今日の予定

このミステリー（？）の中でメイントリックに使われました！

1. 展開図の基礎的な知識

1. 正多面体の共通の展開図

2. 複数の箱が折れる共通の展開図：2時間目
3. Rep-Cube：最新の話題

4. 正多面体に近い立体と正4面体の共通の展開図
5. ペタル型の紙で折るピラミッド型：2時間目～3時間目

Some nets are available at http://www.jaist.ac.jp/~uehara/etc/origami/nets/index-e.html

Common Developments of Three Different Orthogonal Boxes

Ryuhei UEHARA @ JAIST
http://www.jaist.ac.jp/~uehara/
uehara@jaist.ac.jp
and
Toshihiro Shirakawa
(Amateur puzzle solver)

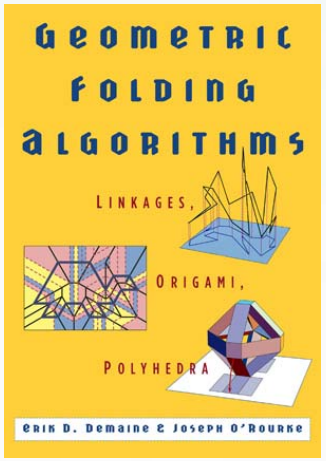
主な文献

Toshihiro Shirakawa and Ryuhei Uehara
Common Developments of Three Different Orthogonal Boxes,
The 24th Canadian Conference on Computational Geometry (CCCG 2012), pp. 19-23, 2012/8/8-10, PEI, Canada.

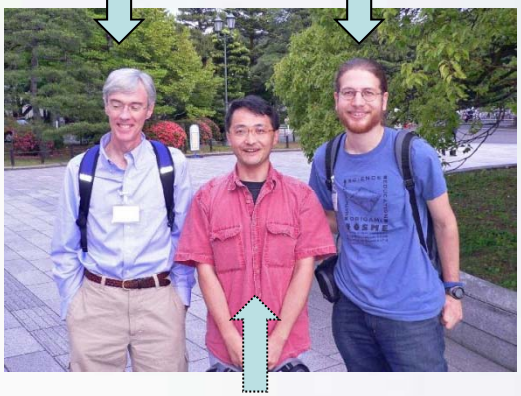
The bible of this topic…

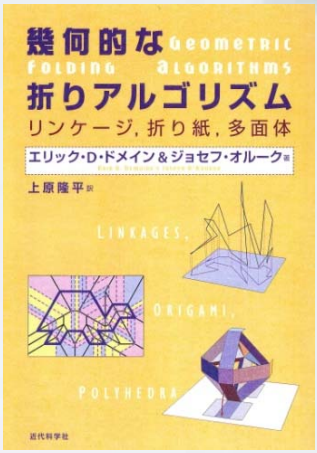
Geometric Folding Algorithms: Linkages, Origami, Polyhedra

by J. O'Rourke and E. D. Demaine, 2007.



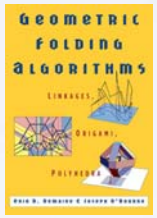
Authors





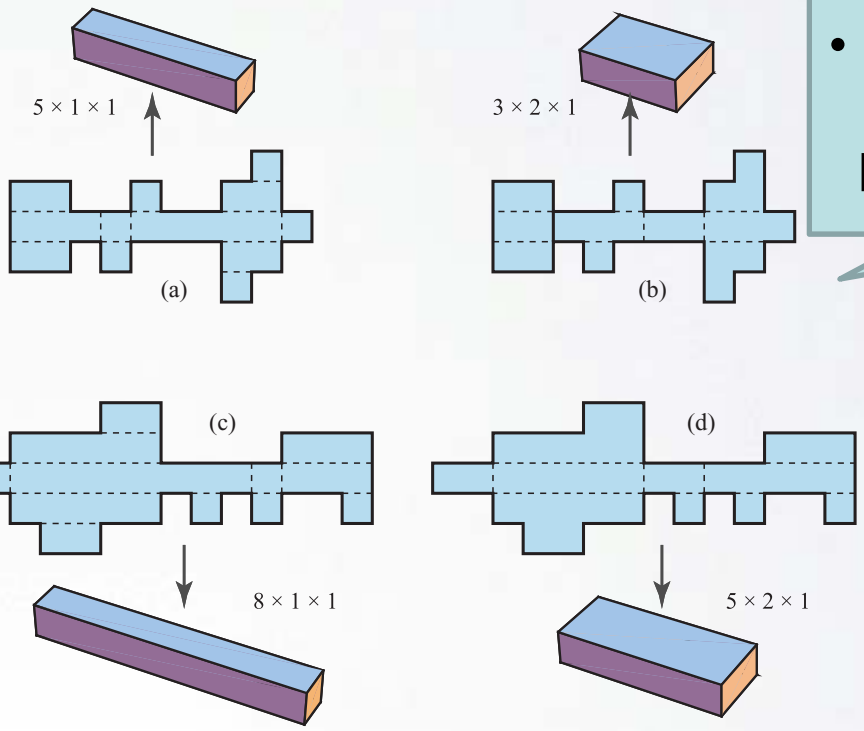
(2009)

I, translated it to Japanese (2009).

In 

,

- There were two developments that fold into two boxes;



- Are they exceptional?
- Is there any development that fold to 3 or more boxes??

