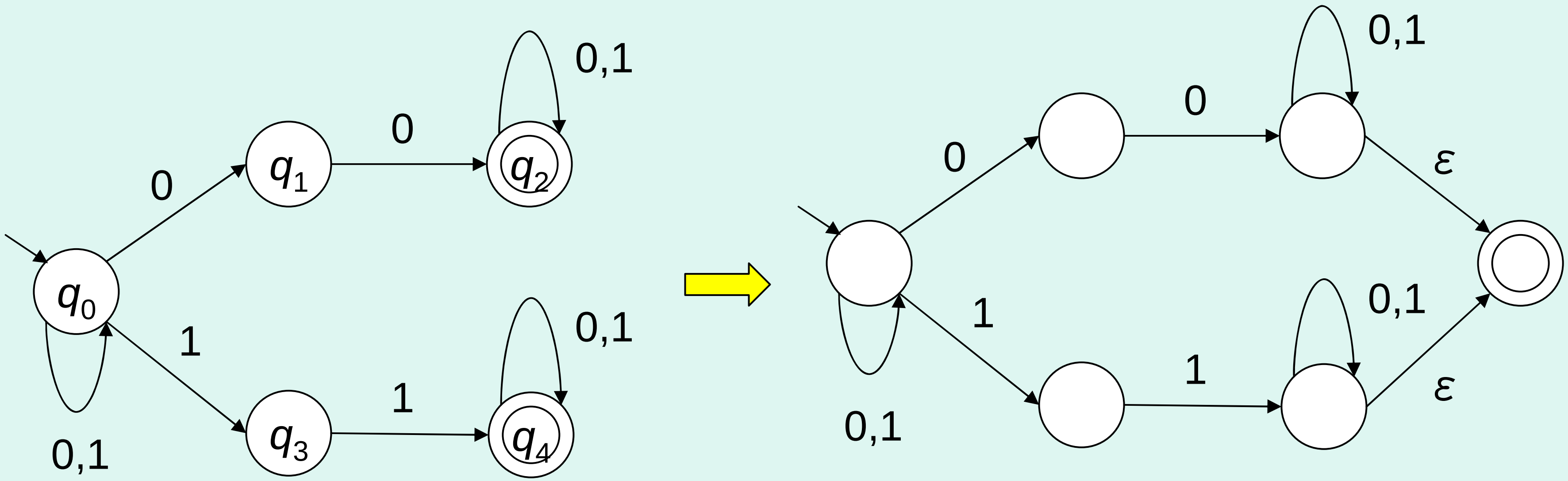
[Problem 1] Let A be an ε-NFA $(\{q_0, q_1, q_2, q_3, q_4\}$, $\{0,1\}$, δ, q_0, $\{q_2, q_4\})$ given in Fig. 1. Give a regular expression that generates $L(A)$.

[Fig. 1]



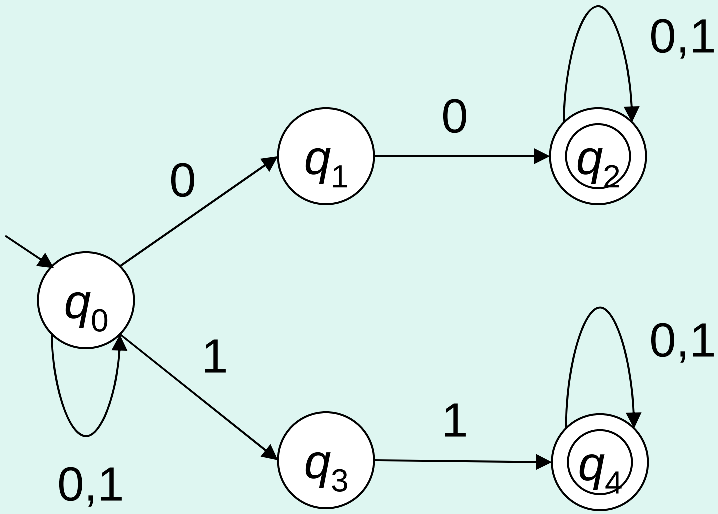
[問題1] 図1で与えられるε-NFA $A=(\{q_0,q_1,q_2,q_3,q_4\}, \{0,1\}, \delta, q_0, \{q_2, q_4\})$が受理する言語と同じ言語を受理する正則表現を構成せよ。

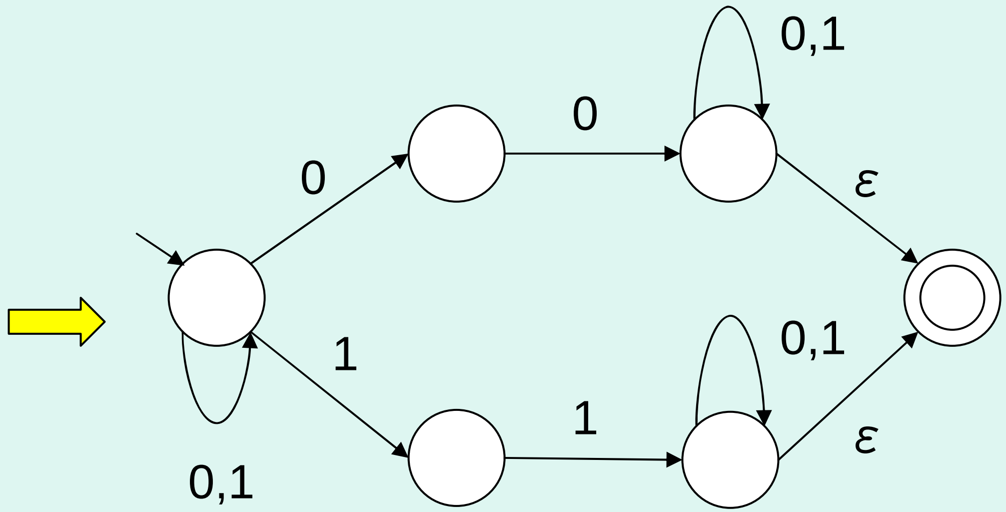
[準備]「受理状態1つ」「受理状態から遷移しない」「無駄な状態が存在しない」ようにする



[Problem 1] Let A be an ε-NFA ($\{q_0, q_1, q_2, q_3, q_4\}$, $\{0,1\}$, δ, q_0, $\{q_2, q_4\}$) given in Fig. 1. Give a regular expression that generates $L(A)$.

[Preliminary] Make A to 'one accepting state', 'no transitions from the accepting state', and 'no redundant states.'





