計算幾何学特論：計算折り紙入門

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11月26日（水）
10:30-12:00
13:00-14:30
14:50-16:20

11月27日（木）
10:30-12:00
13:00-14:30
14:50-16:20

今日のトピック

昨日：複数の凸多面体が折れる展開図の研究

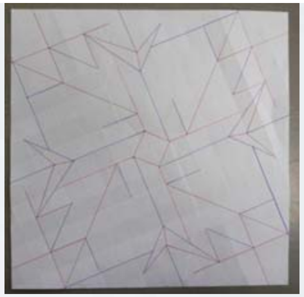
- 展開図と立体のとても悩ましい関係：最大の未解決問題
- 与えられた「展開図」を折って作れる（凸）「立体」

今日：「折り」のアルゴリズムと計算量の関係

- 折り紙の基本操作
- 折り紙のアルゴリズムと計算量
  - 1次元の紙における効率のよい折り方（アルゴリズムと計算量）
  - 1次元の紙における計算不能性（計算の理論）

まだ2次元にすら達してません．．．

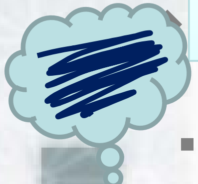
今日の話の背景



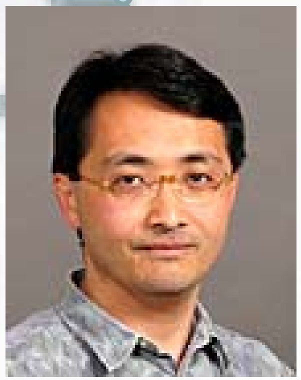
川崎ローズ

- 2008年6月22日、
  第4回「折り紙の科学・数学・教育研究会」にて、、、
  - 川崎敏和氏(数学者だけど川崎ローズの作者として有名)いわく:
  - 「数学者としては、解の存在さえわかれば、あとはどうでもいい」

どうでもいい余談
九州大学の川崎英文先生とは双子です.



- 計算機科学者である上原は…
  - 解の求め方とその
    計算コストが大切!!
    - 良いアルゴリズム
    - 計算量的な困難さ



計算量の理論とアルゴリズム理論

➢理論計算機科学の基礎理論

- 基準となるマシンモデル:

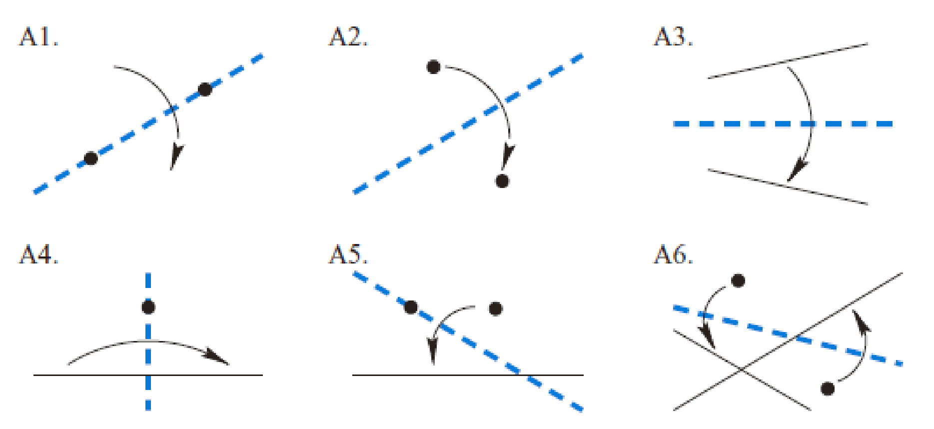
  共通の「基本演算」に関する合意が必要

  - チューリングマシン
  - VRAMモデル

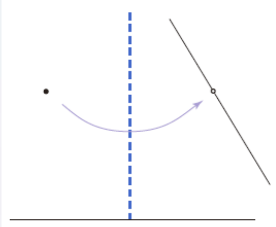
- <u>アルゴリズム</u>
  - 基本演算をどのような手順で組み合わせるか？
- アルゴリズムの<u>計算量</u>
  - 時間計算量:基本演算の回数で効率を測る
  - 領域計算量:計算に必要な記憶領域で効率を測る

“Complexity” of folding…?

- Computation Model:
  - We have “7 axioms” in origami society
  - We have “standard operation set”



Huzita’s six axioms.



Hatori’s axiom A7..

These 7 axioms can solve quartic equations. (More powerful than “ruler and compass” construction)

“Complexity” of folding…?

- We may measure it by…
  1. The number of folding operations
     - Natural analogue of “time complexity”
  2. The number of paper layers
     - Many layers cause problem.
       “Folding paper in half 12 times”
  3. The area you need to fold??…

We may have another measure corresponding to “space complexity”

Folding Paper in Half 12 Times
The story of an impossible challenge solved at the Historical Society office

Alice laughed: "There's no use trying," she said; "one can't believe impossible things." "I daresay you haven't had much practice," said the Queen.