[Biedl, Chan, Demaine, Demaine, Lubiw, Munro, Shallit, 1999]

Developments of two boxes

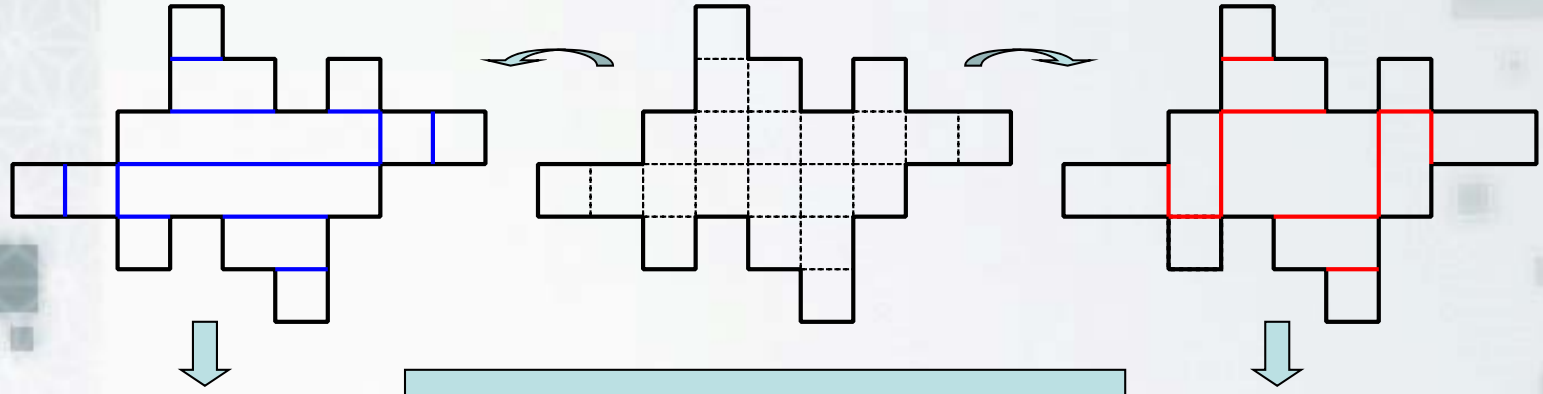
In [Uehara, Mitani 2007], randomized algorithm that looks for such polygons by *brute force*;

- Polygons folding into 2 boxes:
  1. There are many (∼9000) (by supercomputer (SGI Altix 4700))
  2. Theoretically, infinitely many

Note:

Example:
1×1+1×5+1×5
=1×2+2×3+1×3
=11 (surface area: 22)

Polygons folding to 2 different orthogonal boxes



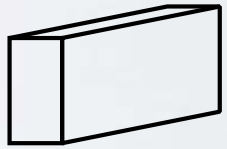
1×1×5
= a × b × c

- We fold/(cut) at an edge of **unit squares**
- Surface area:

$$2(ab+bc+ca)$$

- Necessary condition:

$$ab+bc+ca=a'b'+b'c'+c'a'$$



1×2×3
= a' × b' × c'

It seems to be better to have many combinations…

Note:

If you try to find for three boxes,

If you try to find for four boxes,

- Surface areas;

Area	Trios	Area	Trios
22	(1,1,5),(1,2,3)	46	(1,1,11),(1,2,7),(1,3,5)
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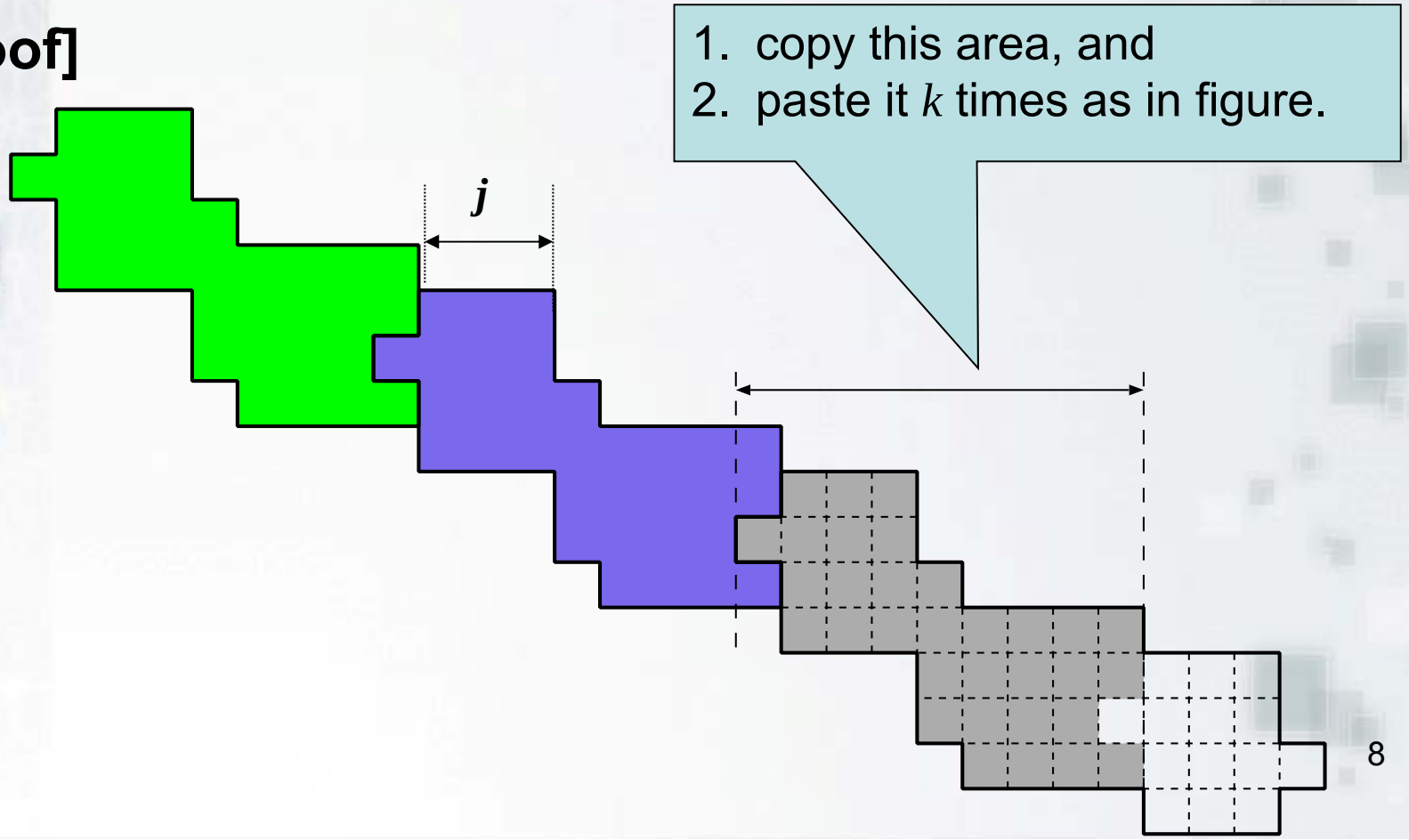
known results