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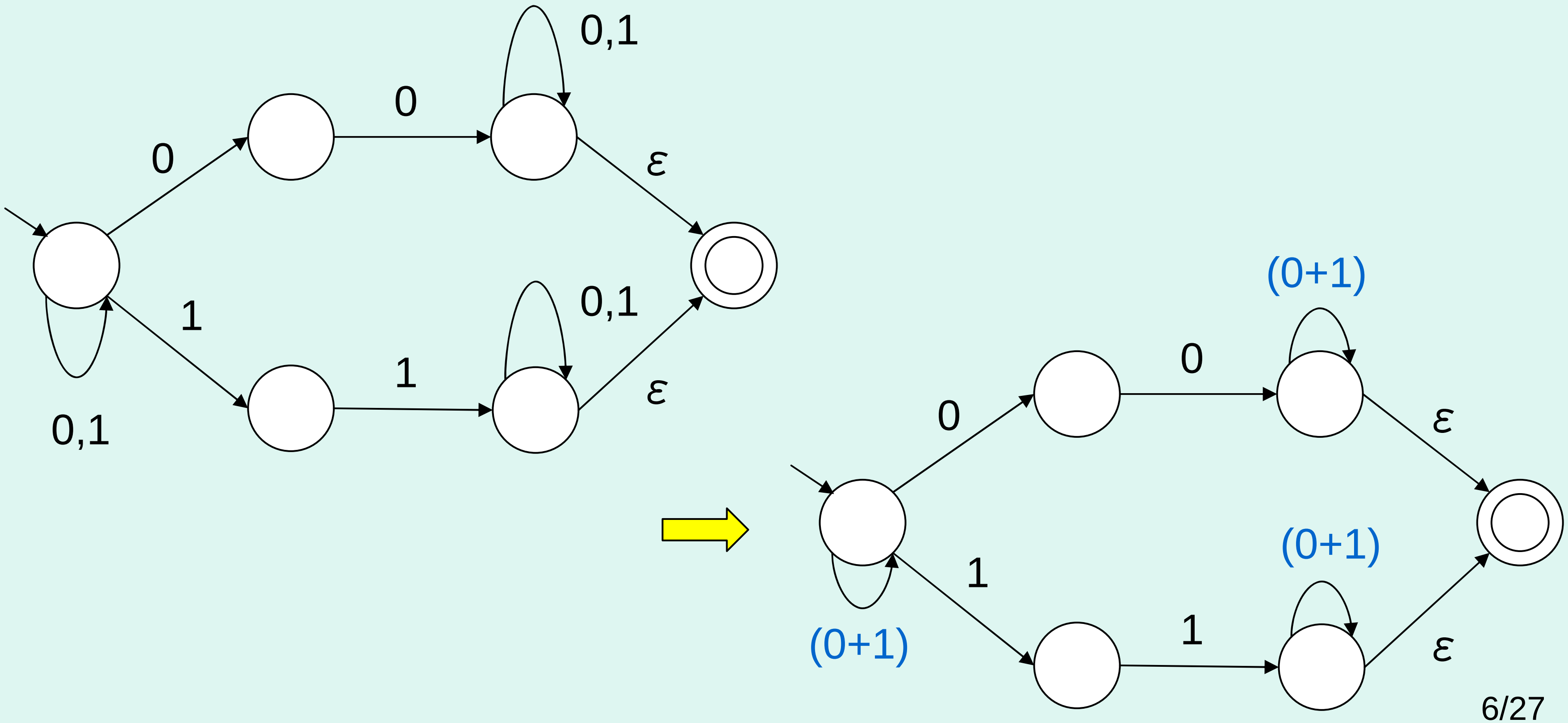
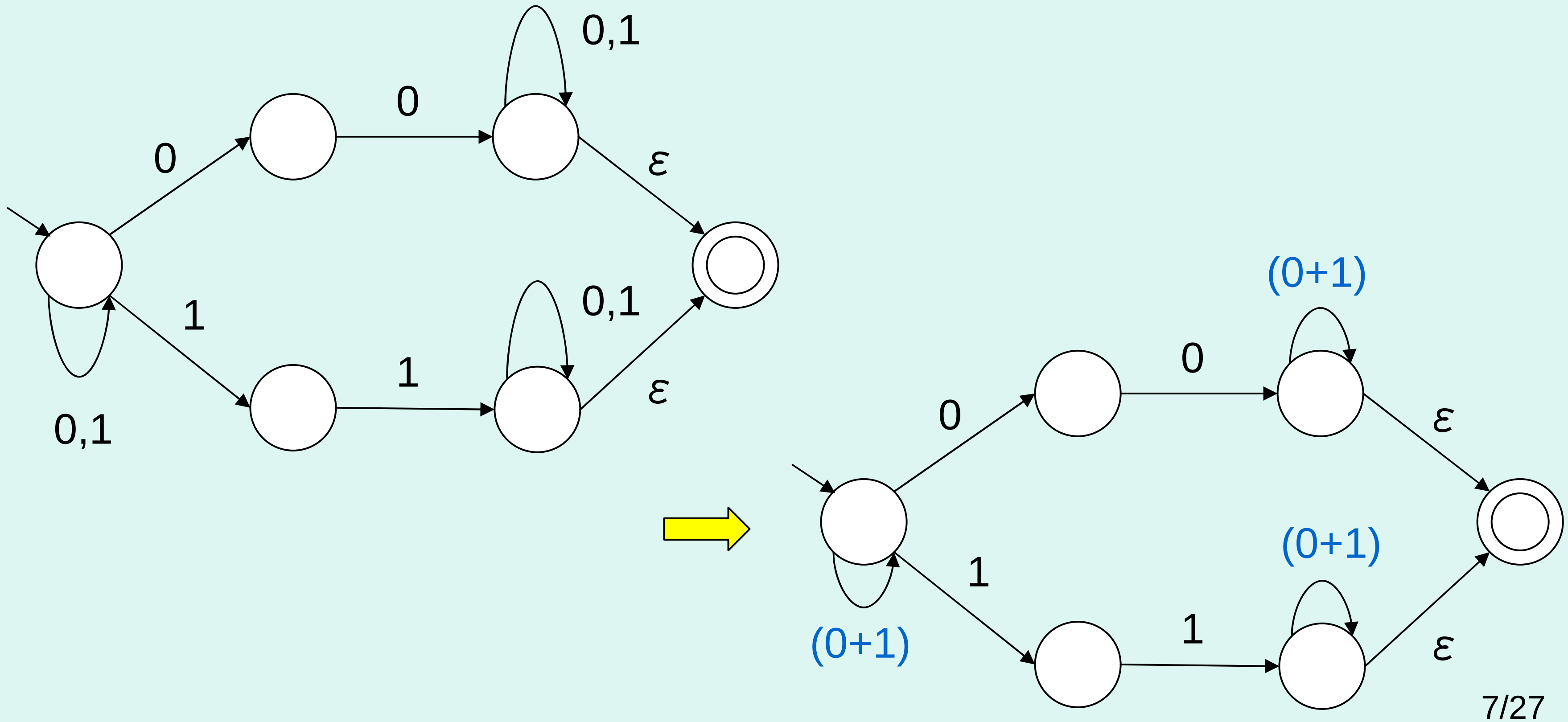
[T1] 多重辺の除去



[Problem 1] Let A be an ε-NFA ($\{q_0, q_1, q_2, q_3, q_4\}$, $\{0,1\}$, δ, q_0, $\{q_2, q_4\}$) given in Fig. 1. Give a regular expression that generates $L(A)$.

[T1] Remove multiple edges.



