レポート(4)補足
Supplementary Explanation of the 4th report

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オーダー記法の補足
Supplementary Exp. for Order Notation

[定義(Definition)]

自然数上の関数 f, g に対して、

For two functions f and g on natural numbers, if

$$\exists c, d > 0, \forall n (> 0) [f(n) \leq cg(n) + d]$$

となるとき, f は<u>オーダーgである</u>といい, $f = O(g)$ と記述する.

then we say f is <u>in the order of g</u> and denote it by $f = O(g)$.

オーダー記法の補足
Supplementary Exp. for Order Notation

定義に従って$f(x)=O(g(x))$を示すには、定義におけるc,dを明示して議論する必要がある。

If you show $f(x)=O(g(x))$ according to its definition, you have to specify the constants c and d.

[Example] $f(x)= 5x^3+2x+1$ なら($f(x)>x^3$だけど) $f(x) = O(x^3)$

[Proof] c=7, d=1 とおくと、任意の正整数 x>0 に対して
(letting c=13 and d=0, for any positive integer x>0,)

$$f(x) = 5x^3 + 2x + 1 \leq 5x^3 + 2x^3 + 1 = 7x^3 + 1$$

オーダー記法の補足
Supplementary Exp. for Order Notation

$$f(n) = O(g(n)) \Leftrightarrow \exists c, d > 0, \forall n(> 0)[f(n) \le cg(n) + d]$$

定義に従って$f(x) \neq O(g(x))$を示すには次を示す必要がある。

If you show $f(x) \neq O(g(x))$ according to its definition, you have to show the following

$$f(n) \neq O(g(n)) \Leftrightarrow \forall c, d > 0, \exists n(> 0)[f(n) > cg(n) + d]$$

オーダー記法の補足
Supplementary Exp. for Order Notation

定義に従って$f(x) \neq O(g(x))$を示すには次を示す必要がある。

If you show $f(x) \neq O(g(x))$ according to its definition, you have to show the following

$$f(n) \neq O(g(n)) \Leftrightarrow \forall c, d > 0, \exists n (> 0) [f(n) > cg(n) + d]$$

[Example] $f(x) = x^2/100$ なら $f(x) \neq O(x)$

[Proof] どんな定数 c, d に対しても、$x > 100c + 100d$ を満たすなら、
(for any positive constants c and d, letting $x > 100c + 100d$,)

$$f(x) = \frac{x^2}{100} > \frac{(100c + 100d)x}{100} = (c + d)x > cx + d$$

おまけ(全称記号∀と存在記号∃)
More (Universal symbol ∀ & Existential symbol ∃)

1. 指定された順番によって意味が違う
   The ordering of the symbols are crucial.

[Example]

F: Set of women, *M*: Set of men, $P(x,y)$= "*x* shakes *y*'s hand at the party."

A: $\exists x \in M\ \forall y \in F\ [P(x,y)]$ … *y* can depend on *x*, but no meaning

B: $\forall y \in F\ \exists x \in M\ [P(x,y)]$ … *x* can depend on *y*, which differs from *A*!

A is true, and so is *B*.

A is not true, but *B* is true.

- *A* implies *B*, but *B* does not.
- *B* properly contains *A*.

おまけ(全称記号∀と存在記号∃)
More (Universal symbol ∀ & Existential symbol ∃)

2. 否定の作り方

How to make negation

1. The ordering is not changed.
2. ∃ and ∀ are exchanged.

[Example]

$$\neg(\exists x \forall y \exists z [R(x, y, z)]) = \forall x \exists y \forall z [\neg R(x, y, z)]$$

A: $\exists x \in M \ \forall y \in F \ [P(x,y)]$

$\neg A$: $\forall x \in M \ \exists y \in F \ [\neg P(x,y)]$

B: $\forall y \in F \ \exists x \in M \ [P(x,y)]$

$\neg B$: $\exists y \in F \ \forall x \in M \ [\neg P(x,y)]$