Developments of two boxes

[Thm] There exist an infinitely many developments that fold to 2 boxes.

[Proof]



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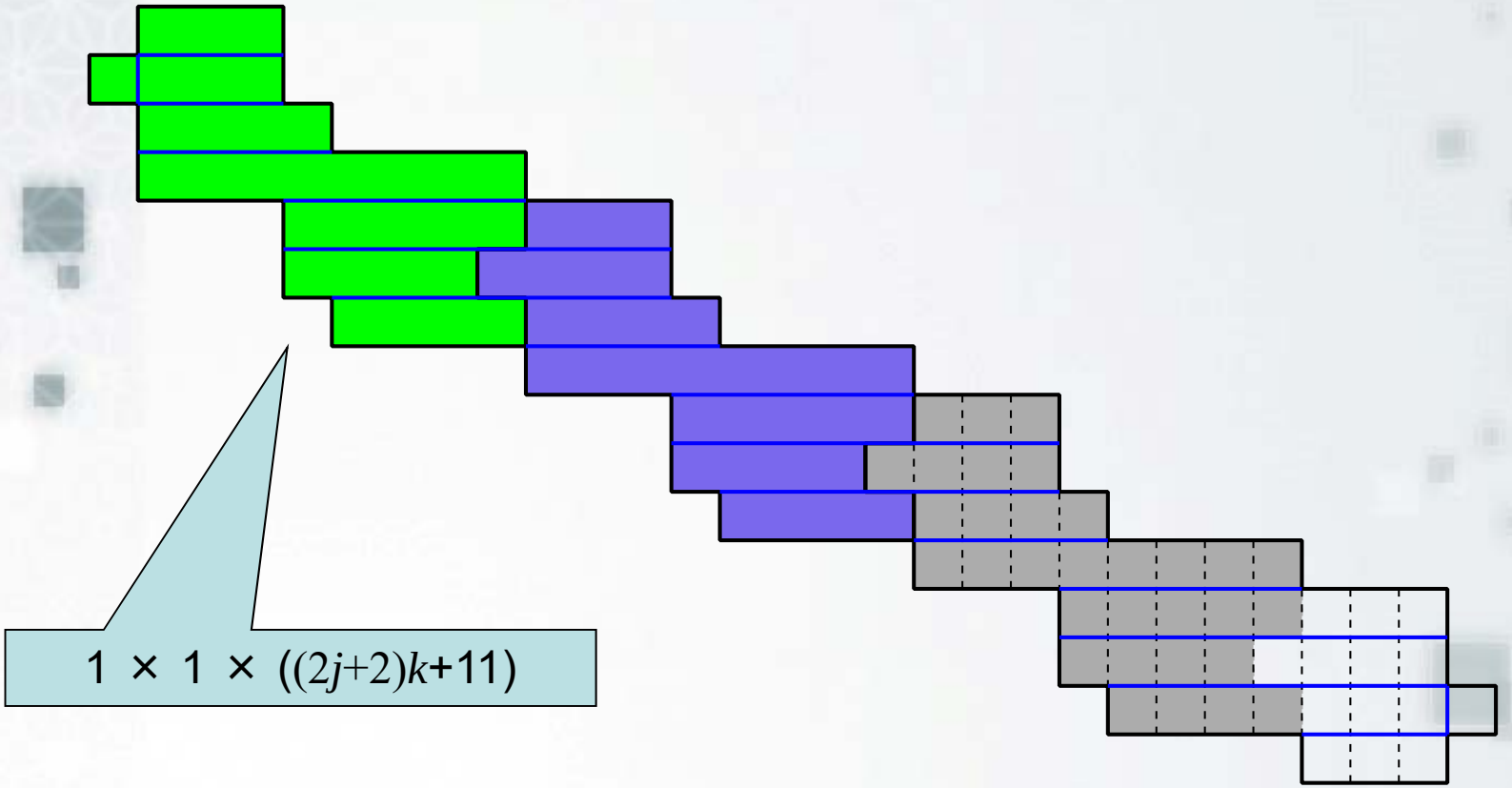

