

# 今日の予定

1. 展開図の基礎的な知識

1. 正多面体の共通の展開図

1時間目～2時間目

2. 正多面体に近い立体と正4面体の共通の展開図: 2時間目～3時間目

3. 複数の箱が折れる共通の展開図: 3時間目

このミステリー(?)の  
中でメインリックに使  
われました!



Some nets are available at <http://www.jaist.ac.jp/~uehara/etc/origami/nets/index-e.html>

# Common Developments of Three Different Orthogonal Boxes

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<http://www.jaist.ac.jp/~uehara/>  
[uehara@jaist.ac.jp](mailto:uehara@jaist.ac.jp)  
and  
Toshihiro Shirakawa  
(Amateur puzzle solver)

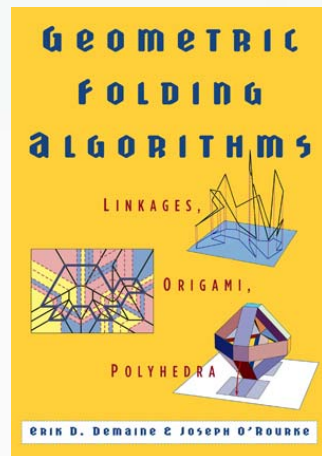
## 主な文献

Toshihiro Shirakawa and Ryuhei Uehara  
Common Developments of Three Different Orthogonal Boxes,  
*The 24th Canadian Conference on Computational Geometry*  
(CCCG 2012), pp. 19-23, 2012/8/8-10, PEI, Canada.

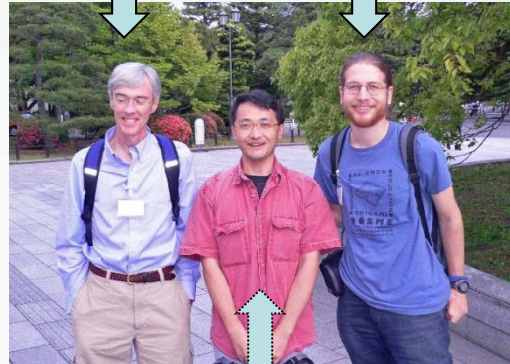
# The bible of this topic...

*Geometric Folding Algorithms: Linkages, Origami, Polyhedra*

by J. O'Rourke and E. D. Demaine, 2007.



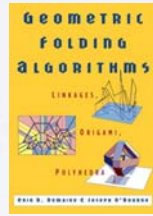
Authors



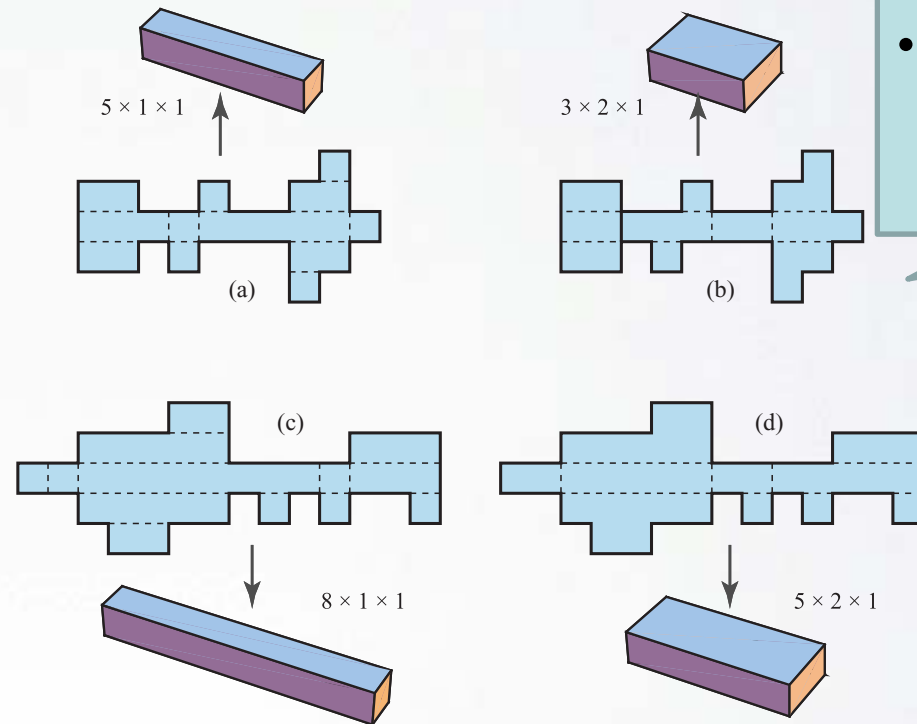
(2009)

I, translated it to Japanese (2009).

In ,



➤ There were two developments that fold into two boxes;



- Are they exceptional?
- Is there any development that fold to 3 or more boxes??

[Biedl, Chan, Demaine, Demaine, Lubiw, Munro, Shallit, 1999]

# Developments of two boxes

In [Uehara, Mitani 2007], randomized algorithm that looks for such polygons by *brute force*;

➤ Polygons folding into 2 boxes:

1. There are  
many (~9000)  
(by supercomputer (SGI Altix 4700))
2. Theoretically,  
infinitely many





# Note:

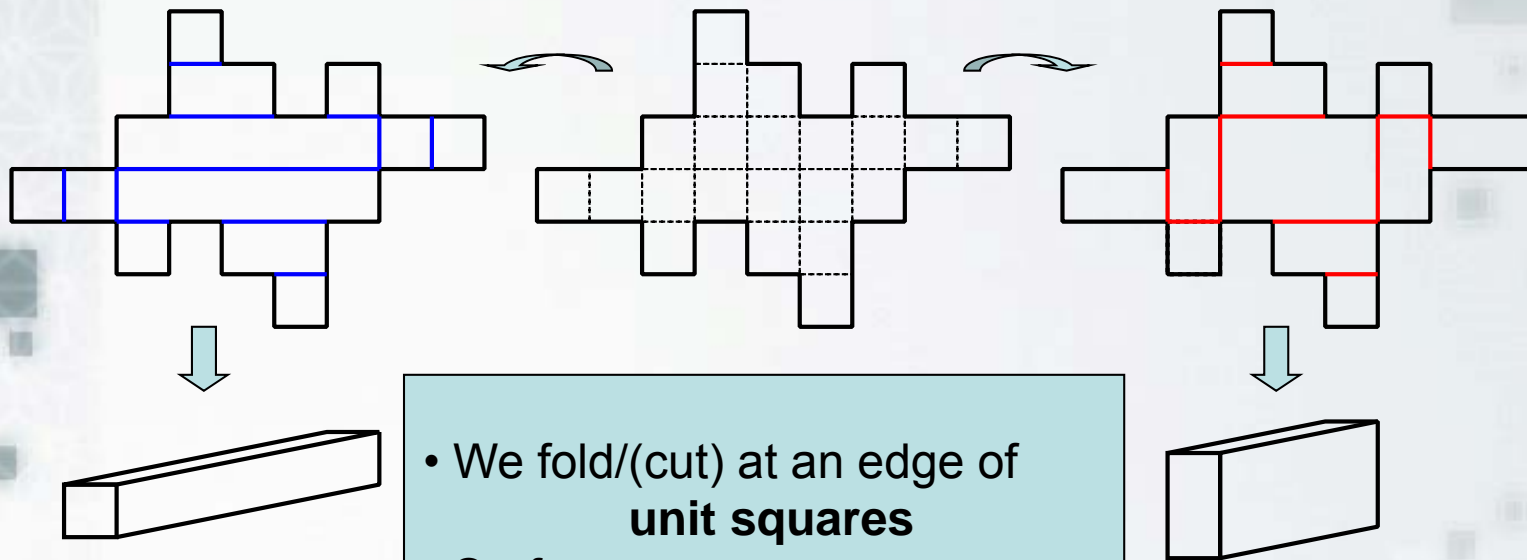
## Polygons folding to 2 different orthogonal boxes

