

平成26年12月5日~12月7日  
日米先端科学(JAFoS)シンポジウム



# ORIGAMI

## Paper Folding and Its Generalization

@Ori-zuru!

Japan Advanced Institute of Science and Technology  
(JAIST)

School of Information Science  
Ryuhei Uehara

# What's ORIGAMI?

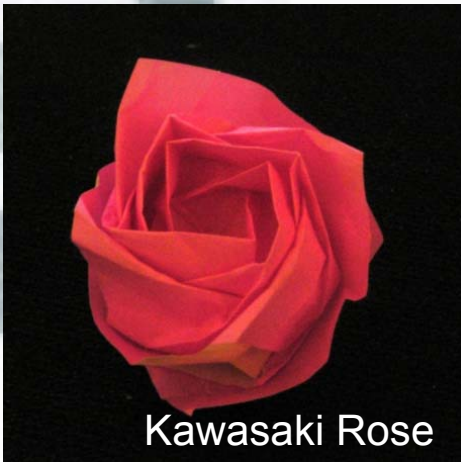
- “Ori” means **folding**, and “kami(=gami)” means **paper**, which may be born in 1500s in Asia?
- As you know, **ORIGAMI** is now available even in English...

Sometimes, ORIGAMI seems to be **wider** than original Japanese... which may be better to understand our work!!



# “ORIGAMI”...?

- Normal Origami
- Difficult Origami
- Impossible Origami



Kawasaki Rose



Maekawa Devil



By Satoshi Kamiya (TV Champion)

# “ORIGAMI”...?

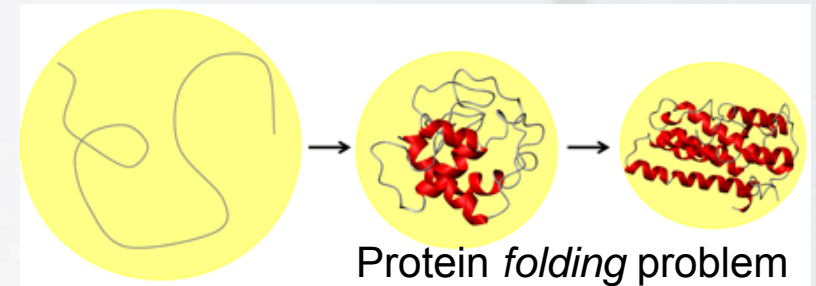
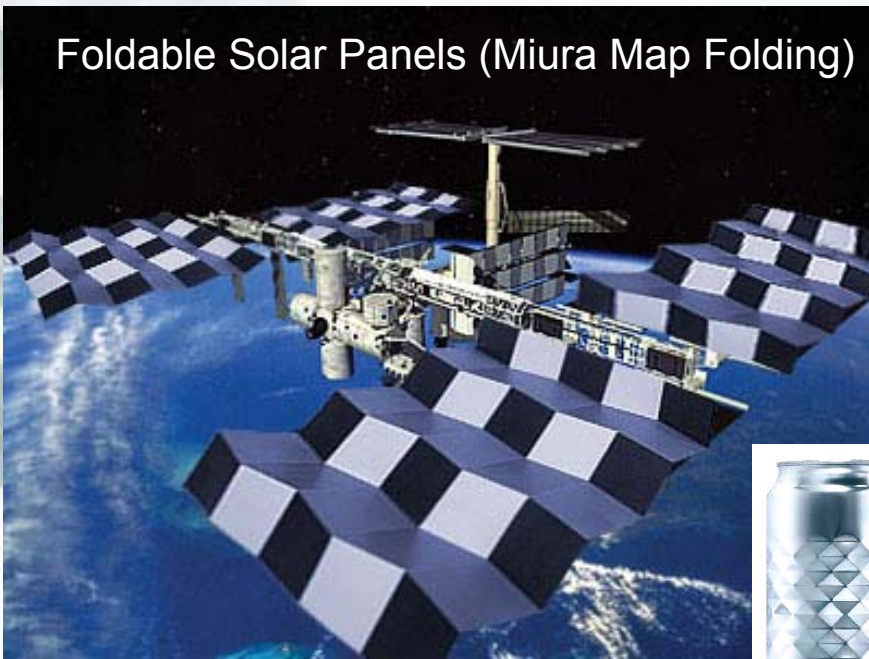
- Normal Origami
- Useful/important Origami