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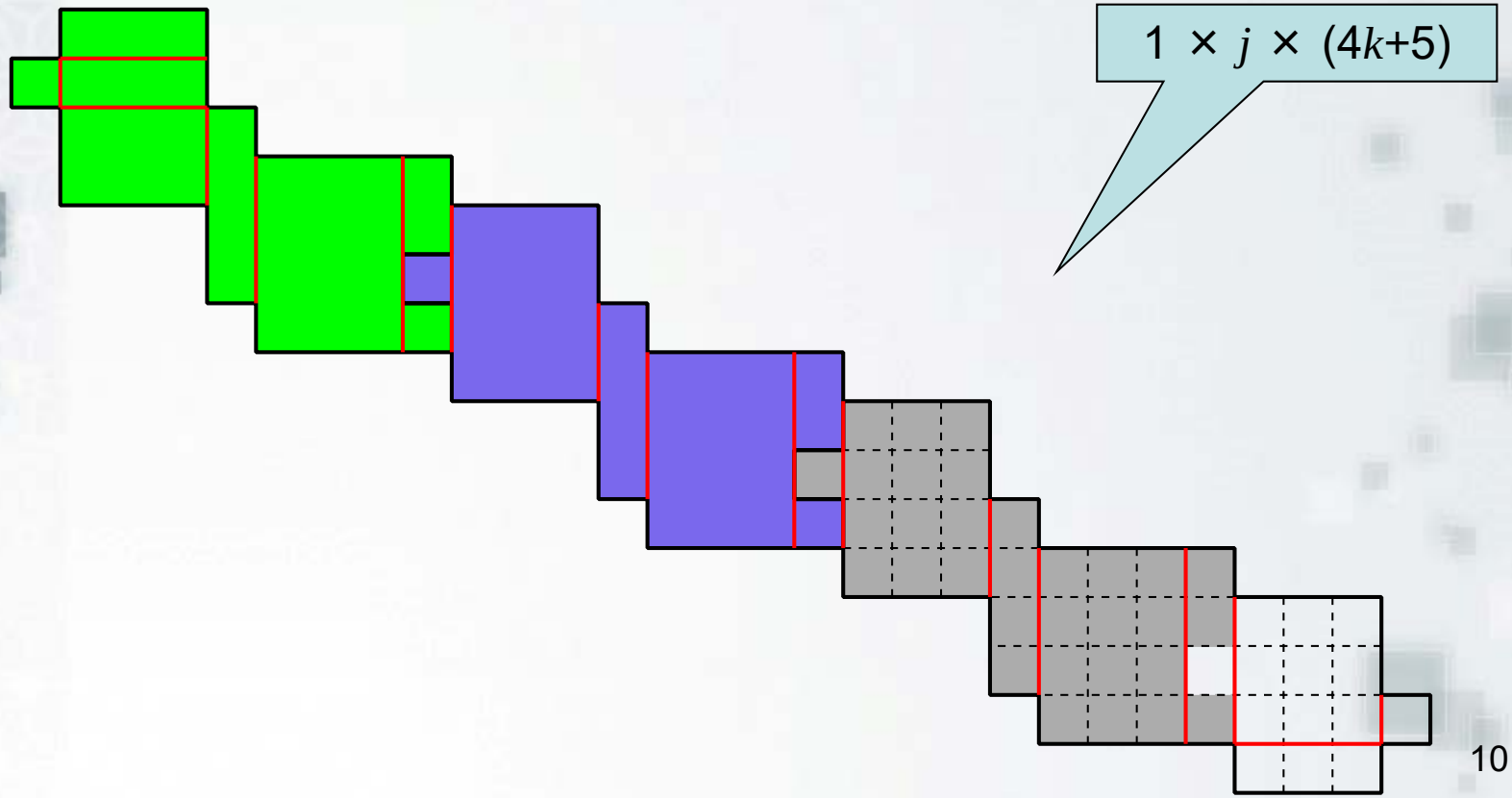
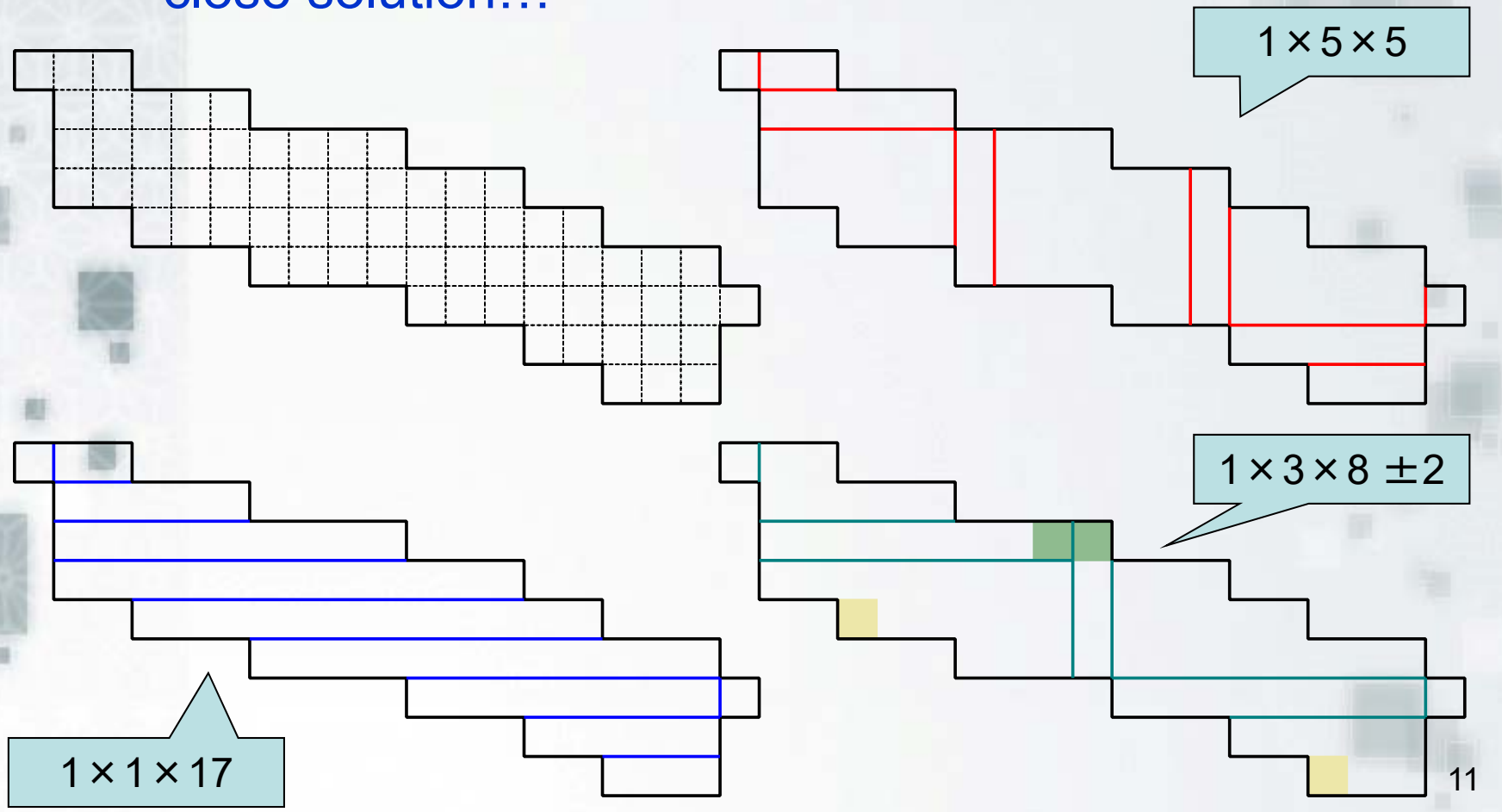