Example:

$$1 \times 1 + 1 \times 5 + 1 \times 5$$

$$= 1 \times 2 + 2 \times 3 + 1 \times 3$$

$$= 11 \text{ (surface area: 22)}$$



$$1 \times 1 \times 5 \\ = a \times b \times c$$

- We fold/(cut) at an edge of **unit squares**
- Surface area:  
$$2(ab + bc + ca)$$
- Necessary condition:

$$1 \times 2 \times 3 \\ = a' \times b' \times c'$$

$$ab + bc + ca = a'b' + b'c' + c'a'$$

It seems to be better to have many combinations...

# Note:

Surface areas;

If you try to find for three boxes,

If you try to find for four boxes,

Area	Trios	Area	Trios
<b><u>22</u></b>	(1,1,5),(1,2,3)	46	(1,1,11),(1,2,7),(1,3,5)
30	(1,1,7),(1,3,3)	70	(1,1,17),(1,2,11),(1,3,8),(1,5,5)
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38	(1,1,9),(1,3,4)	118	(1,1,29),(1,2,19),(1,3,14), (1,4,11),(1,5,9),(2,5,7)

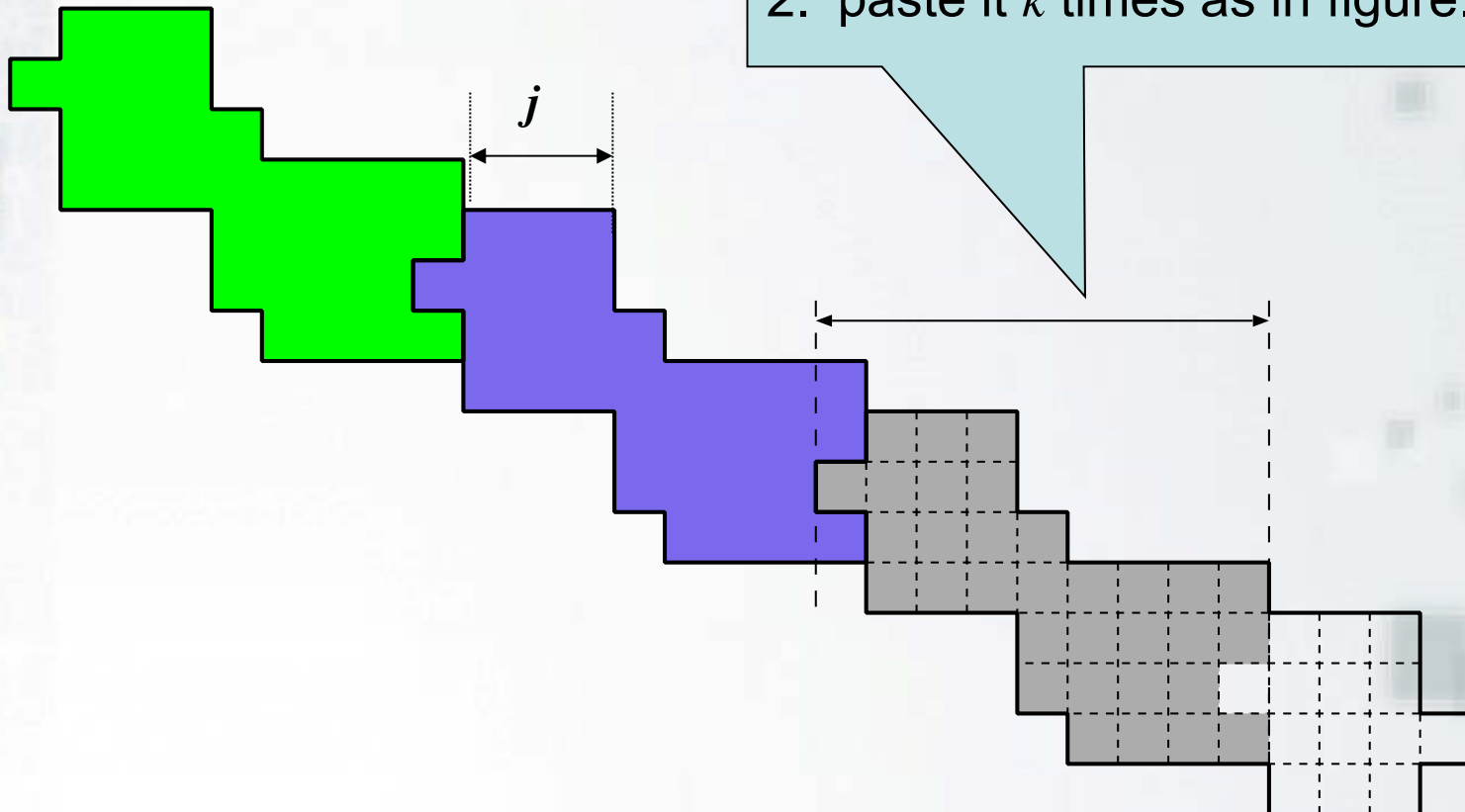
known results

# Developments of two boxes

[Thm] There exist an infinitely many developments that fold to 2 boxes.

**[Proof]**

1. copy this area, and
2. paste it  $k$  times as in figure.