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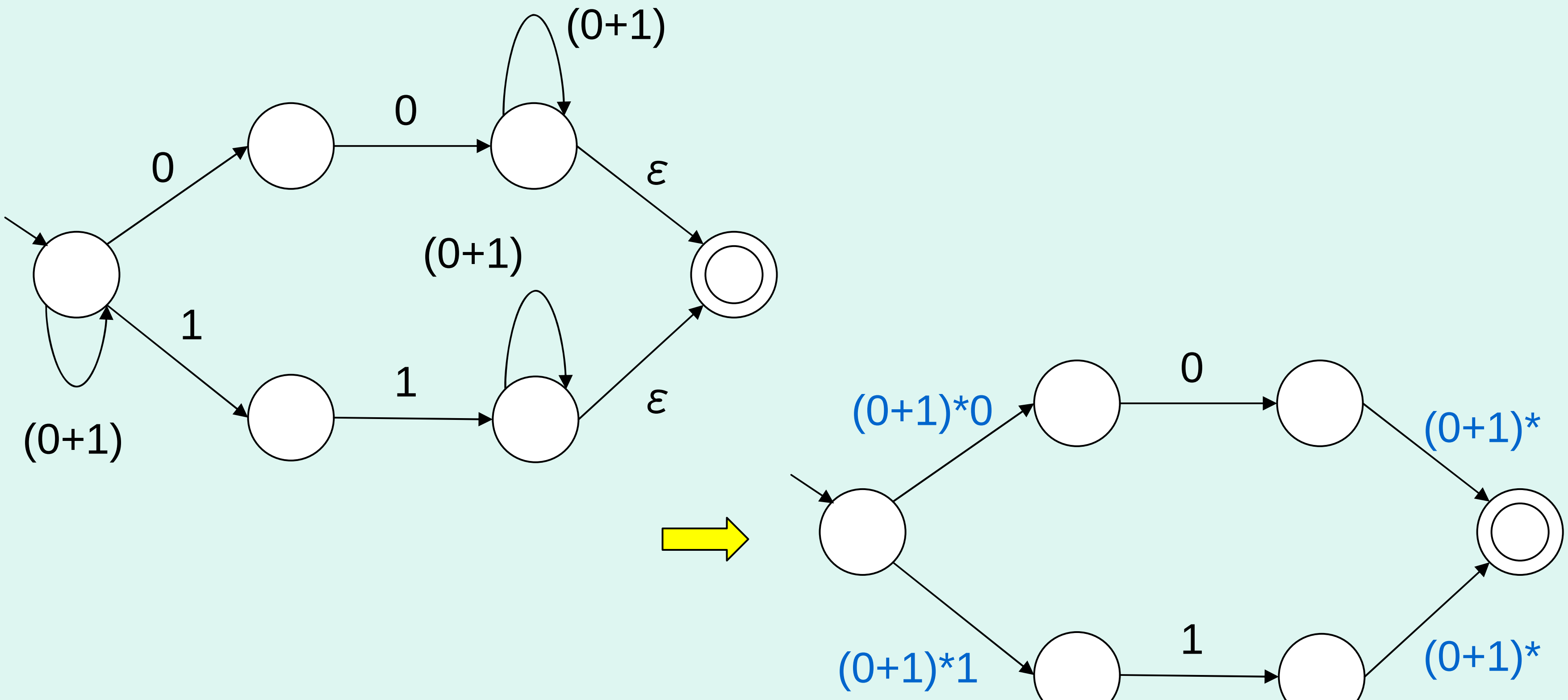
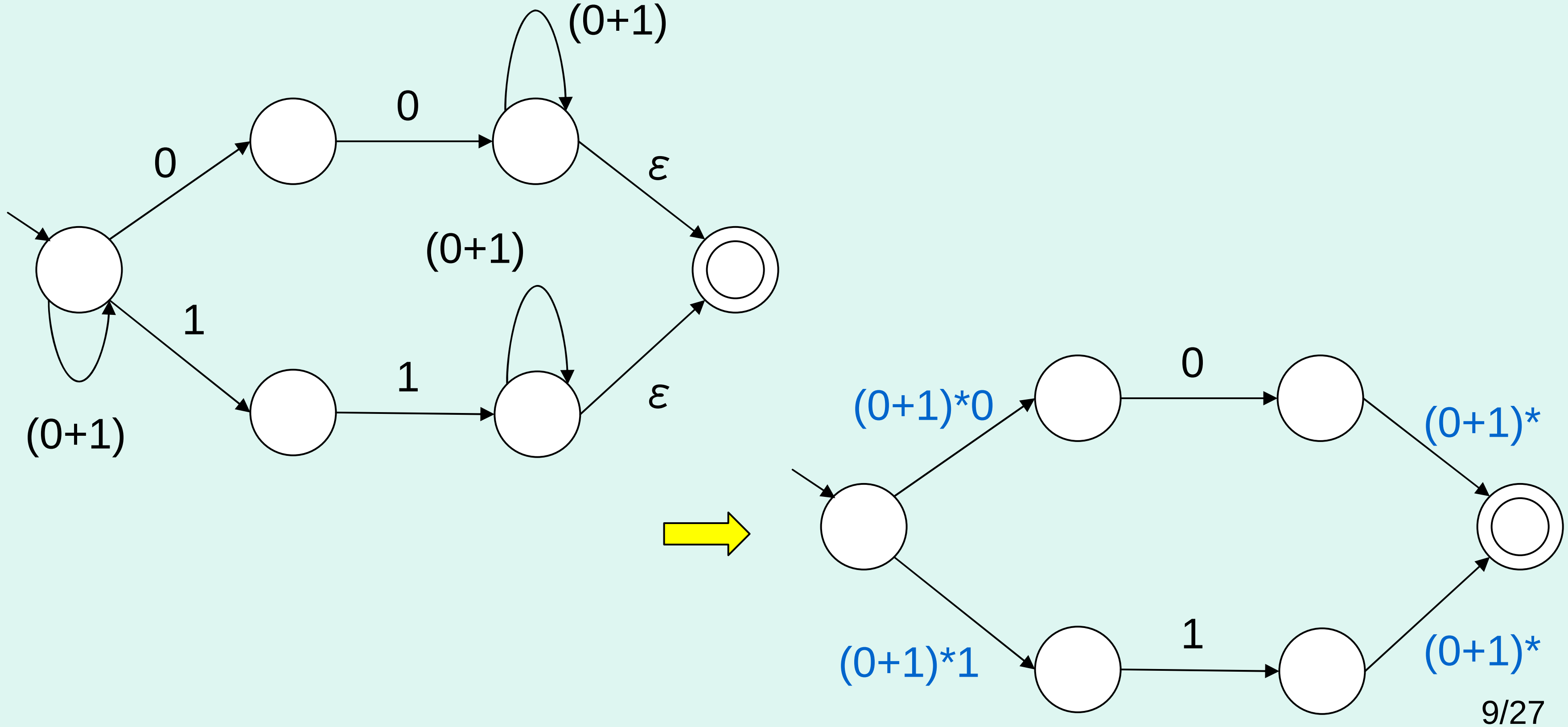
[T2] ループの除去



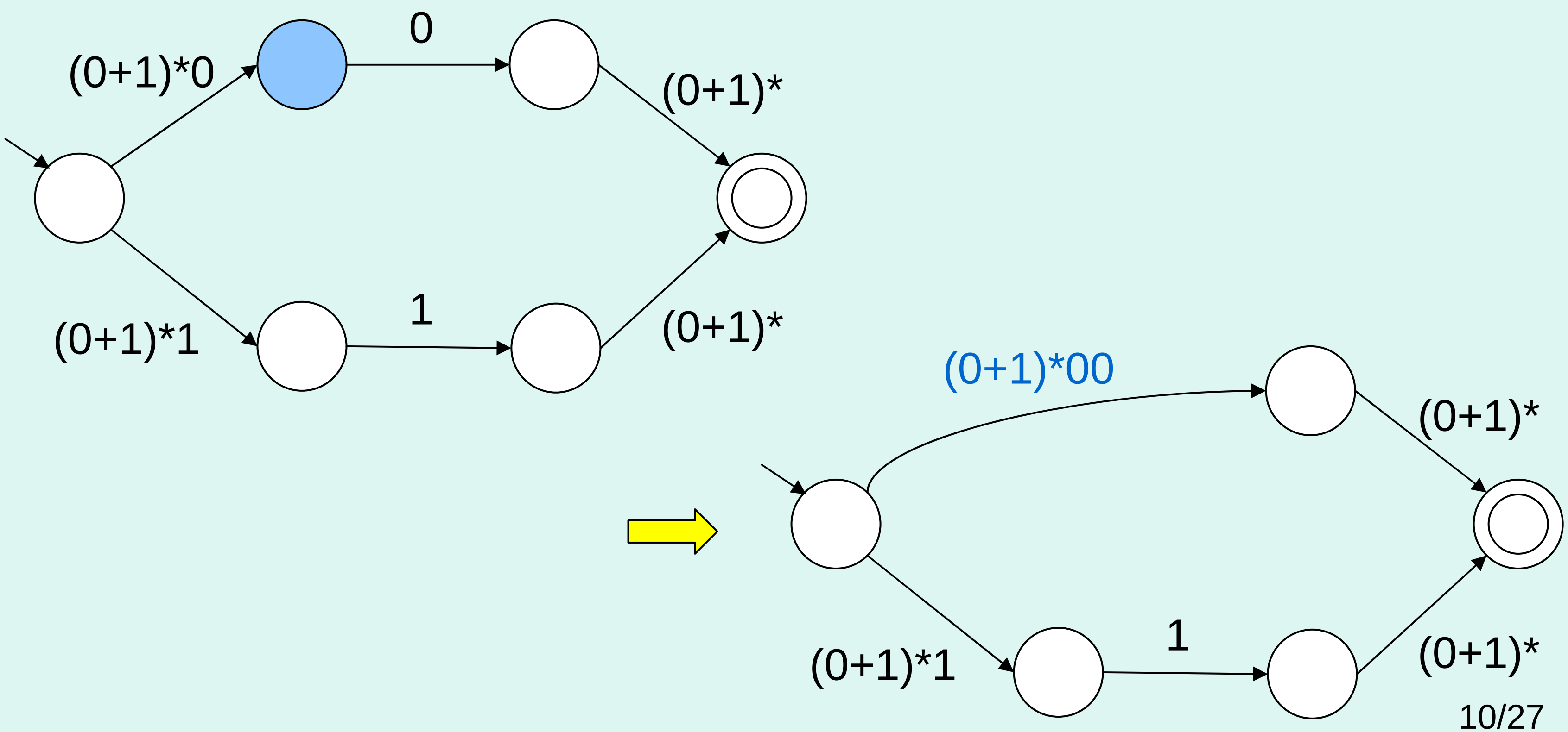
[Problem 1] Let A be an ε-NFA $(\{q_0, q_1, q_2, q_3, q_4\}, \{0,1\}, \delta, q_0, \{q_2, q_4\})$ given in Fig. 1. Give a regular expression that generates $L(A)$.