There are many unsolved problems and applications of “folding”



“Origami Science” based on the basic operations of “folding”

Foldable Solar Panels (Miura Map Folding)



# “ORIGAMI”...?

## ➤ Useful/important Origami

There are many unsolved problems and applications of “folding”

**Tomohiro Tachi:**

Working on

“Rigid Origami” architecture

E.g., *Generalization of Mira-ori*

**Thomas Hull:**

Working on

“Mathematical Origami”

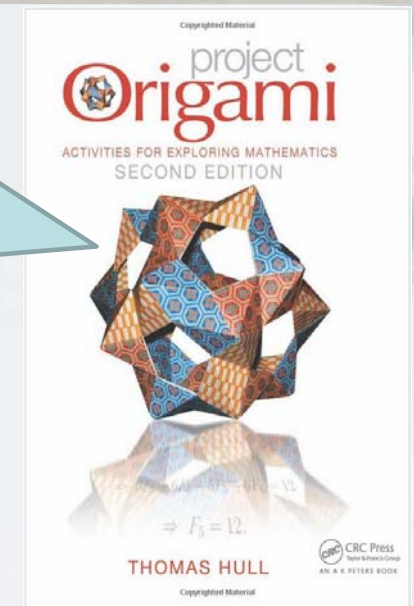
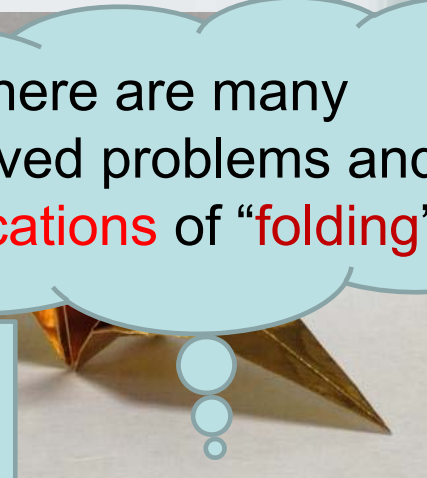
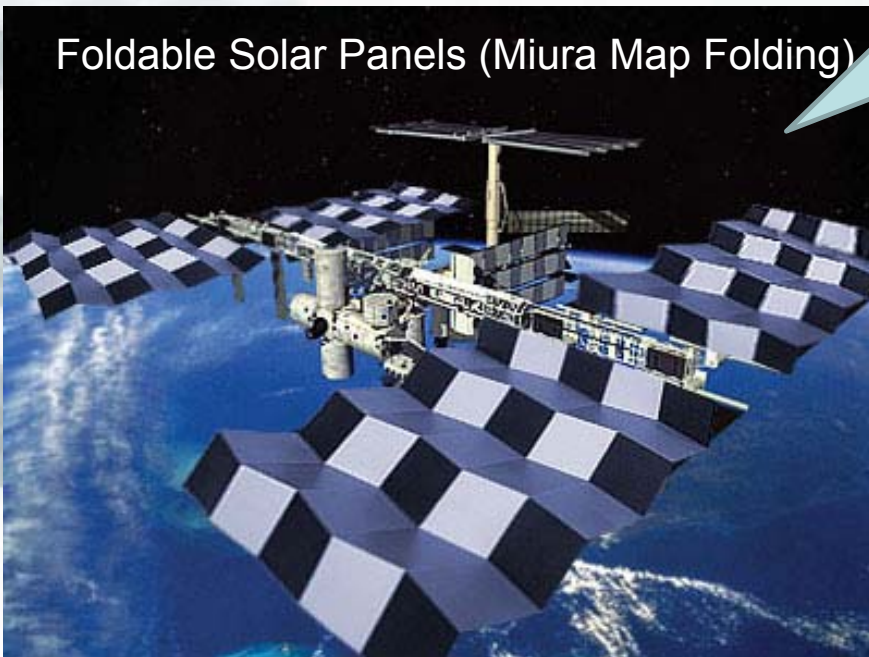
*Modern Origami* from  
Mathematical viewpoint

**Ryuhei Uehara:**

Working on

“Computational Origami”

Foldable Solar Panels (Miura Map Folding)



# Who's Ryuhei?

**Name:** Ryuhei Uehara

**Affiliation:** School of Information Science,

Japan Advanced Institute of Science and Technology (JAIST)

**Title:** Professor, **Director of JAIST Gallery**

JAIST close  
to Kanazawa



You are here



JAIST Gallery



In this gallery, we have **around ten thousands of puzzles** due to NOB Yoshigahara, one of the most popular three puzzle collections (another is in England, and the last one is in USA.)