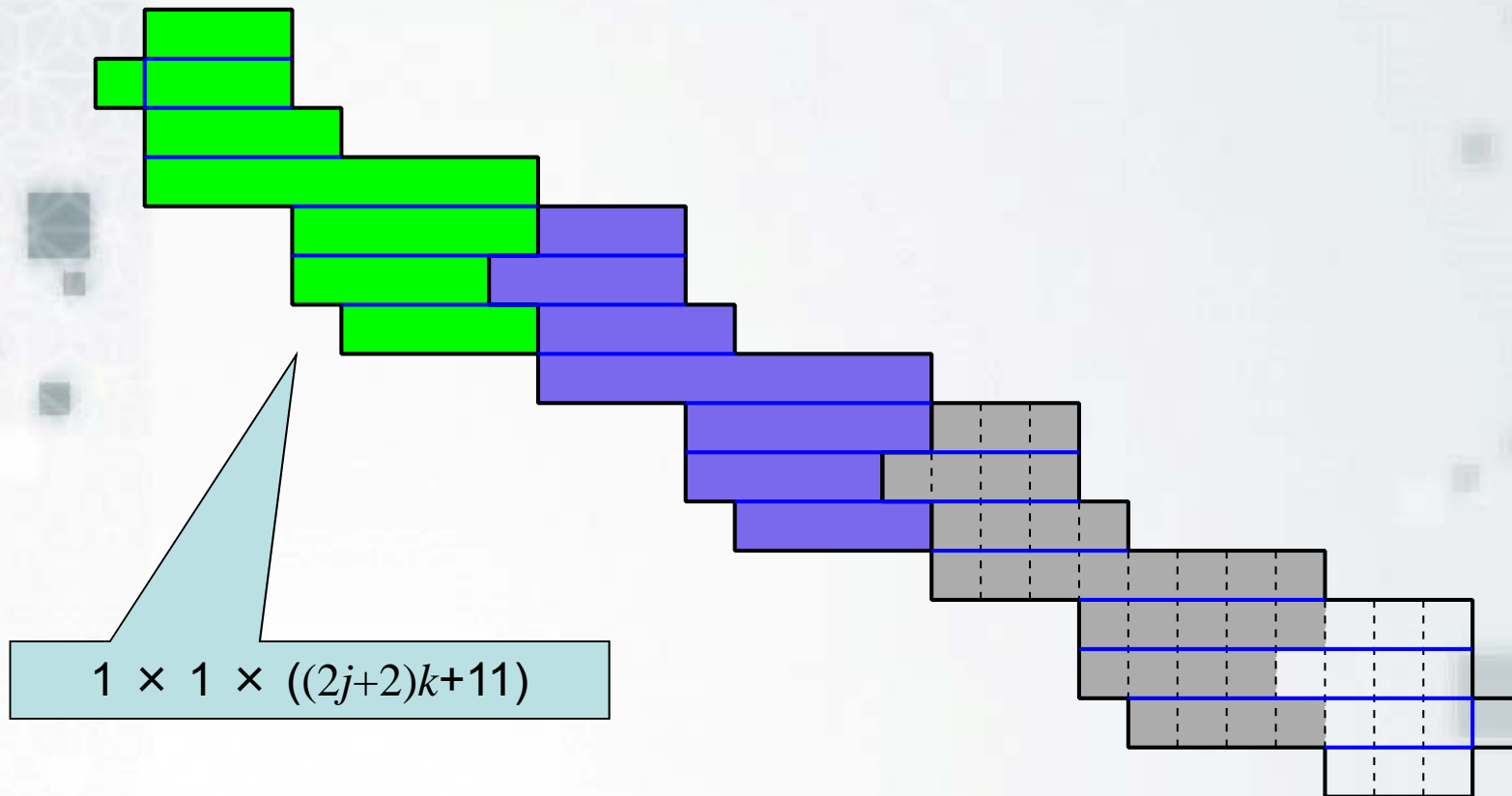




# Developments of two boxes

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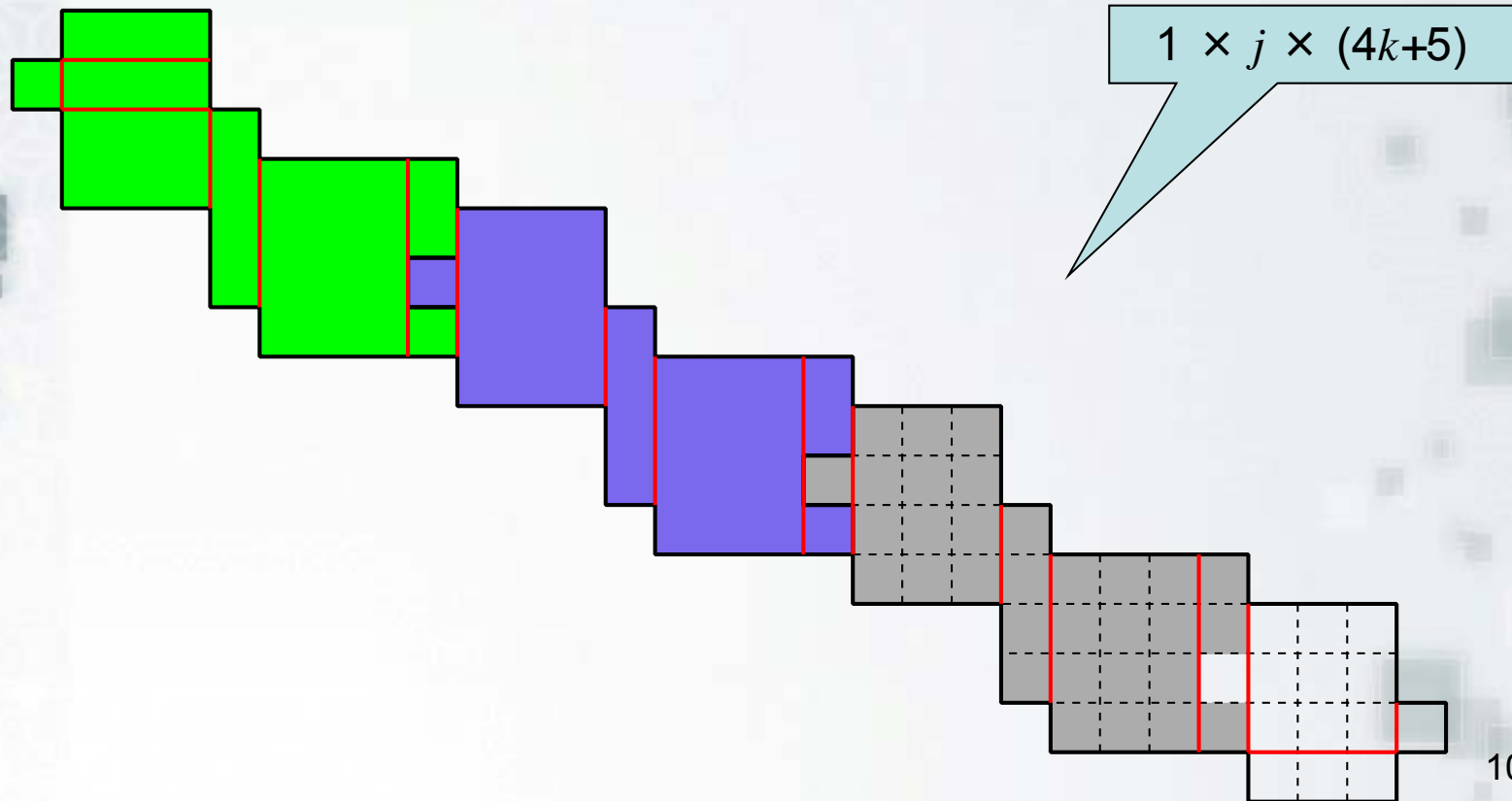
[Proof]