[T2] Remove loops.



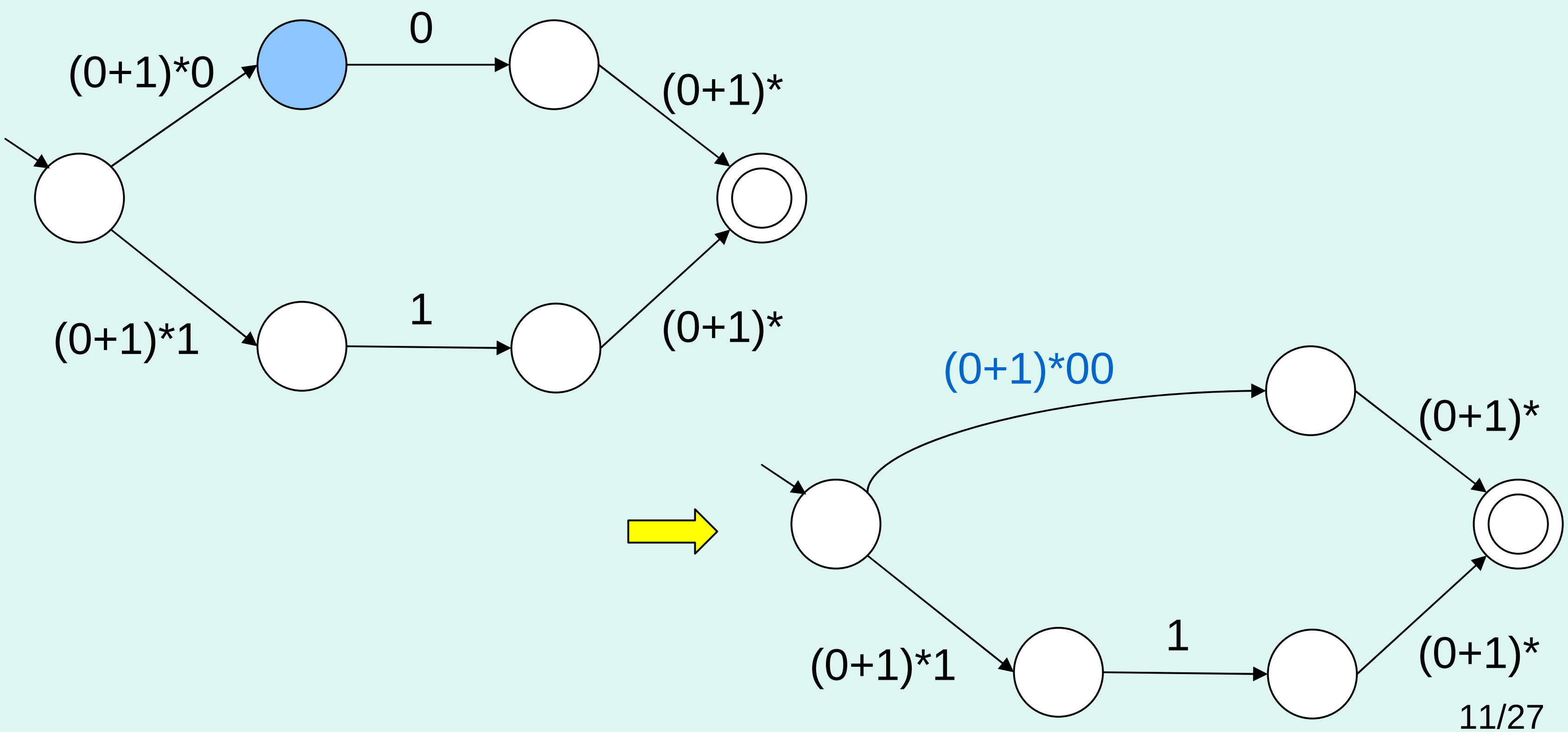
［問題1］ 図1で与えられるε-NFA $A=(\{q_0,q_1,q_2,q_3,q_4\},$ $\{0,1\},\ \delta,\ q_0,\ \{q_2,\ q_4\})$が受理する言語と同じ言語を受理する正則表現を構成せよ。

[T3] (初期状態、受理状態を除く)頂点の除去



[Problem 1] Let A be an ε-NFA $(\{q_0, q_1, q_2, q_3, q_4\}, \{0,1\}, \delta, q_0, \{q_2, q_4\})$ given in Fig. 1. Give a regular expression that generates $L(A)$.

[T3] Remove a node except initial/accepting states.



