

# I216 Computational Complexity and Discrete Mathematics Report (2)

2011, Term 2-1

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**Propose(出題):** October 17 (Mon)

**Deadline(提出期限):** October 27 (Thu), 9:20

**Note(注意):** Do not forget to handwrite your name, student ID, problems, and answers on your report.  
(レポートには氏名, 学生番号, 問題, 解答を, すべて手書きで書くこと.)

Answer one of the following three problems. (以下の問題から1問選んで答えよ.)

**Problem 1 (5 points):** Determine if each of the following equations is correct or wrong. If it is correct, prove it. If it is wrong, disprove it. You can use l'Hospital's rule if you need it. (以下の式は正しいか. 正しいければ証明し, 間違っていれば反証せよ. 必要ならロピタルの定理を使ってもよい.)

1.  $5n^2 + 3n + 8 = O(2n^2 + 1)$
2.  $3n^3 + 4n^2 = O(n^2)$
3.  $n^5 = O(1.5^n)$
4.  $O(n^2) = O(n^3)$

**Problem 2 (5 points):** For any given string  $s$ , we denote by  $lo(s)$  the index of  $s$  in the pseudo-lexicographical ordering with length preferred. For example, we have  $lo(\epsilon) = 1$ ,  $lo(0) = 2$ , and  $lo(1) = 3$ . We also denote by  $n < \infty$  when a number  $n$  is finite. Now, declare if each of the followings is true or false. If it is false, show a counterexample. In the followings,  $s$  denotes a string and each of  $n, x$ , and  $y$  denotes an integer. (文字列  $s$  に対して, 長さ優先辞書式順序における  $s$  の出現順序を  $lo(s)$  と書くことにする. 例えば  $lo(\epsilon) = 1$ ,  $lo(0) = 2$ ,  $lo(1) = 3$  である. またある数  $n$  が有限であることを  $n < \infty$  と書くことにする. このとき以下の記述が正しいか誤りかを判定せよ. 誤りである場合は反例を示せ. ただし以下の記述中,  $s$  は文字列,  $n, x, y$  は整数である.)

$$\forall x \exists y [x + y = 0] \tag{1}$$

$$\exists x \forall y [x + y = 0] \tag{2}$$

$$\forall x \exists n [ |x| < \infty \rightarrow lo(x) < n ] \tag{3}$$

$$\exists n \forall x [ |x| < \infty \rightarrow lo(x) < n ] \tag{4}$$

**Problem 3 (5 points):** It is nonsense that defining the class  $co-P$  because  $co-P = P$ . Prove  $co-P = P$ . (クラス  $co-P = P$  なので, クラス  $co-P$  を定義しても意味がない.  $co-P = P$  を証明せよ.)