# Developments of two boxes

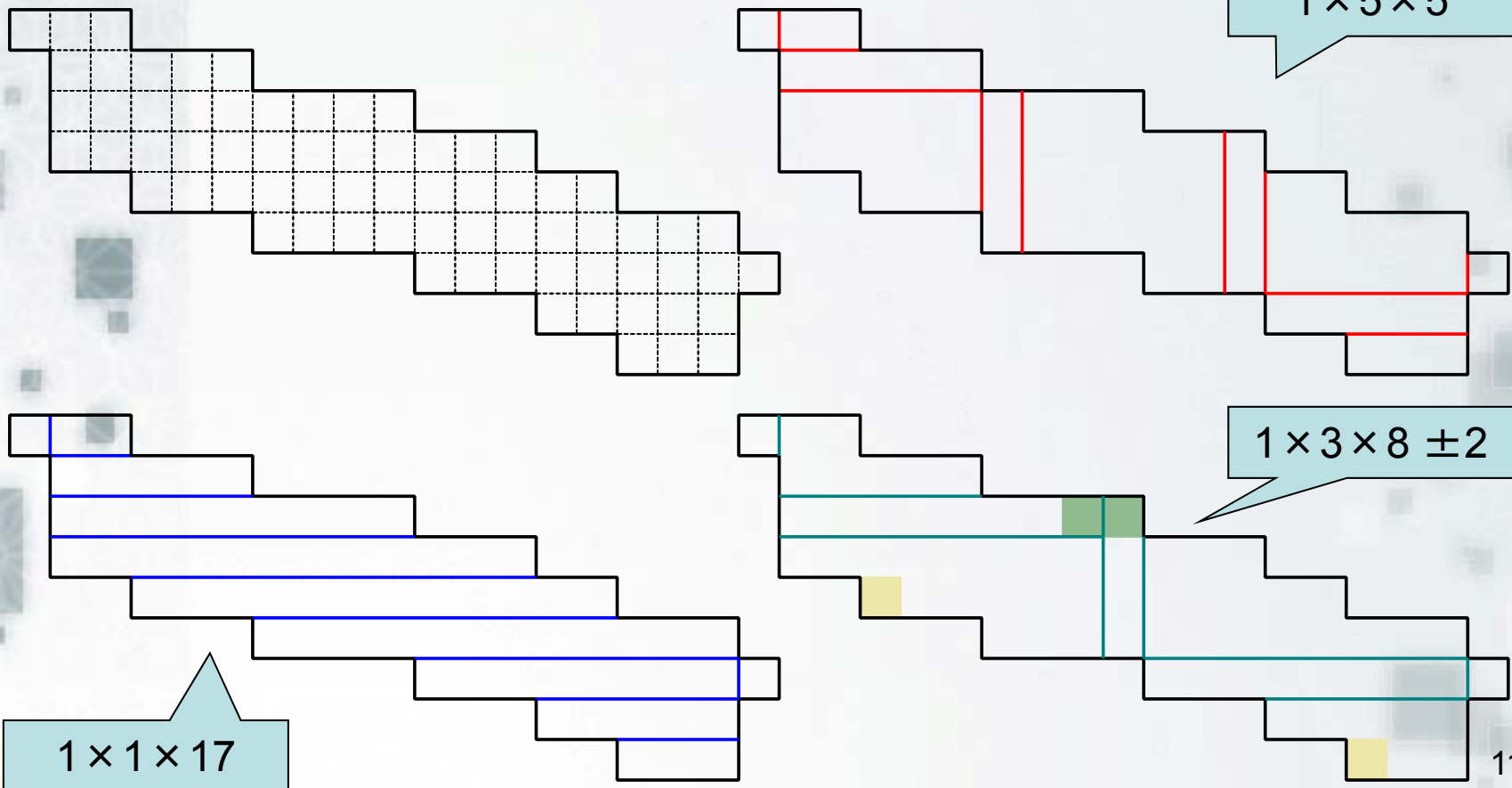
[Thm] There exists an infinitely many polygons...

[Proof]



# Developments of *three* boxes(?)

- A polygon that can fold to three distinct boxes...?
  - close solution...



# Developments of *three* boxes(?)

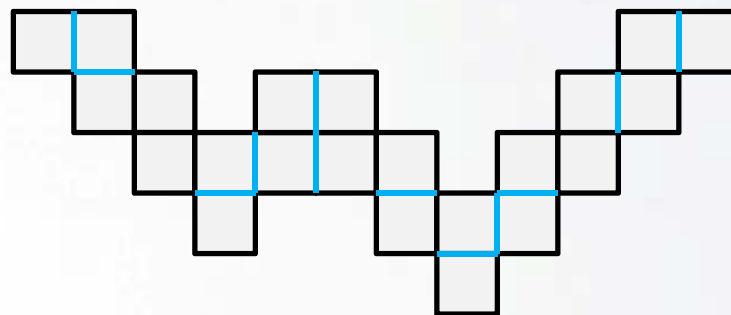
In [Abel, Demaine, Demaine, Matsui, Rote, Uehara 2011],

- The number of developments that fold to  $1 \times 1 \times 5$  box and  $1 \times 2 \times 3$  box is 2263.
- ◆ the latest algorithm runs in around 10 hrs.
- Among them, there is only one **pearl** development...

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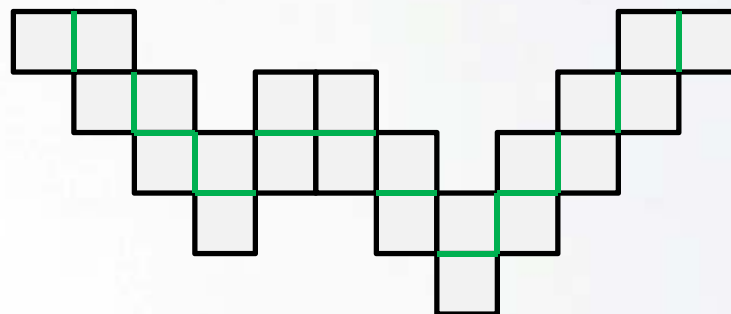


$1 \times 2 \times 3$

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$1 \times 1 \times 5$



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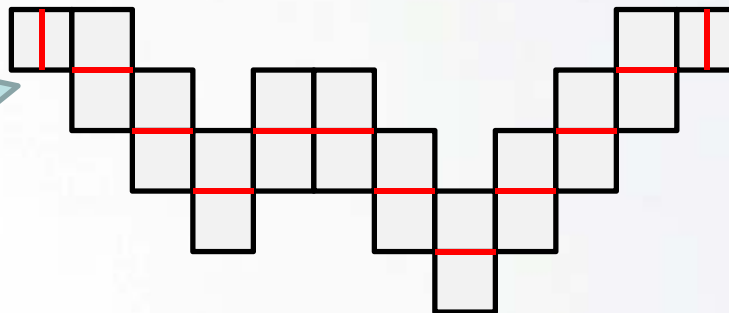
おまけ問題:  
3通り折ってみよう

The number of developments that fold to  $1 \times 1 \times 5$  box and  $1 \times 2 \times 3$  box is 2263.