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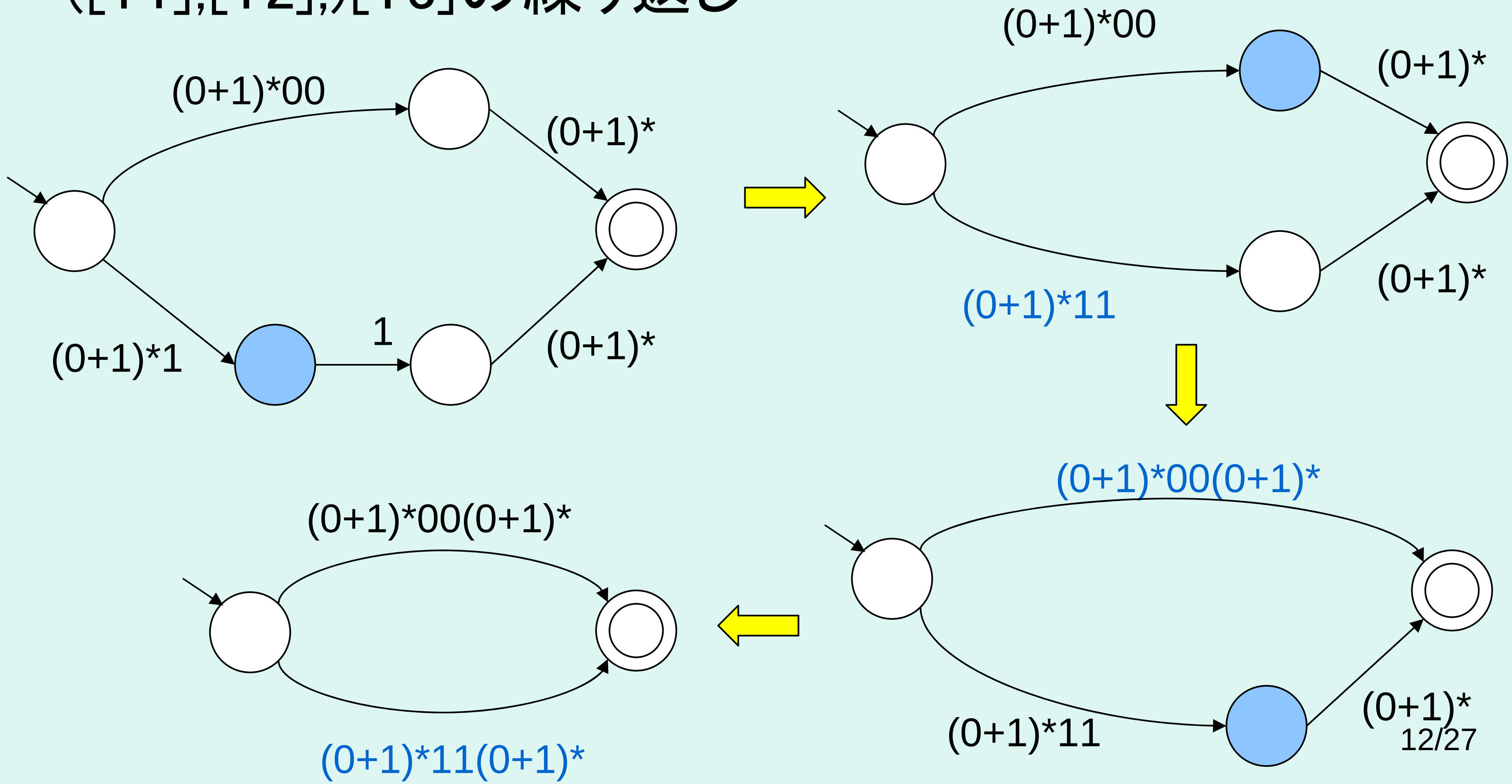
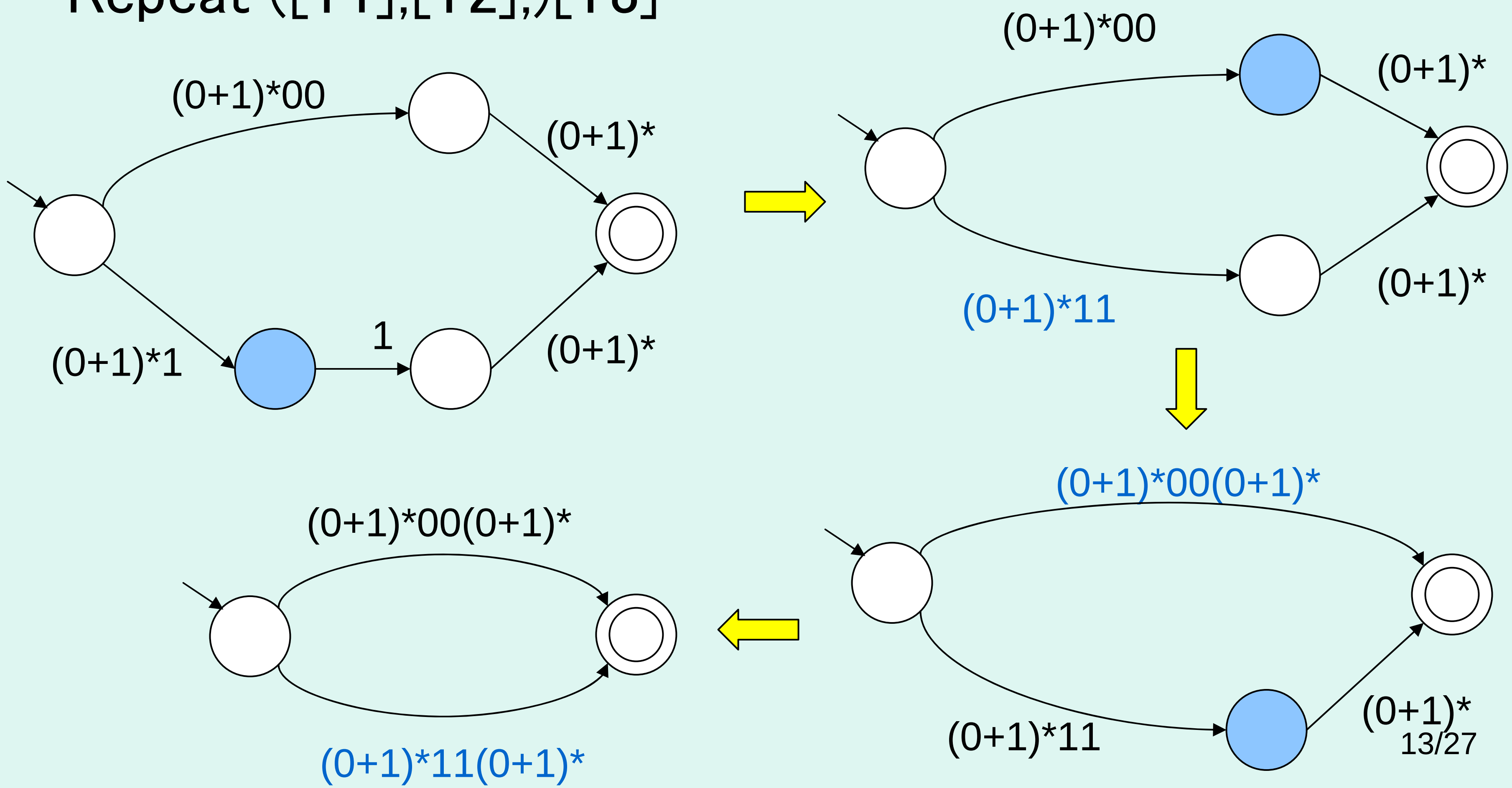
([T1],[T2],)[T3]の繰り返し…



[Problem 1] Let A be an ε-NFA $(\{q_0, q_1, q_2, q_3, q_4\}, \{0,1\}, \delta, q_0, \{q_2, q_4\})$ given in Fig. 1. Give a regular expression that generates $L(A)$.

Repeat ([T1],[T2],)[T3]···



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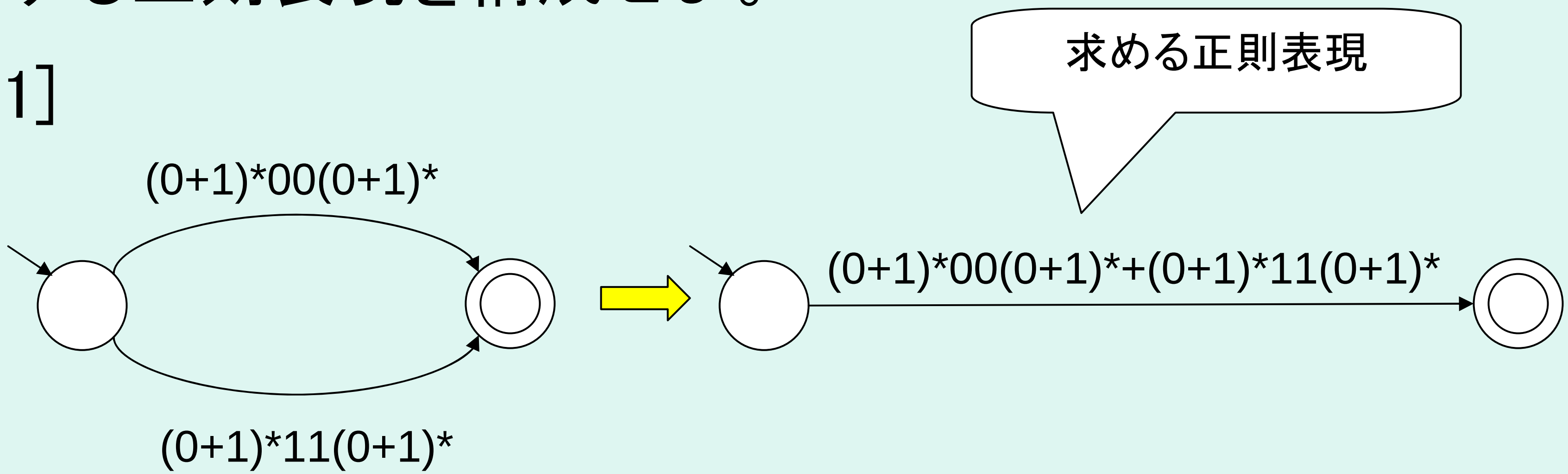
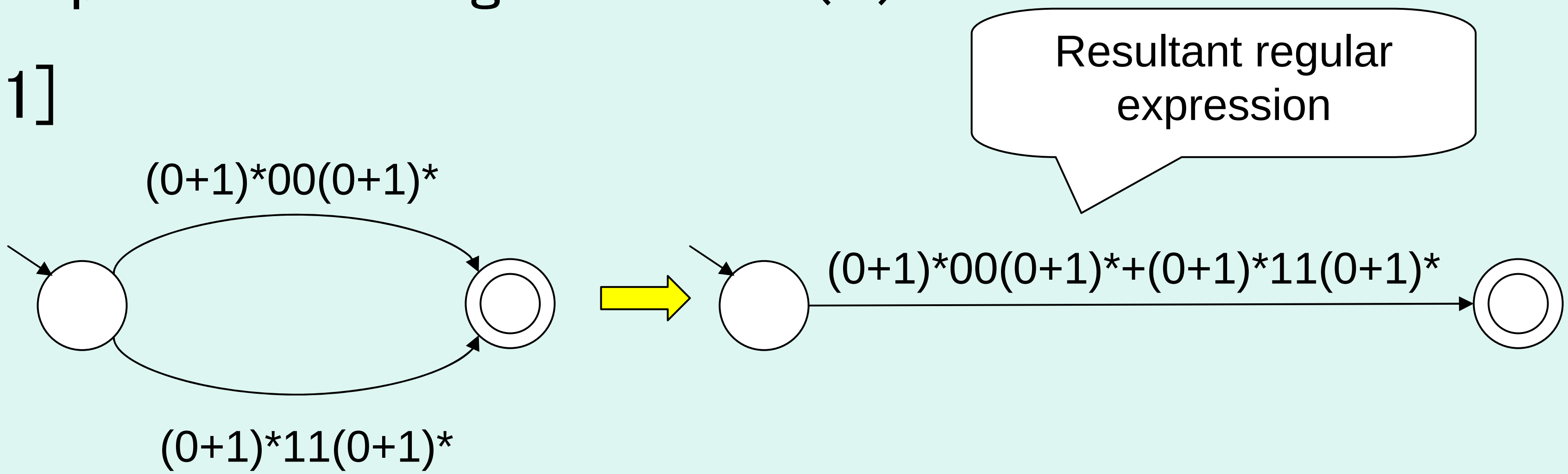
[T1]



[Problem 1] Let A be an ε-NFA $(\{q_0,q_1,q_2,q_3,q_4\}, \{0,1\}, \delta, q_0, \{q_2, q_4\})$ given in Fig. 1. Give a regular expression that generates $L(A)$.