This gallery itself is designed as a big puzzle, so you will enjoy the nice experience of being in the big puzzle. Of course you can also enjoy some puzzles, including ones in Uehara-lab 😊

This "latin-cross" can fold into 23 different convex shapes in 85 different folding ways!!

Experimental results  
(by computer!!)

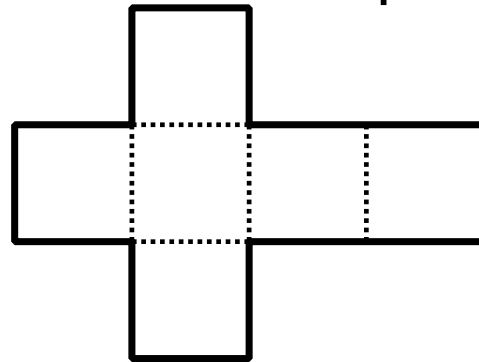
tational Q  
t favor  
ygon  $P$  and polyhedron  $Q$

As... does  $P$  folds into  $Q$  (or vice versa)?

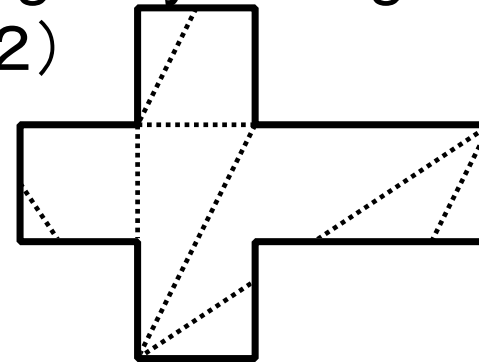
This problem is quite counterintuitive in general...

**Exercise:** What shape do you get by folding...

(1)



(2)



# Background

➤ My most favorite unsettled problem:

Given: polygon  $P$  and polyhedron  $Q$

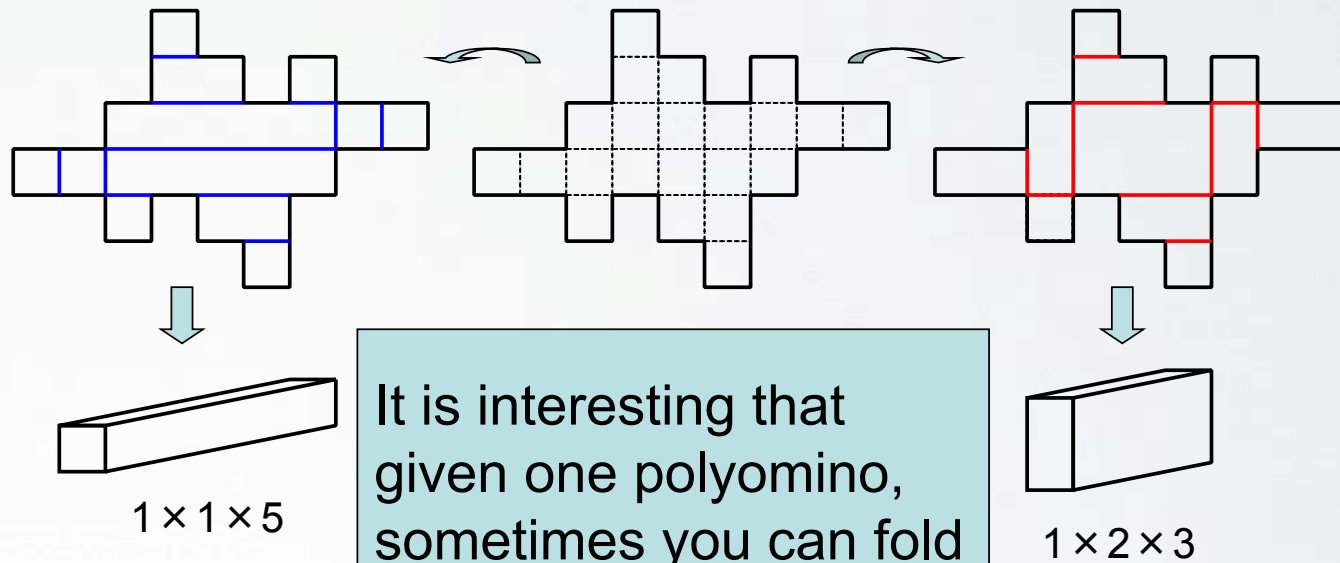
Asked: does  $P$  folds into  $Q$  (or vice versa)?