Is the “box” *cheat* having volume 0?

the latest algorithm runs in around 10 hrs.  
Among them, there is only one **pearl** development...

If you don't like  $\frac{1}{2}$ , refine each square into 4 squares



Since each column has height 2 except both sides

$1 \times 11 \times 0$

!

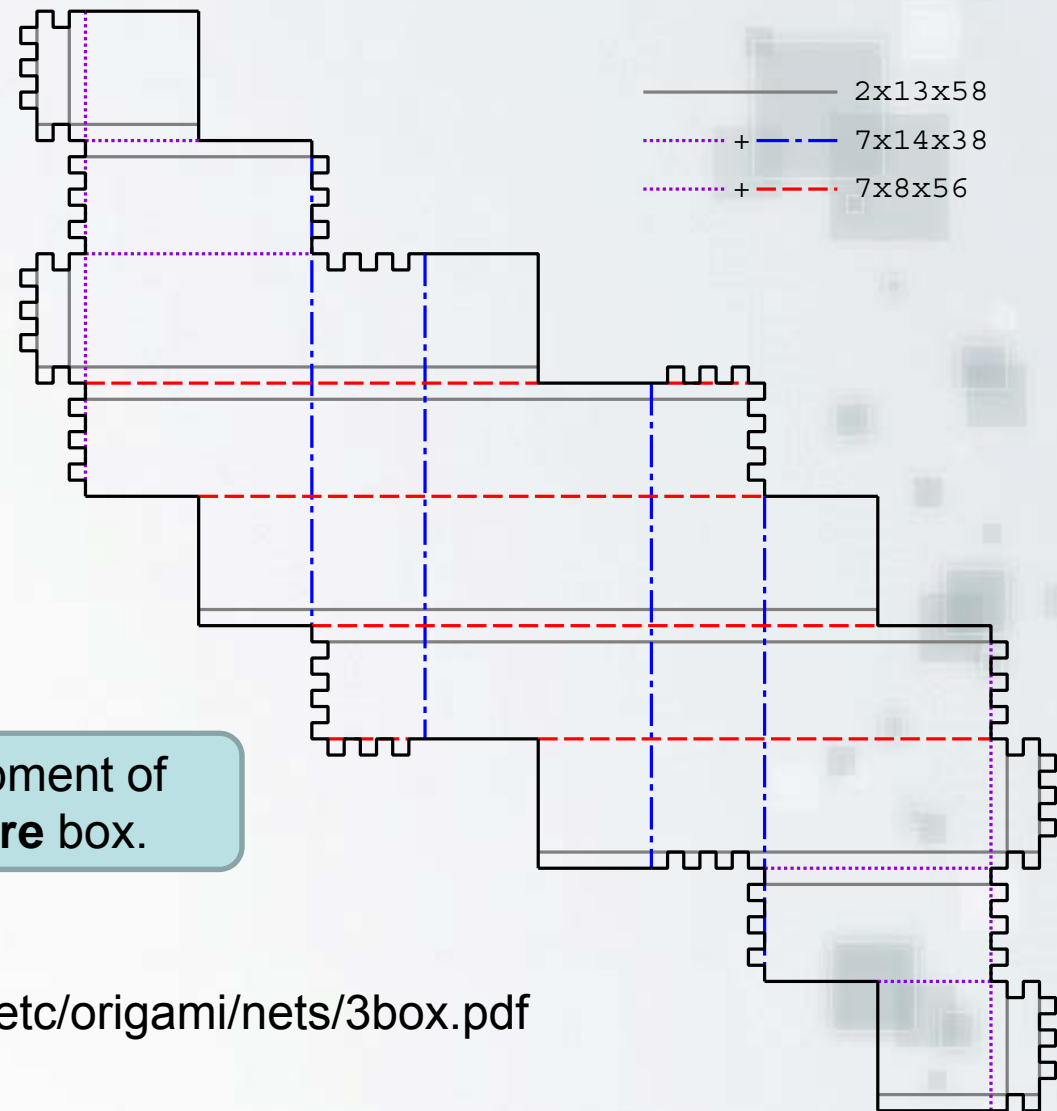
# Developments of three boxes(!)

In February 2012,  
Shirakawa (and I)  
finally found that:

There exists  
a polygon that  
folds to **3 boxes!!**

[Basic Idea] From a development of  
2 boxes, we make **one more** box.