[T1]



[問題2] $\Sigma=\{a,b\}$上の言語Lを次のように定める:

$L = \{ w \mid w \in \Sigma^*,\ w$ 中のaの個数とbの個数は等しい$\}$

このとき L は正則言語でないことを証明せよ。

[鳩の巣原理に基づいた証明]

[反復補題に基づいた証明]

[Problem 2] Let L be a language over $\Sigma = \{a, b\}$ defined as follows:

$L = \{ w \mid w \in \Sigma^*,\ w$ consists of the same number of a and b.$\}$

Prove that L is not a regular language.

[Proof based on the pigeon hole principle]

[Proof based on the pumping lemma]

[問題2] $\Sigma=\{a,b\}$上の言語Lを次のように定める:

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このとき L は正則言語でないことを証明せよ。

[鳩の巣原理に基づいた証明]

L が正則言語であると仮定すると、Lを受理するオートマトンAが存在する。Aの状態数をnとする。ここで m を $m \geqq n$ を満たす任意の数とする。

すると鳩ノ巣原理より、A が a^i を読み込んだときも、a^j を読み込んだときも、同じ状態qとなる整数i,j $(0<i<j \leqq m)$ が存在する。

したがって A に入力文字列として

$$w = a^m b^m$$

$$w' = a^{m-(j-i)} b^m$$

と与えると、Aはどちらの文字列に対しても同じ動作をする。

しかし $w \in L$ かつ $w' \notin L$ なので、矛盾。よってLは正則ではない。

[Problem 2] Let L be a language defined as follows:

$L = \{ w \mid w \in \Sigma^*$, w consists of the same number of a and b.$\}$

Prove that L is not a regular language.

[Proof based on the pigeon hole principle]

Suppose that L is regular. Then there is a DFA A that accepts L.

Let n be the number of states in A, and m an integer with $m \geqq n$.

Then, by the pigeon hole principle, there are two distinct integers i and j ($0 < i < j \leqq m$) such that A becomes the same state q after reading a^i and a^j.

Hence, when we give two distinct inputs

$w = a^m b^m$

$w' = a^{m-(j-i)} b^m$

to A, A has to transit to the same state. However, we have $w \in L$ and $w' \notin L$, which is a contradiction. Hence L is not regular.

[問題2] $\Sigma=\{a,b\}$上の言語Lを次のように定める:

$L = \{ w \mid w \in \Sigma^*, w$ 中のaの個数とbの個数は等しい$\}$

このとき L は正則言語でないことを証明せよ。

[反復補題を用いた証明]

Lが正則言語であると仮定すると、反復補題より以下を満たす定数nが存在する: $|w| \geqq n$, $w \in L$ を満たす任意の文字列は以下の文字列x, y, zに分解できる。(1) $y \neq \varepsilon$ (2) $|xy| \leqq n$ (3) $xy^kz \in L (k \geqq 0)$。

$w=a^nb^n$を考える。すると $w \in L$ より、反復補題が成立し、上記の x, y は $x=a^i$, $y=a^j$ と書ける。したがって $k=0$ とおけば、$a^ib^n \in L$, $i<n$ となり、矛盾。したがってLは正則ではない。

[Problem 2] Let L be a language defined as follows:

$L = \{ w \mid w \in \Sigma^*, w$ consists of the same number of a and b.$\}$

Prove that L is not a regular language.

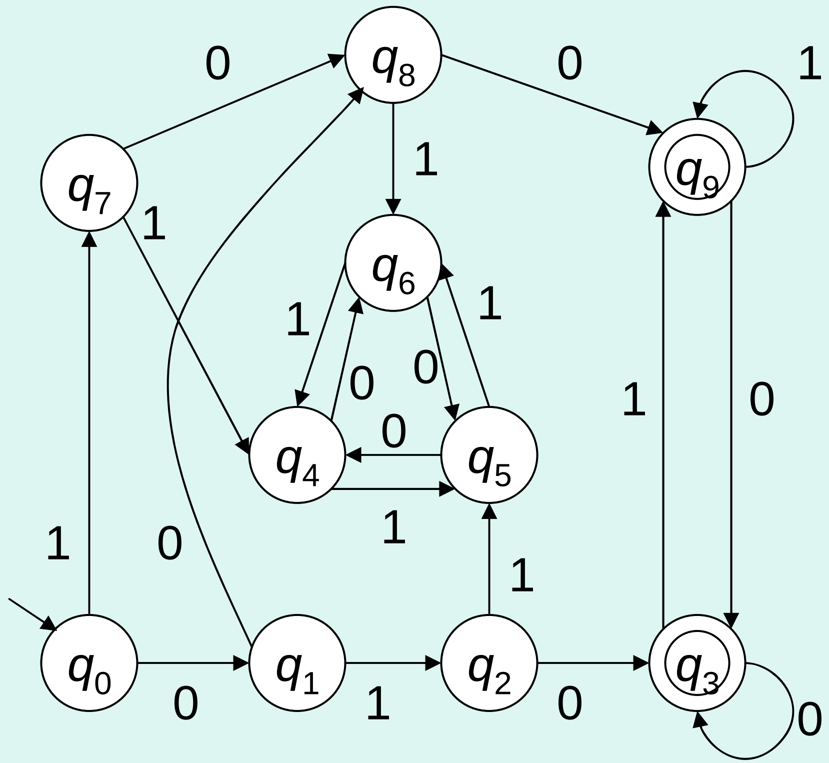
[Proof based on the pumping lemma]

Suppose that L is regular. Then, by the pumping lemma, there is a constant n such that: any $w \in L$ with $|w| \geqq n$ can be decomposed to three strings x, y, z with (1) $y \neq \varepsilon$ (2) $|xy| \leqq n$ (3) $xy^kz \in L (k \geqq 0)$。

Let $w = a^n b^n$. Then, since $w \in L$, we have the pumping lemma, and the strings x and y satisfy $x = a^i$ and $y = a^j$ for some $j > 0$. Hence, letting $k=0$, we have $a^i b^n \in L$ with $i < n$, which is a contradiction. Therefore, L is not regular.

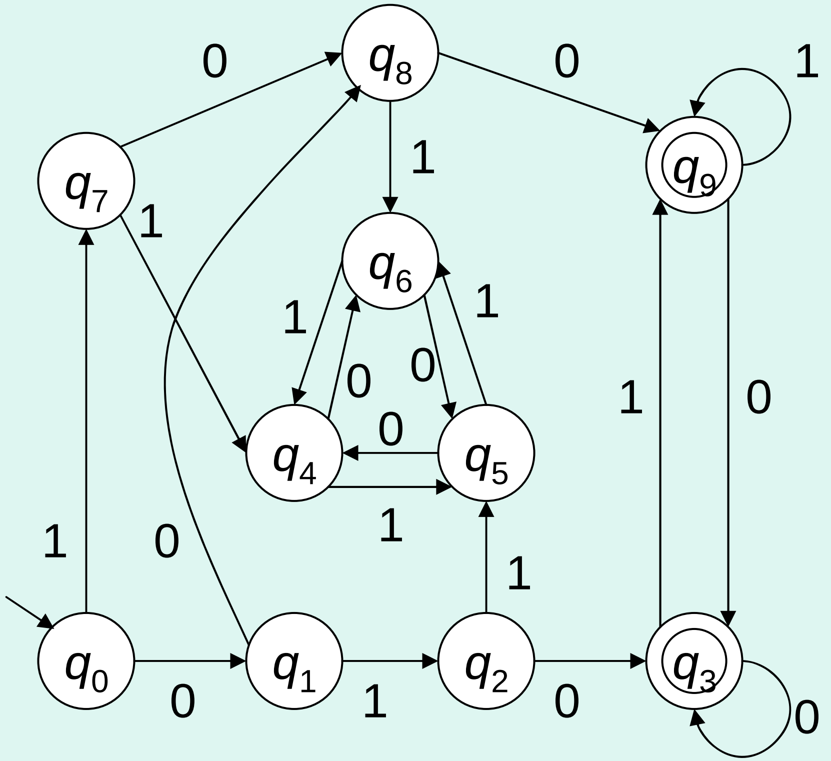
[問題3] 図2で与えられる DFA $A_3 = (\{q_0, q_1, \cdots, q_9\}, \{0,1\}, \delta, q_0, \{q_3, q_9\})$ の状態数を最小化せよ。A_3はどんな言語を受理するか。