Developments of *three* boxes(?)

- A polygon that can fold to **three** distinct boxes…?
  - close solution…



Developments of *three* boxes(?)

In [Abel, Demaine, Demaine, Matsui, Rote, Uehara 2011],

- The number of developments that fold to $1 \times 1 \times 5$ box and $1 \times 2 \times 3$ box is 2263.
  - the latest algorithm runs in around 10 hrs.
- Among them, there is only one pearl development…

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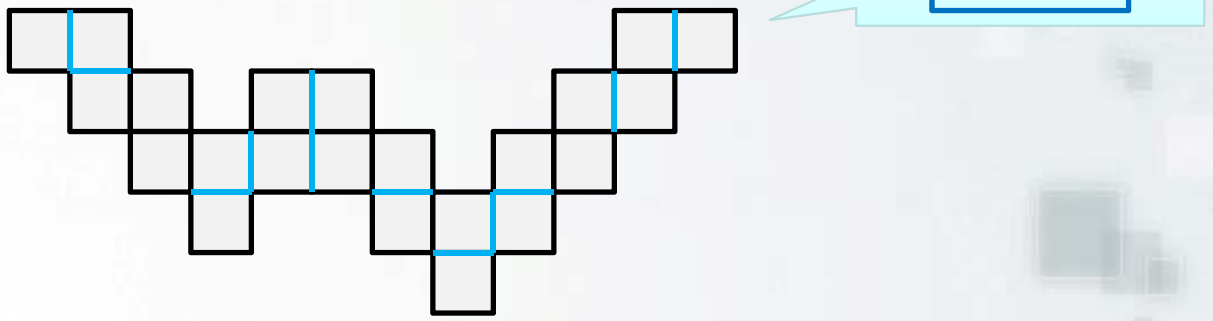

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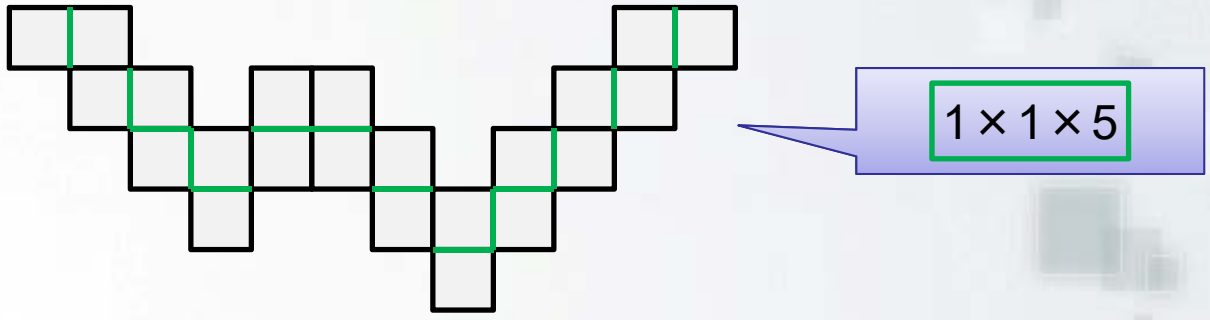


Developments of *three* boxes(?)

In [Abel, Demaine, Demaine, Matsui, Rote, Uehara 2011],

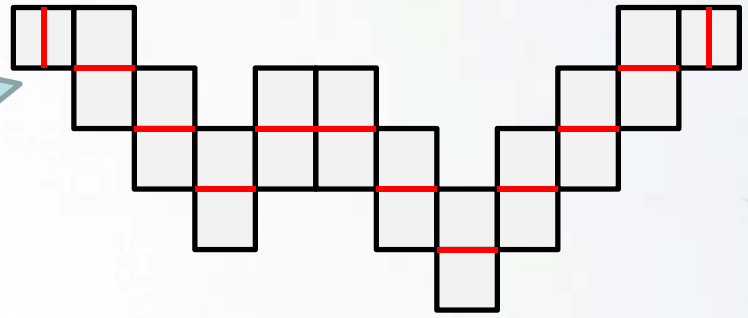
おまけ問題:
3通り折ってみよう

The number of developments that fold to $1 \times 1 \times 5$ box and $1 \times 2 \times 3$ box is 2263.

the latest algorithm runs in around 10 hrs.