Through the Looking Glass by L. Carroll

BRITNEY'S FOLDING RECORD STILL HOLDS

The long standing challenge was that a *single* piece of paper, no matter the size, cannot be *folded* in half more than 7 or 8 times. Recently, reports have been made that Britney's paper folding record of folding a piece of paper in half 12 times has been broken. These current attempts, though laudable and will eventually be

Photo of the 11th Fold. One More to go.

“Complexity” of folding…?

- 折り紙の計算量的な複雑さを考えるにあたって，妥当なモデルとは？
  1. 最も単純なモデル:1次元等間隔折り紙
     - 長い紙テープ上に，等間隔に折り目をつける/与えられる
  - 拡張の方向は二つ
     - 折り目が等間隔でなくてもよい
        - （斜めも許す？）
     - 2次元への拡張

今は，
まだほぼ
このあたり！

1. The number of operations

- …corresponds to "time complexity"
- My paper:
  - J. Cardinal, E. D. Demaine, M. L. Demaine, S. Imahori, T. Ito, M. Kiyomi, S. Langerman, R. Uehara, and T. Uno: Algorithmic Folding Complexity, *Graphs and Combinatorics*, Vol. 27, pp. 341-351, 2011.

The 7th EATCS/LA Presentation Award!

1. The number of operations

- 1D origami
- Creases are at regular intervals

Algorithmic Folding Complexity:

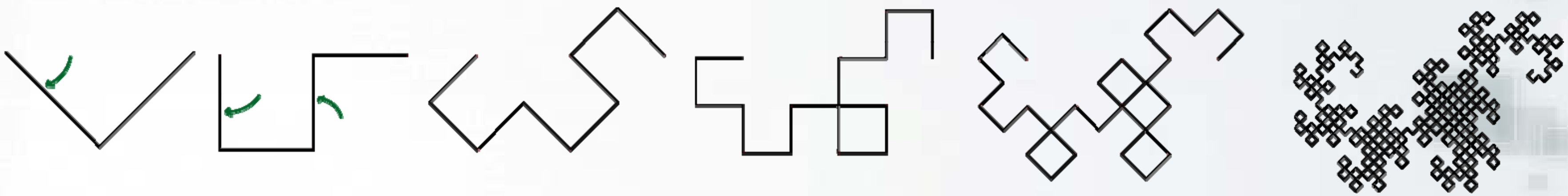
- Input: paper strip of length $n+1$ with M/V assignment of the strip
- Question: How many folding operations you need to make the given creases?
- Rules:
  - Simple folding (flat → flat)
  - All unfolding at once with no cost!
  - Each crease remembers the last direction.

1. The number of operations

➢ *Algorithmic Folding Complexity:*

- Rules:
  - Simple folding (flat → flat)
  - All unfolding (at once with no cost!)
  - Each crease remembers the last direction.
- Two trivial bounds:
  - Lower bound: $\log n$ operations needed to make n creases.
  - Upper bound: any pattern can be made by n operations.

Dragon curves



1. The number of operations

Our 3 log² n /2 algorithm for pleats is not so simple… ;-)

of it. The algorithm consists of three steps:

Step 0. Repeat folding at the middle crease $k-3$ times. After that, we have the patt[ern] "[xxx]" of length 4 on the paper. Then mountain-fold three times and obtain "[MMM]". Next, unfold the paper and obtain the sequence "+MMM+VVV+MMM + VVV+MMM+VVV+· · ·."

Step 1. In order to pile all patterns "VVV" at the same place, repeat folding $k-3$ times at the middle crease in the "MMM" pattern which is the closest one to the center of the paper. (Note that this step requires $k-3$ foldings; since the "MMM" pattern closest to the center of the paper is not on the center exactly. Hence the algorithm first folds $k-4$ times, and the last one folding is done to fix the length to 8.) After that, we have the pattern "[M+VVV+M]" of length 8. Then mountain-fold five times at each place with label "V" [or] "+", and obtain "[MMMMMMMM]." Unfold the paper and obtain the sequence "VM+MMMMMMMM+MVVVVVM+MMMMMMMM+MVVVVVM+M· · ·."

Step 2. Repeat Step 1; precisely, (1) pile all patterns "VVV· · ·V" at the same place by repeatedly folding at the middle crease in the pattern "MMM· · ·M" that is the closest to the center, and (2) mountain-fold at each place with label "V" or "+", and unfold the paper.

Step 1 is repeatedly performed for each $i = 2, 3, 4, \ldots, k-2$. Finally, when $i = k-2$, the number of consecutive patterns of "V" becomes one, fix it, and algorithm halts.