[図2]



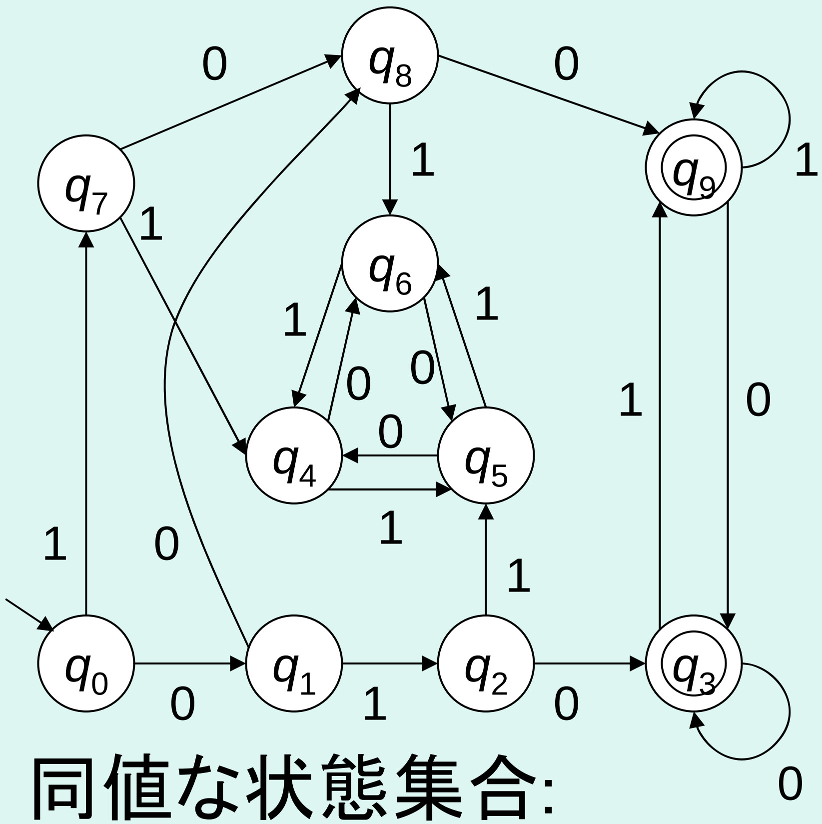
[Problem 3] Minimize the DFA A_3=($\{q_0,\ q_1,\ \cdots,\ q_9\}$, {0,1}, δ, q_0, $\{q_3,\ q_9\}$) in Fig. 2. Also, describe the language accepted by A_3.

[Fig. 2]



[問題3] 図2で与えられる DFA A_3=({q_0, q_1, ･･･, q_9}, {0,1}, δ, q_0, {q_3, q_9}) の状態数を最小化せよ。A_3はどんな言語を受理するか。

[図2]



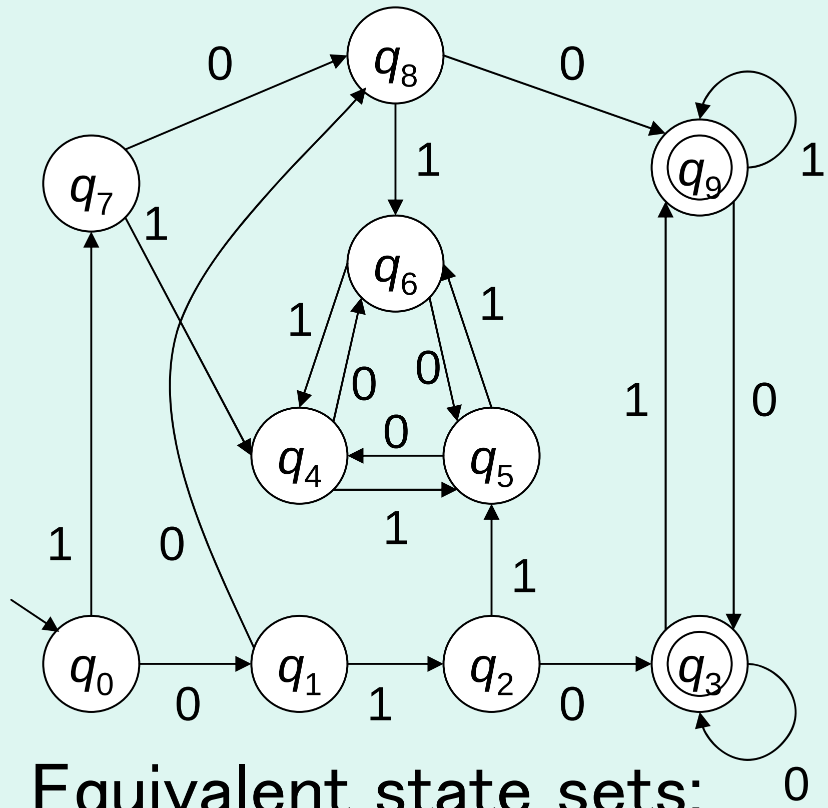
	q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9
q_0	X	X	X	X	X	X	X	X	X	X
q_1	2	X	X	X	X	X	X	X	X	X
q_2	1	1	X	X	X	X	X	X	X	X
q_3	0	0	0	X	X	X	X	X	X	X
q_4	3	2	1	0	X	X	X	X	X	X
q_5	3	2	1	0		X	X	X	X	X
q_6	3	2	1	0			X	X	X	X
q_7	2	2	1	0	2	2	2	X	X	X
q_8	1	1		0	1	1	1	1	X	X
q_9	0	0	0		0	0	0	0	0	X

同値な状態集合:

{q_2,q_8},{q_3,q_9},{q_4,q_5,q_6}

[Problem 3] Minimize the DFA A_3=({q_0, q_1, ⋯, q_9}, {0,1}, δ, q_0, {q_3, q_9}) in Fig. 2. Also, describe the language accepted by A_3.

[Fig. 2]



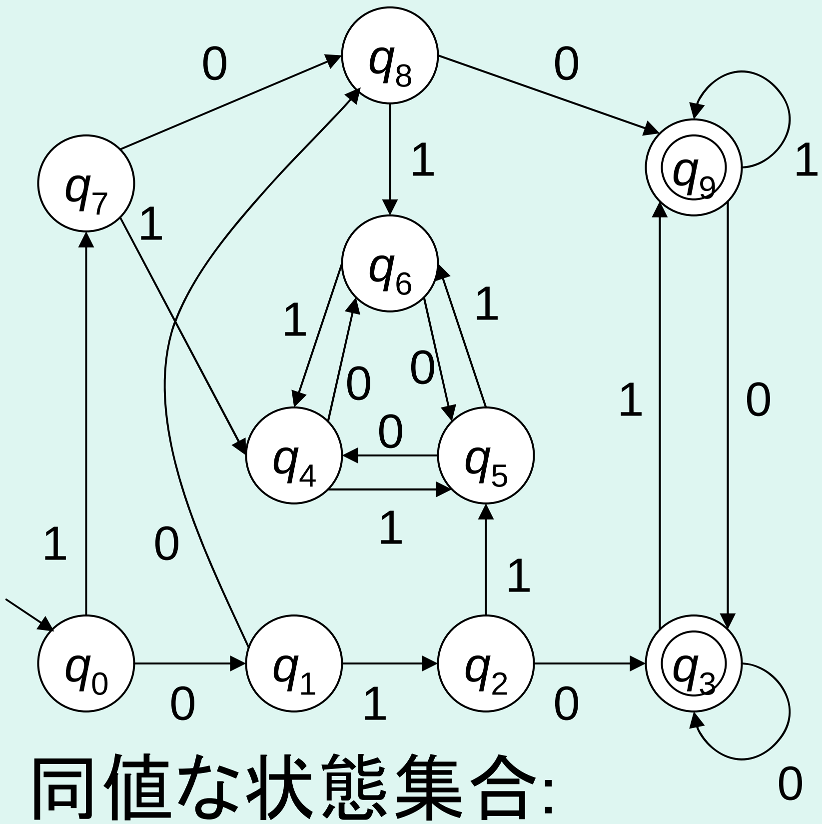
	q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9
q_0	X	X	X	X	X	X	X	X	X	X
q_1	2	X	X	X	X	X	X	X	X	X
q_2	1	1	X	X	X	X	X	X	X	X
q_3	0	0	0	X	X	X	X	X	X	X
q_4	3	2	1	0	X	X	X	X	X	X
q_5	3	2	1	0		X	X	X	X	X
q_6	3	2	1	0			X	X	X	X
q_7	2	2	1	0	2	2	2	X	X	X
q_8	1	1		0	1	1	1	1	X	X
q_9	0	0	0		0	0	0	0	0	X