Is the "box" *cheat* having volume 0?

Among them, there is only one pearl development…



If you don't like ½, refine each square into 4 squares

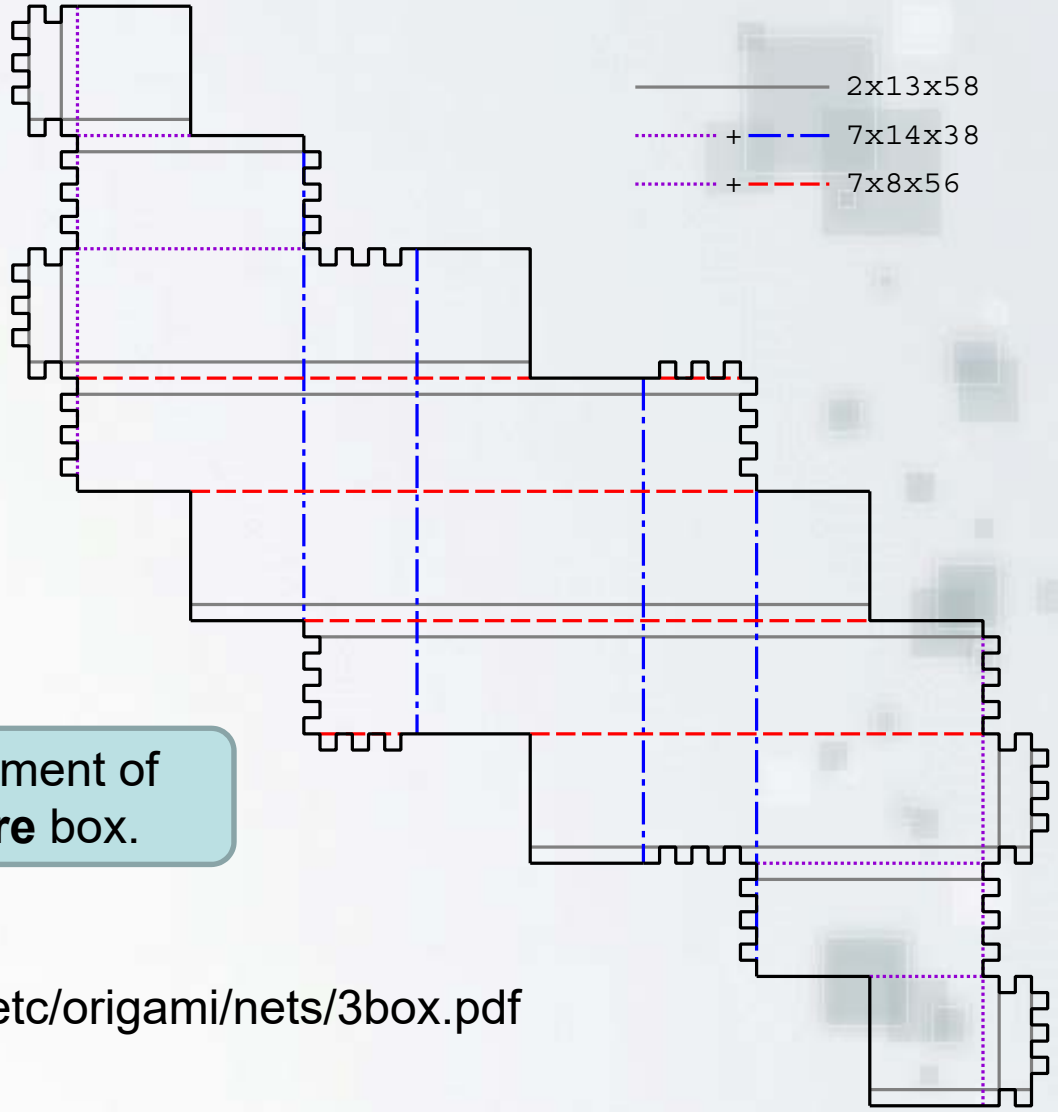
Since each column has height 2 except both sides

$1 \times 11 \times 0$!

Developments of *three* boxes(!)

In February 2012, Shirakawa (and I) finally found that:

- There exists a polygon that folds to 3 boxes!!



[Basic Idea] From a development of 2 boxes, we make **one more** box.