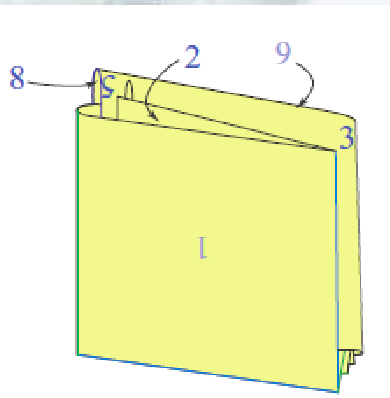
Algorithmic Folding Complexity:

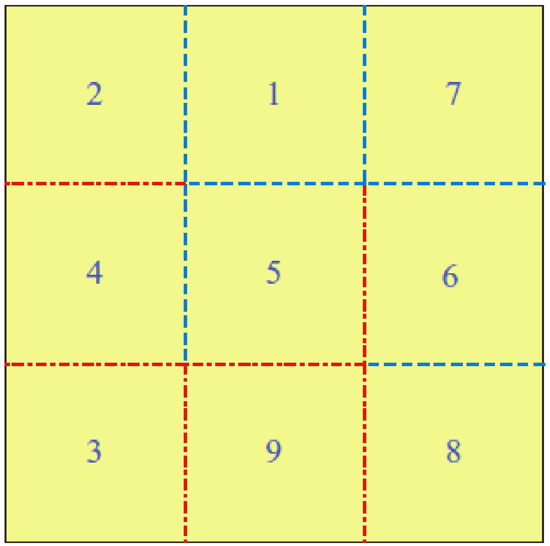
- Non trivial results:
  - For pleats:
    - Upper bound: $O(\log^2 n)$
    - Lower bound: $\Omega(\log^2 n/\log \log n)$
  - General pattern:
    - Almost all patterns require $\Omega(n/\log n)$ folds
    - General algorithm folds any pattern in $O(n/\log n)$ folds

Exceptionally simple pattern.

Gap≒4

1. The number of operations





- *Algorithmic Folding Complexity:*
  - Future works
    - Fill the gaps between upper/lower bounds
    - Different folding model (not 'simple' fold model)
      - How about counting "unfold operations"?
    - Non-regular creases
    - 2 dimensional (~ "map folding problem")

[Map Folding Problem]

Given M/V assignment for a map of size $n \times m$, determine if it can be folded into size 1×1.

- Poly time if only simple folds are allowed
- Poly time for $2 \times n$. but it takes $O(n^9)$ [Morgan 2012]

2. The number of paper layers

- 1D origami
- Creases are at regular intervals

"Crease width"の導入

2.1. The number of paper layers at the crease;

- Minimize the total/maximum crease width.
- My papers:
  - T. Umesato, T. Saitoh, R. Uehara, H. Ito, and Y. Okamoto: Complexity of the stamp folding problem, Theoretical Computer Science, Vol. 497, pp. 13-19, 2012.
  - R. Uehara, On stretch minimization problem on unit strip paper, CCCG 2010, 2010, pp. 223–226.

2. The number of paper layers

- そもそもその前に…?
  - 2.1. 与えられたパターンに合致する折り方は…?
    - 与えられた山谷パターンに合う折り方の個数とは？
    - “Stamp folding”問題:

      The number $F(n)$ of folding ways of strip length n
      - Better bounds for long standing open problem as

        $3.06^n \leqq F(n) \leqq 4^n$
    - My paper:
      - R. Uehara, Stamp foldings with a given mountain–valley assignment, in: ORIGAMI[5], CRC Press, 2011, pp. 585–597.

2. The number of paper layers

- How can we estimate thickness…?
  - 2.1. "Crease width": Survey of results
    - Find the "best" folding way to minimize the maximum crease width is NP-complete.
    - We give a simple FPT algorithm w.r.t. the total crease width.

2. The number of paper layers

- How can we estimate thickness…?
  - 2.1. "Crease width": Future works
    - Better bounds for "stamp folding"
    - Better FPT algorithm?
    - 2 dimensional extension…?
    - Non regular creases...?
      - In regular creases, all creases are piled on two endpoints, which makes the problem "local" problem.
  - 2.2. "Non regular creases…"
    - Different criteria (we must consider "global" structure)
    - WALCOM 2015@Bangladesh (02/26-02/28)!!

Future work

3. The area you need to fold? Other measure?

- We need natural analogue of “space complexity?”
  … that have “time-space trade-offs?”
- Many unsolved problems from the viewpoint of theoretical computer science, namely,
  - Algorithms
  - Computational complexity.
- Extension to 2-dimension, non-unit creases, some models of folding ways…

Complexity of Origami folding: Conclusion

<u>Goal</u> would be to answer:
"which is more complex?"

Kawasaki Rose

You have to fold simultaneously in 3D! But I can fold it in 10 min without textbook.

You have to fold many times with accuracy! I need more than 40 min to fold with textbook still now!

Maekawa Devil

今日の予定

「折り」のアルゴリズムと計算量の関係

1. 折り紙の基本操作(済み)
2. 折り紙のアルゴリズムと計算量

- 現状:まだ1次元の紙(細長い紙)を折る問題
  1. 効率のよい折り方(アルゴリズムと計算量)
     - ◆切手折り問題に関する未解決問題(組合せ理論)
     - ◆Folding Complexity (時間計算量に対応する新概念)
     - ◆最適化問題とNP完全問題と Fixed Parameter Tractability
  2. 計算不能性(時間があれば)