- We have no idea so far even if  $P$  and  $Q$  are explicitly given. Especially, we cannot pre-determine *where the folding lines (or creases) are placed on  $Q$* ?
- We only have partial results when  $P$  and  $Q$  are very restricted.



# One of my major works in computational origami

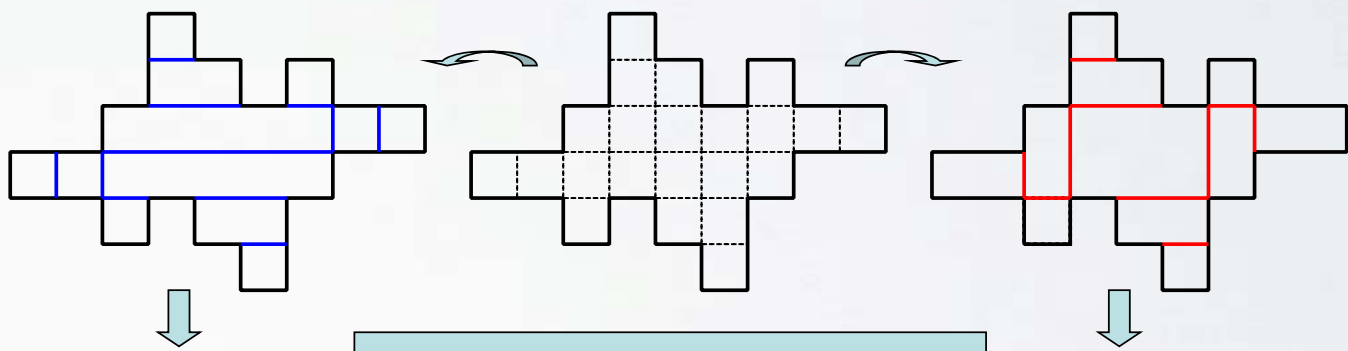
- Polyominoes (set of unit squares) that fold into two or more boxes!



# It is necessary that..

Example:  
 $1 \times 1 + 1 \times 5 + 1 \times 5$   
 $= 1 \times 2 + 2 \times 3 + 1 \times 3$   
 $= 11$  (Area is 22)

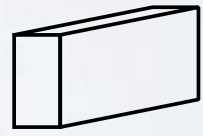
➤ Two boxes have the same surface area!



$$1 \times 1 \times 5$$

$$= a \times b \times c$$

- We fold/(cut) at an edge of **unit squares**
- Surface area:  
 $2(ab + bc + ca)$
- Necessary condition:



$$1 \times 2 \times 3$$

$$= a' \times b' \times c'$$

$$ab + bc + ca = a'b' + b'c' + c'a'$$

It seems to be better to have many combinations...

# Note:

## ➤ Surface areas;

If you try to find for three boxes,

If you try to find for four boxes,

Area	Trios	Area	Trios
22	(1,1,5),(1,2,3)	46	(1,1,11),(1,2,7),(1,3,5)
30	(1,1,7),(1,3,3)	70	(1,1,17),(1,2,11),(1,3,8),(1,5,5)
34	(1,1,8),(1,2,5)	94	(1,1,23),(1,2,15),(1,3,11), (1,5,7),(3,4,5)
38	(1,1,9),(1,3,4)	118	(1,1,29),(1,2,19),(1,3,14), (1,4,11),(1,5,9),(2,5,7)

Using computer programs,

- we can generate all (2263) common developments of two boxes of size  $1 \times 1 \times 5$  and  $1 \times 2 \times 3$  in 5 hours in 2014.
- we could not for area 30 and more...