I put it at  
<http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf>



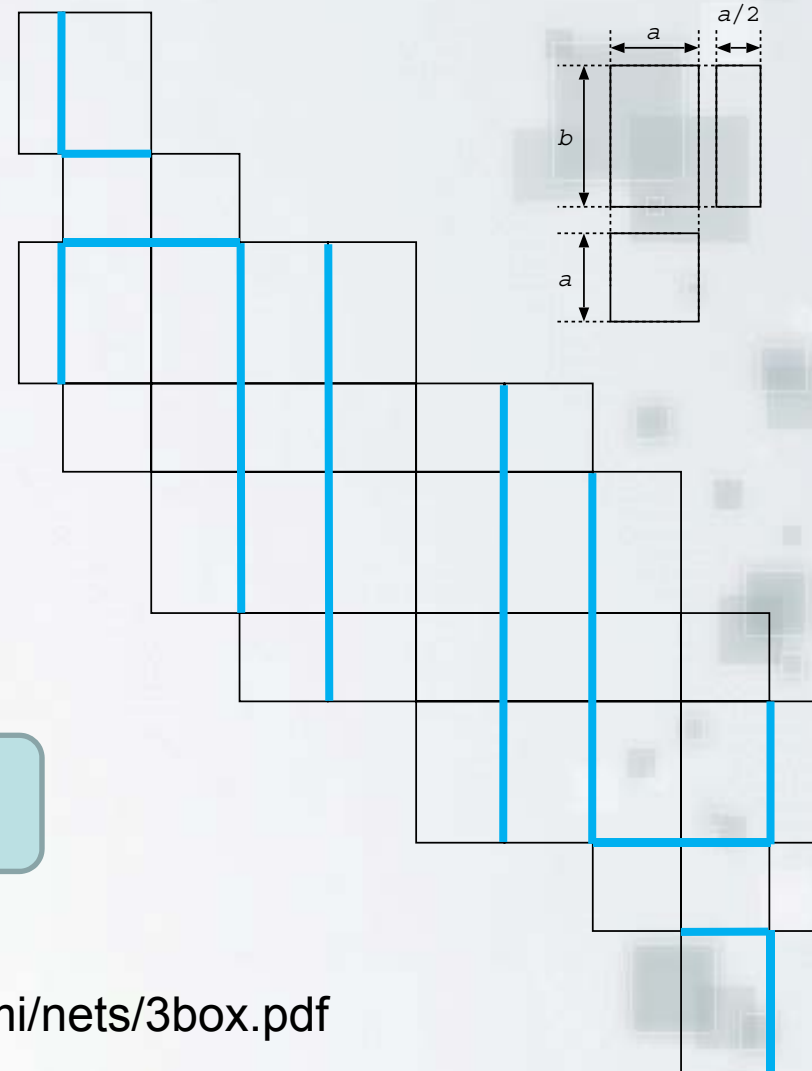
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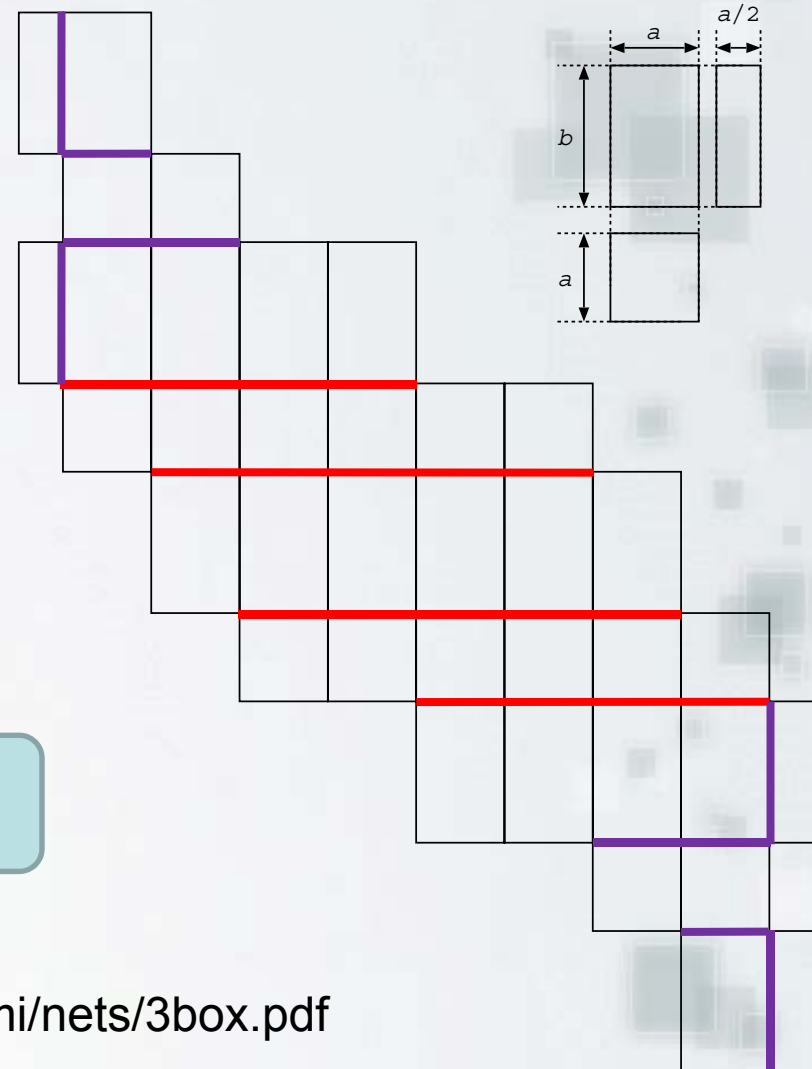
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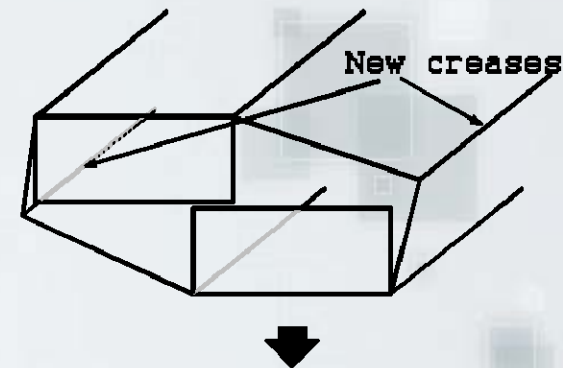
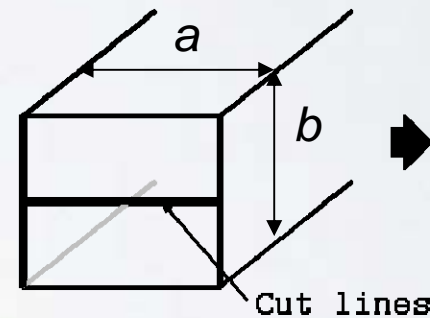


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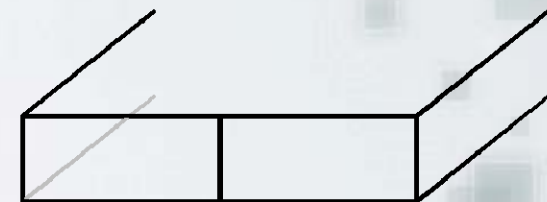
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One more box  
is obtained by  
this *squashing*!?



[No!!]  
This works iff  $a=2b$ , i.e.,  
from  $1 \times 2$  square to  $2 \times 1$  square

I put it at  
<http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf>

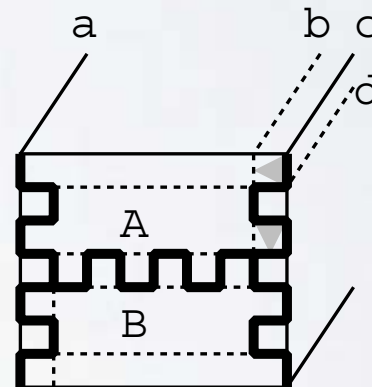
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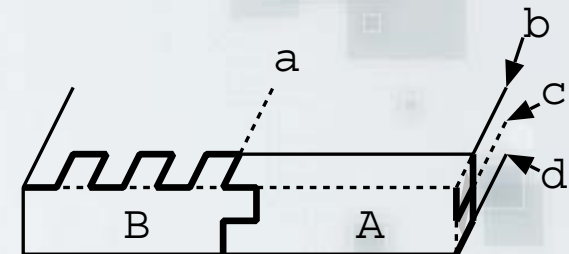
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(a)



(b)

[Yes... with a trick!]  
This idea works;  
move a part of  
the lid to 4 *sides*!