I put it at
http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf

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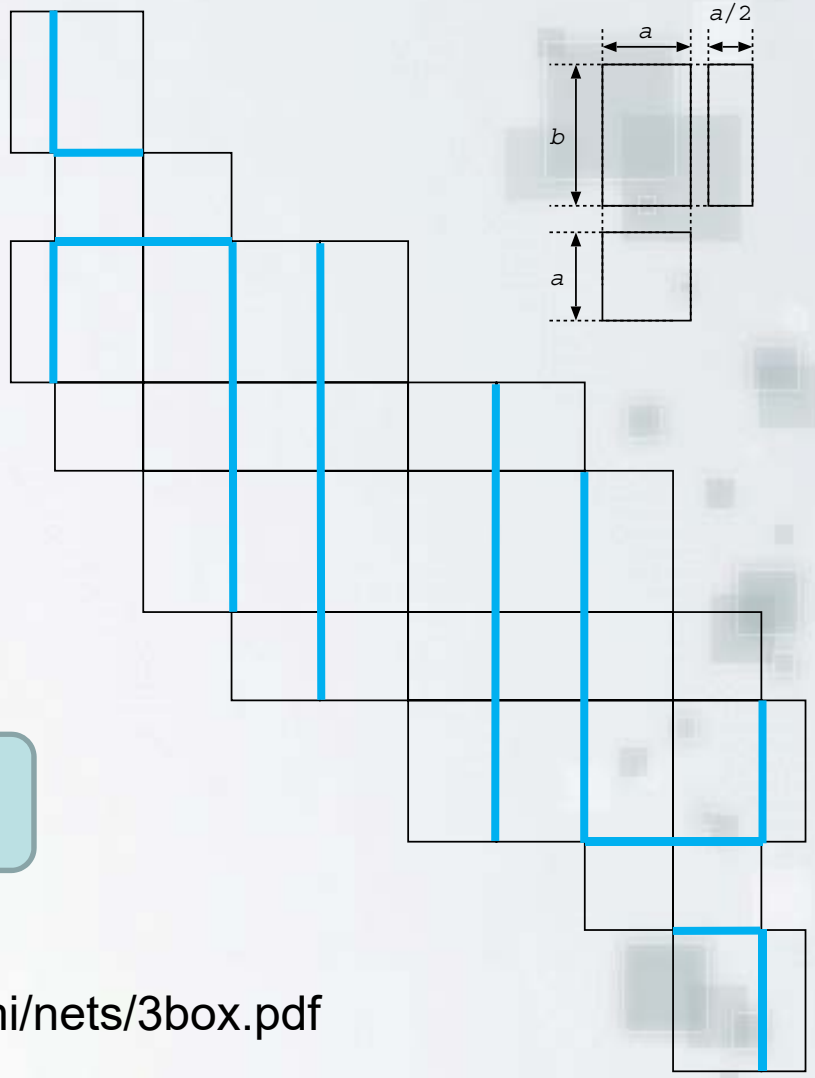

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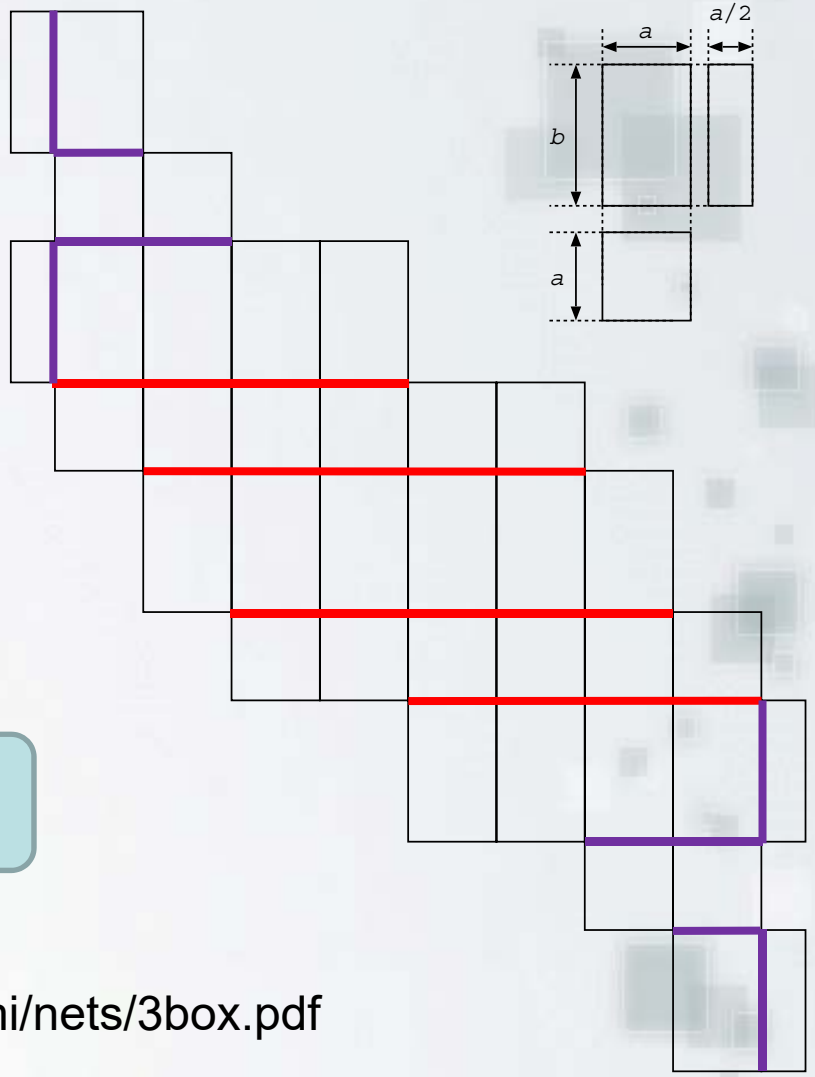

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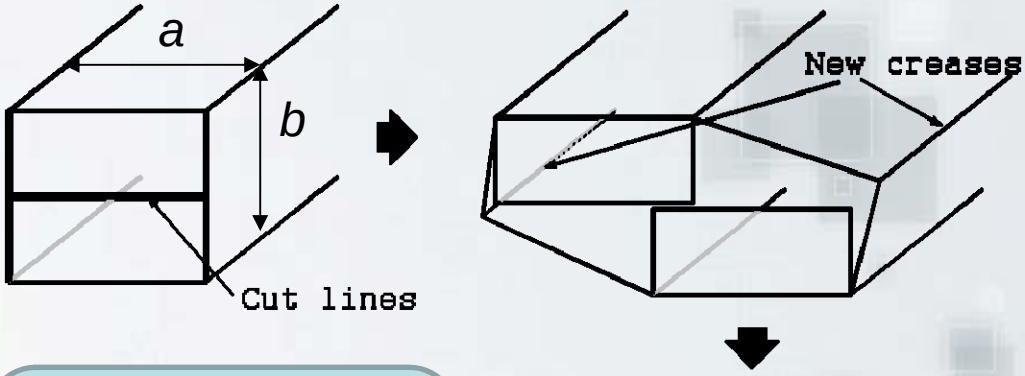

One more box
is obtained by
this *squashing!?*

[Basic Idea] From a development of
2 boxes, we make **one more** box.