# Developments of three boxes(!)

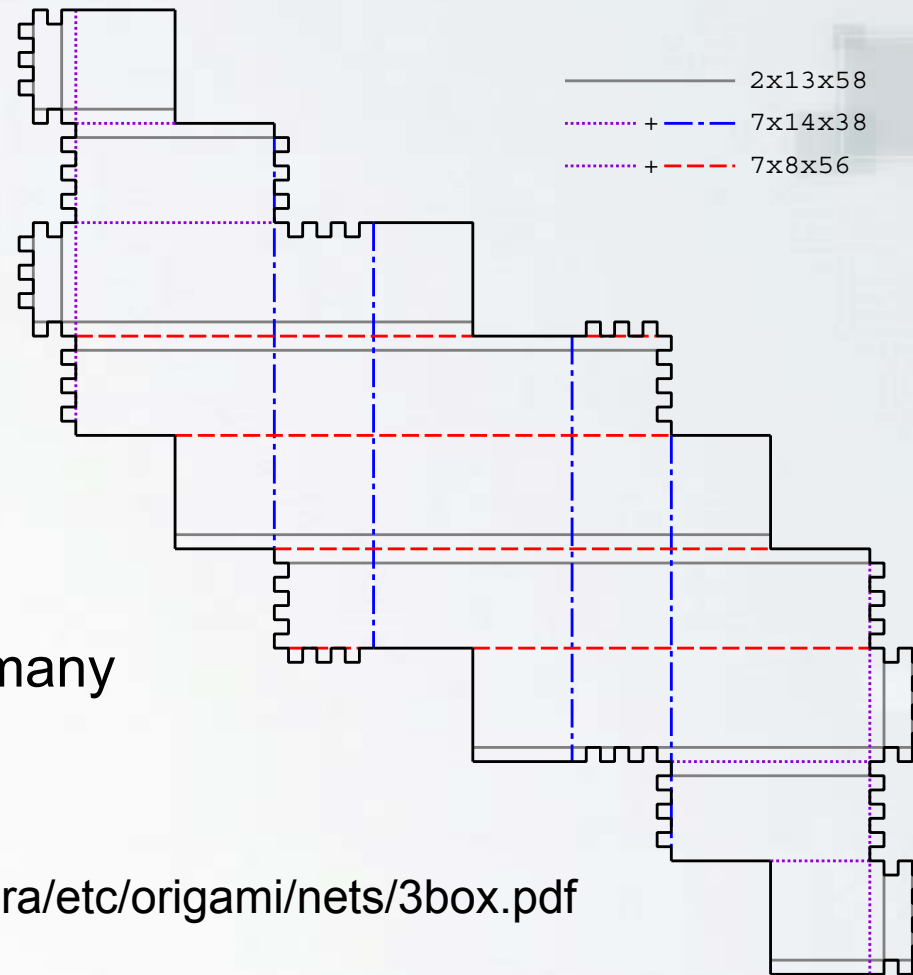
➤ In February 2012,  
our group found:

Without computer 😊

There exists  
a polygon that  
folds to **3 boxes!!**

... although it require many  
unit squares....