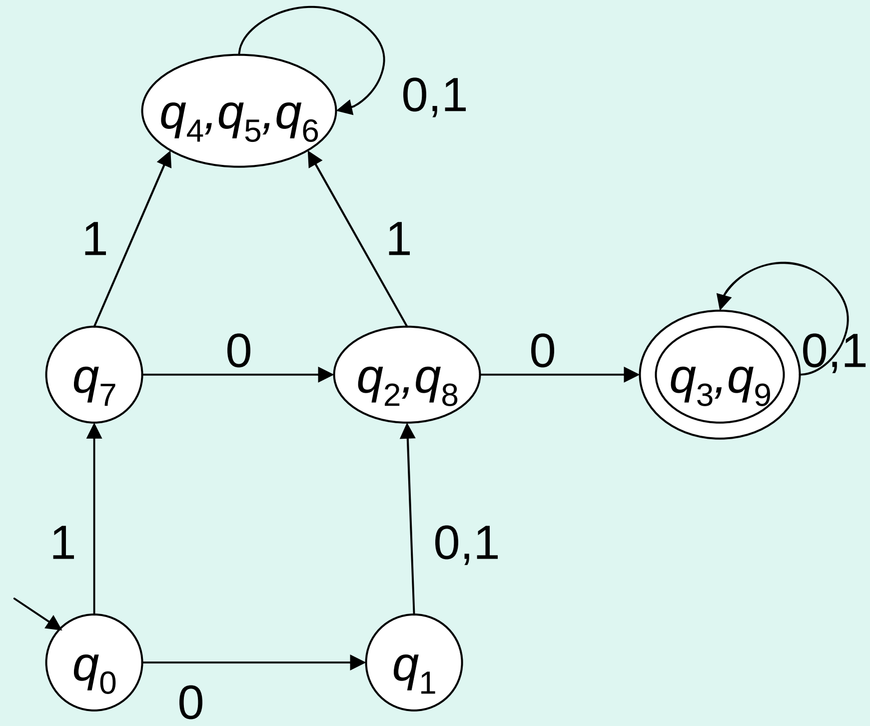
Equivalent state sets:

{q_2,q_8},{q_3,q_9},{q_4,q_5,q_6}

[問題3] 図2で与えられる DFA A_3=({q_0, q_1, ⋯, q_9}, {0,1}, δ, q_0, {q_3, q_9}) の状態数を最小化せよ。A_3はどんな言語を受理するか。

[図2]





同値な状態集合:

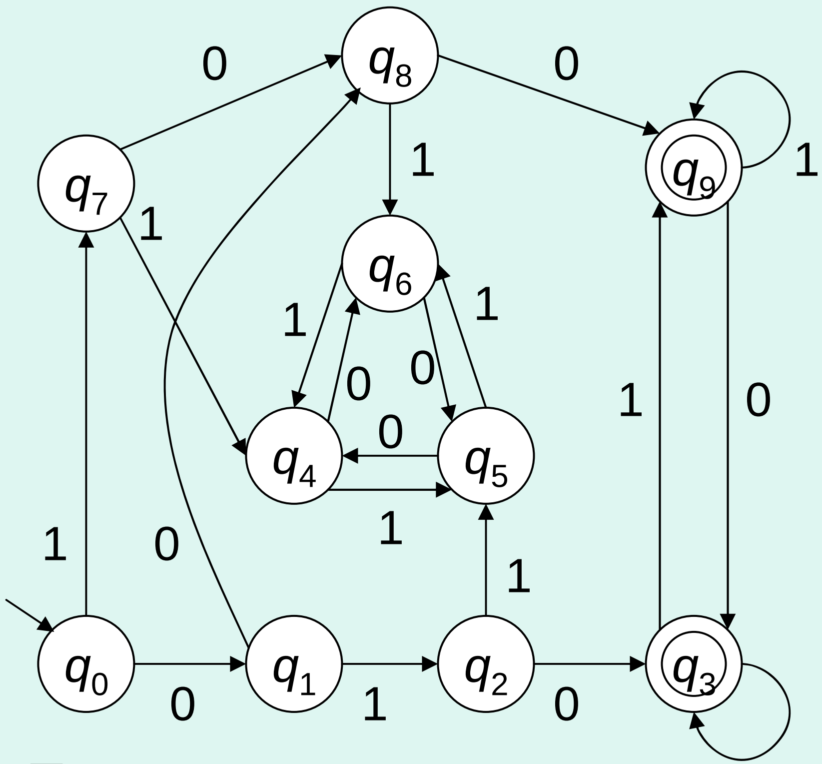
{q_2,q_8},{q_3,q_9},{q_4,q_5,q_6}

上記の最小化されたDFAより、

$L(A_3)$={ w | w は100,000,010のいずれかで始まる文字列}

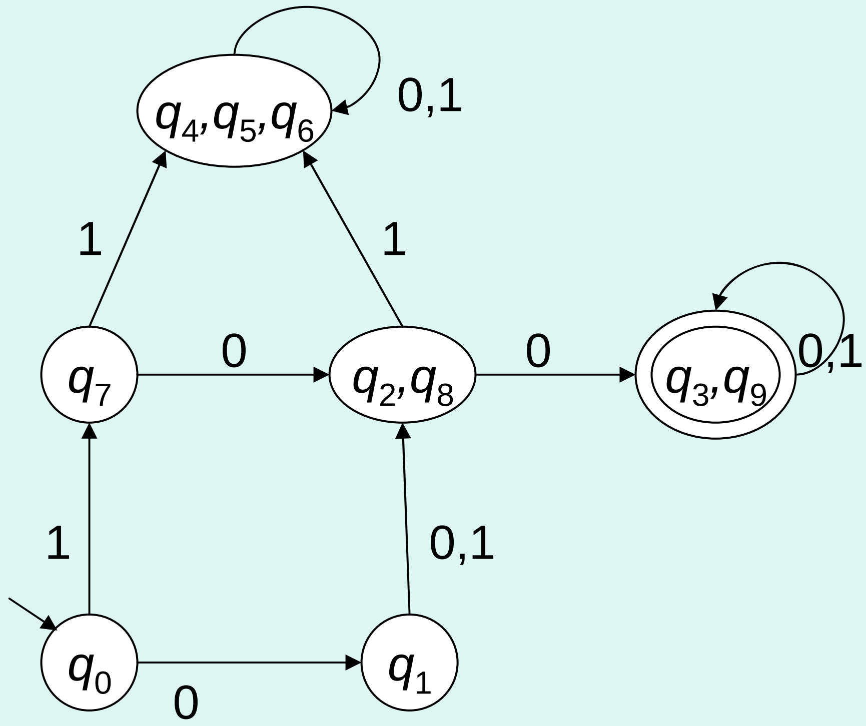
[Problem 3] Minimize the DFA A_3=($\{q_0, q_1, \cdots, q_9\}$, {0,1}, δ, q_0, $\{q_3, q_9\}$) in Fig. 2. Also, describe the language accepted by A_3.

[Fig. 2]



Equivalent state sets:

$\{q_2,q_8\},\{q_3,q_9\},\{q_4,q_5,q_6\}$



By above minimized DFA,

$L(A_3)$={ w | w starts with 100, 000, or 010}