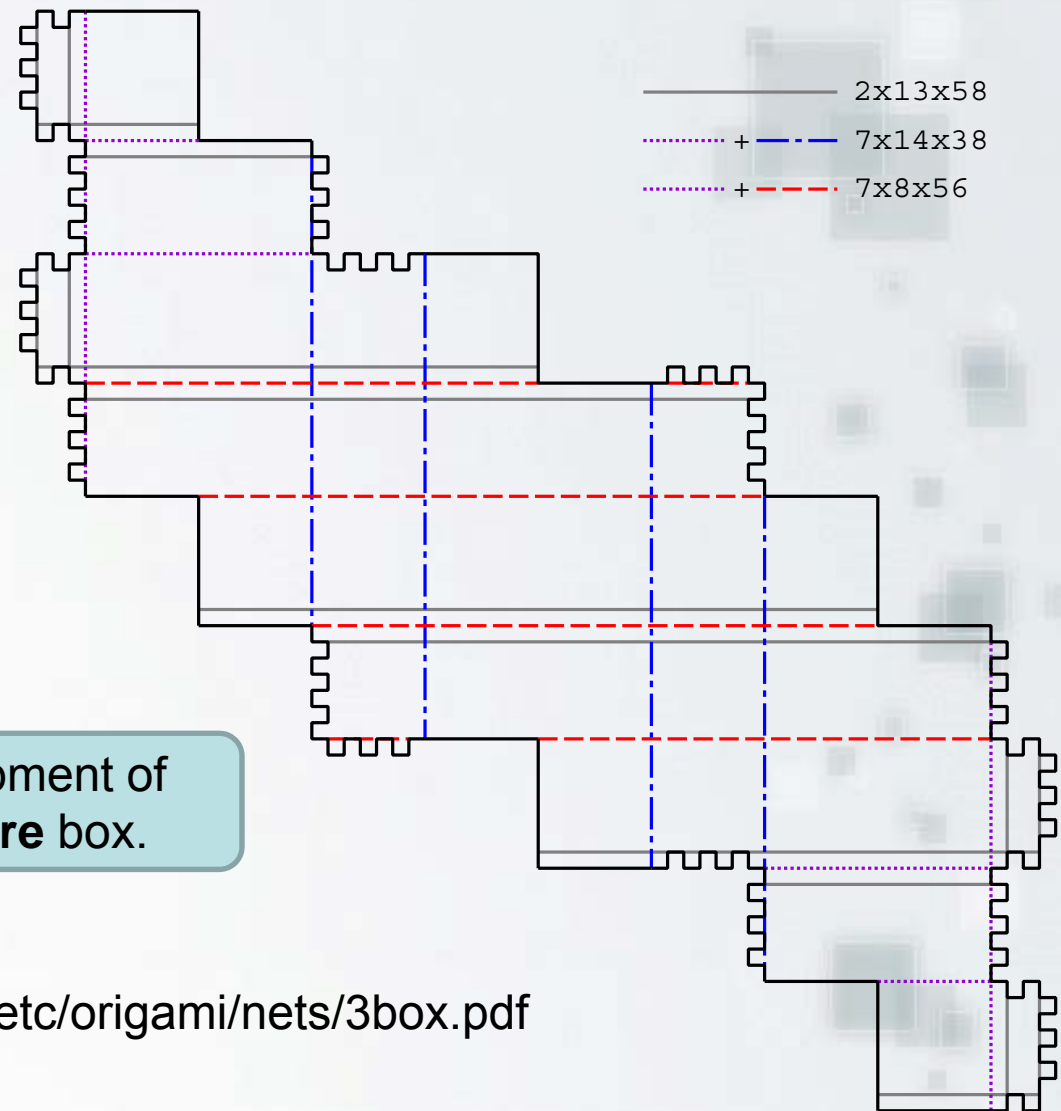
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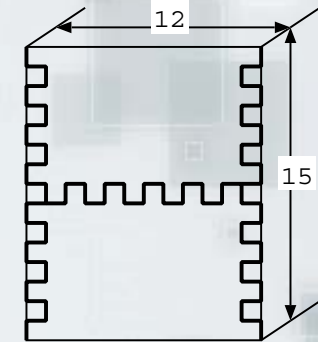
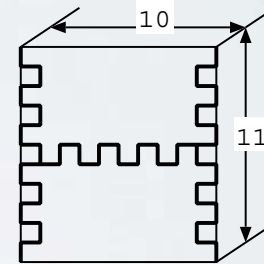
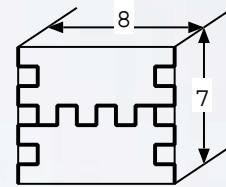
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# Developments of *three boxes*(!)

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[Theorem]

There exist an infinite number of polygons  
that fold into 3 different boxes.

[Generalization]

- Basic box is flexible for the edge lengths.
- Zig-zag pattern can be extended.

I put it at

<http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf>

# Future works

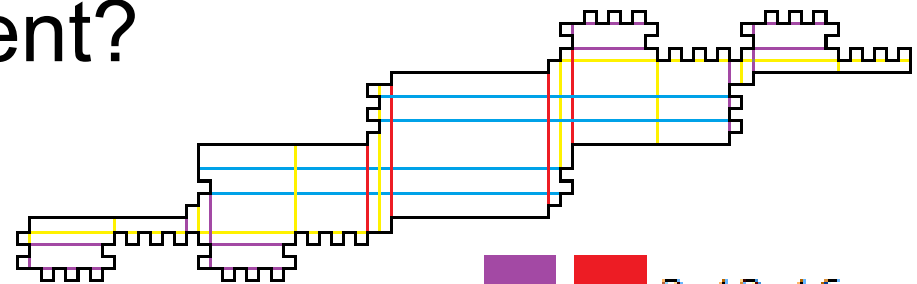
## ➤ Smallest development?

The current “smallest” development requires **532** squares.

>> the smallest area **46** that may produce three boxes of size  $(1,1,11)$ ,  $(1,2,7)$ ,  $(1,3,5)$ .