I put it at  
<http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf>



# Note:

## ➤ Surface areas;

If you try to find for three boxes,

If you try to find for four boxes,

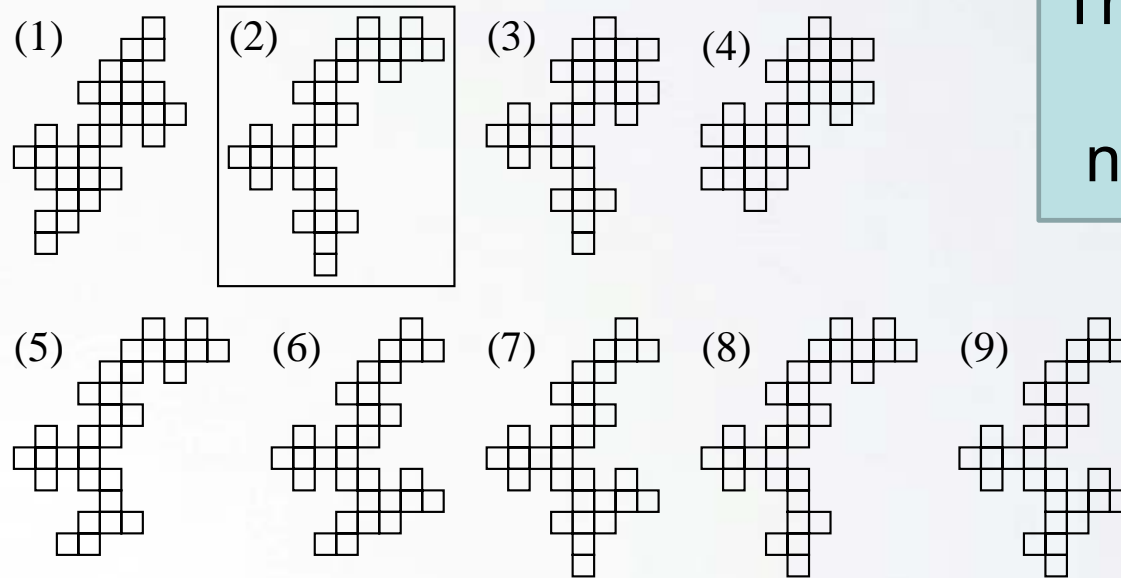
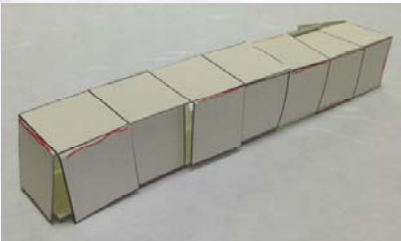
Area	Trios	Area	Trios
22	(1, 1, 5), (1, 2, 3)	46	(1, 1, 11), (1, 2, 7), (1, 3, 5)
30	(1, 1, 7), (1, 3, 3)	70	(1, 1, 17), (1, 2, 11), (1, 3, 8), (1, 5, 5)
34	(1, 1, 8), (1, 2, 5)	94	(1, 1, 23), (1, 2, 15), (1, 3, 11), (1, 5, 7), (3, 4, 5)
38	(1, 1, 9), (1, 3, 4)	118	(1, 1, 29), (1, 2, 19), (1, 3, 14), (1, 4, 11), (1, 5, 9), (2, 5, 7)

Using computer programs,

- we can generate all (2263) common developments of two boxes of size  $1 \times 1 \times 5$  and  $1 \times 2 \times 3$  in 5 hours in 2014.
- we could not for area 30...

# On June, 2014...

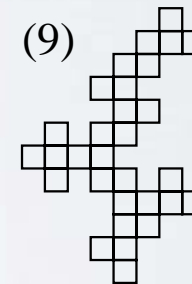
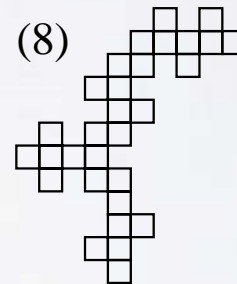
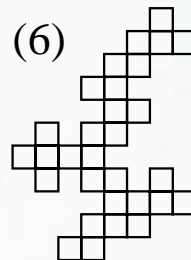
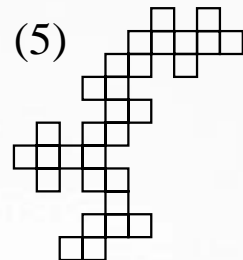
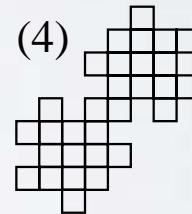
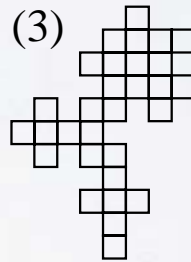
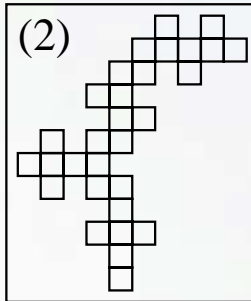
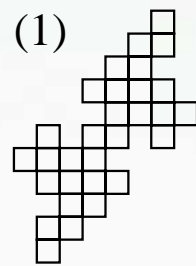
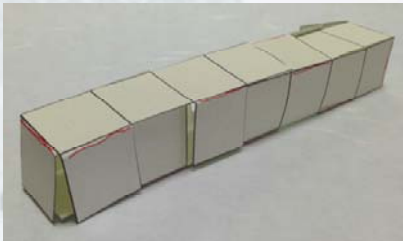
- We completed all common developments of two boxes of size  $1 \times 1 \times 7$  and  $1 \times 3 \times 3$  of area **30**.
- **1080** common developments of these two boxes are found by a **supercomputer** (Cray XC 30) in **three months** 😊



The number **30** is a magic number s.t....

# On June, 2014...

...And 9 of 1080 allow us to fold a cube of size  $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$



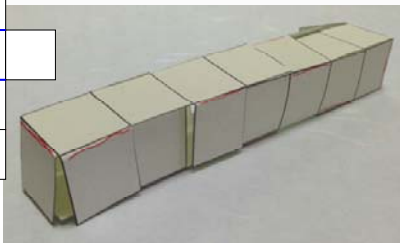
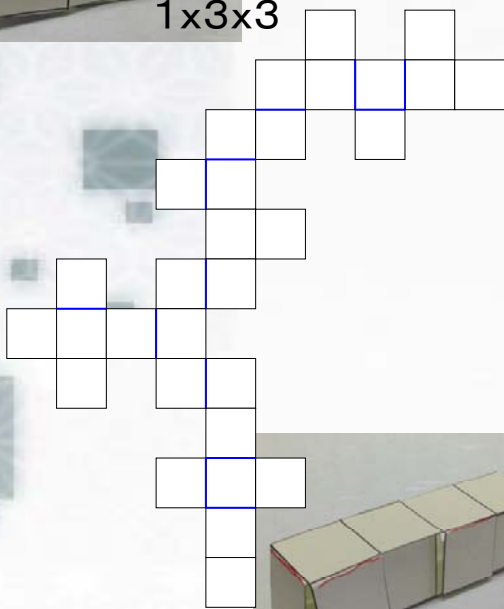
Moreover, (2) has a **special property!!**

# The current nicest polygon...

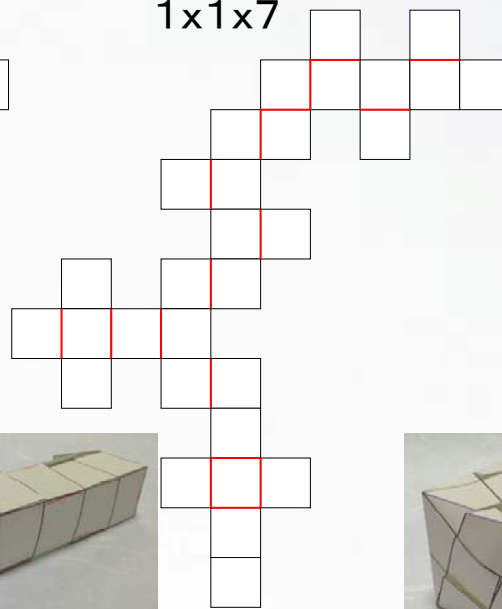
- It admits to fold three different boxes in 4 different folding ways!!



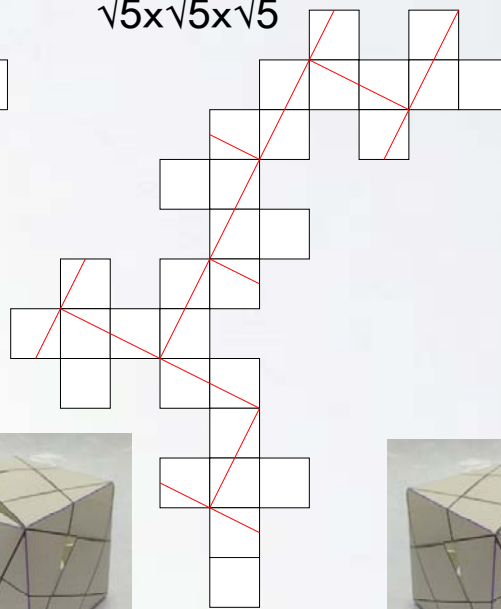
1x3x3



1x1x7



$\sqrt{5} \times \sqrt{5} \times \sqrt{5}$



$\sqrt{5} \times \sqrt{5} \times \sqrt{5}$