(Remind:

2263 polygons of area **22** folding to  $(1,1,5)$ ,  $(1,2,3)$ )



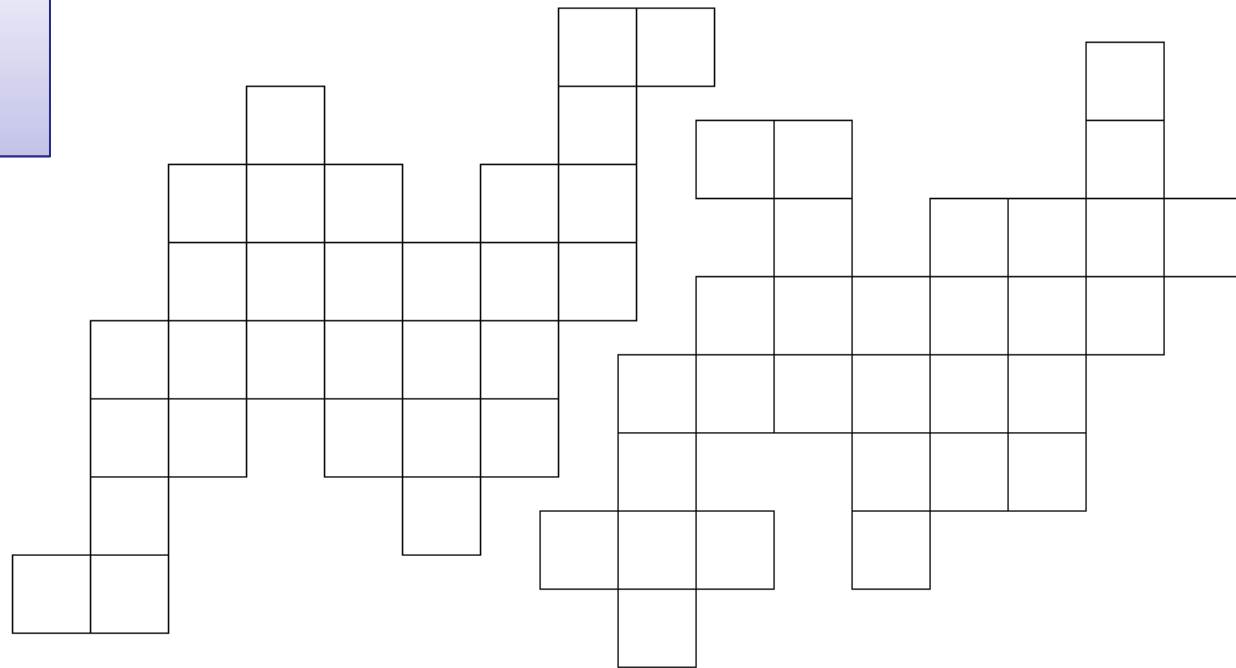
■	■	2x13x16
■	■	2x 4x43
	■	7x 8x14

Is there a polygon that folds to 4 or more boxes?

## 2012年10月23日：白川さんからのメール

「面積30で、 $1 \times 1 \times 7$ と $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$ の2通りの箱を折れる展開図を見つけました。この面積は $1 \times 3 \times 3$ も作れるので、斜めを許した場合3通りの箱が折れる最小のポリオミノになる可能性があります。」

おまけ問題：  
2通り折ってみよう



# Note:

Surface areas;

If you try to find for three boxes,

If you try to find for four boxes,

Area	Trios	Area	Trios
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known results

2011年当時の松井君のプログラム:

- **面積22**の展開図を全探索:
  - $1 \times 1 \times 5$ と $1 \times 2 \times 3$ の箱を折る2263個の展開図
- 実行時間はパソコンで**10時間**

面積30は微妙な数字...

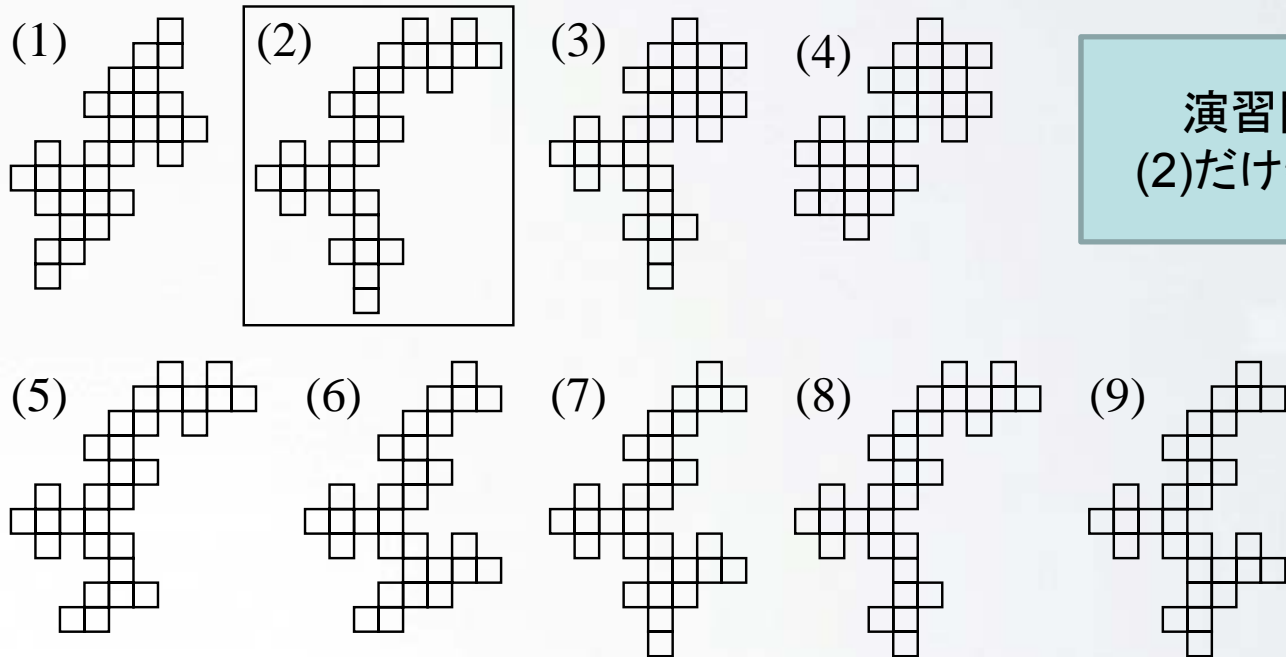
# Dawei君の結果(2014年6月)

- 面積30の展開図の全探索に成功！ [Xu, Horiyama, Shirakawa, Uehara 2015]

追記:その後BDDの使用により[PCで10日]というレベルの驚愕の高速化に成功.

- 大雑把な結果

- JAISTのスパコン(Cray XC 30)で二ヶ月
- $1 \times 1 \times 7$ の箱と $1 \times 3 \times 3$ の箱が折れる共通の展開図は1080個
- そのうち、 $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$ の三つ目の箱が折れる展開図は9個



演習問題(?)  
(2)だけ特別です。



# まとめと課題

Surface areas;

Area	Trios	Area	Trios
<u>22</u>	(1,1,5),(1,2,3)	46	(1,1,11),(1,2,7),(1,3,5)
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known results

If you try to find for three boxes,

If you try to find for four boxes,

2011年, 面積22の展開図の全探索はPCで10時間.

2014年, 面積30の展開図の全探索はスパコンで2ヶ月.

BDDを使うとPCで10日間.

...この調子で46まで行くのは難しいと言わざるをえないが...?27