[No!!]
This works iff $a=2b$, i.e.,
from 1×2 square to 2×1 square

I put it at
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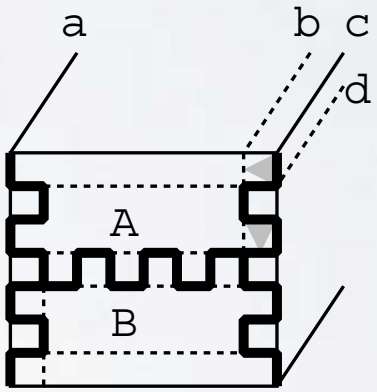
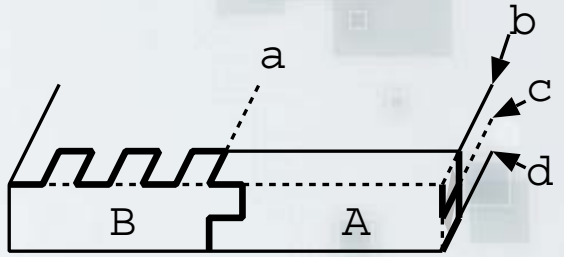


(a)



(b)

[Basic Idea] From a development of 2 boxes, we make **one more** box.

[Yes… with a trick!]
This idea works;
move a part of
the lid to 4 *sides*!

I put it at
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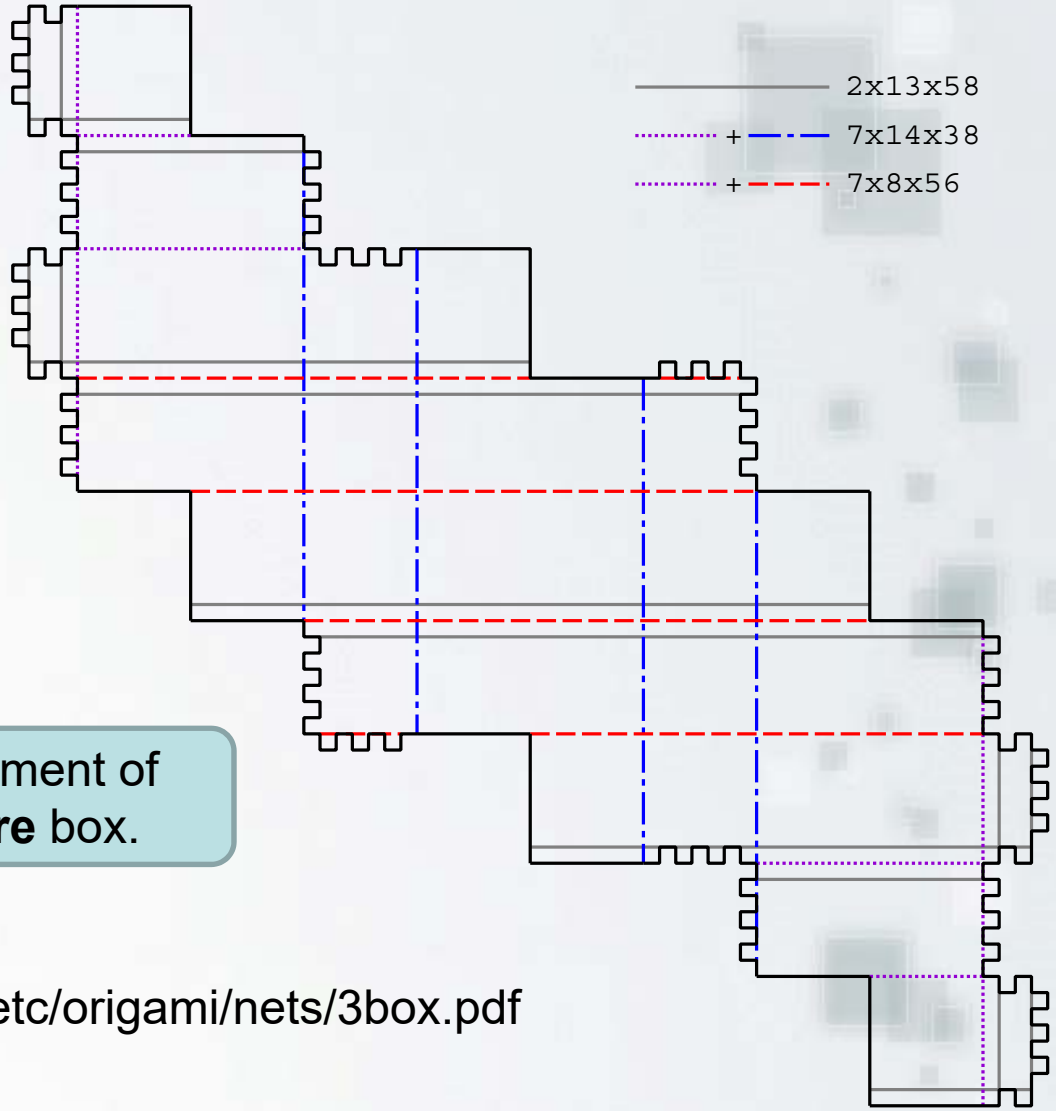

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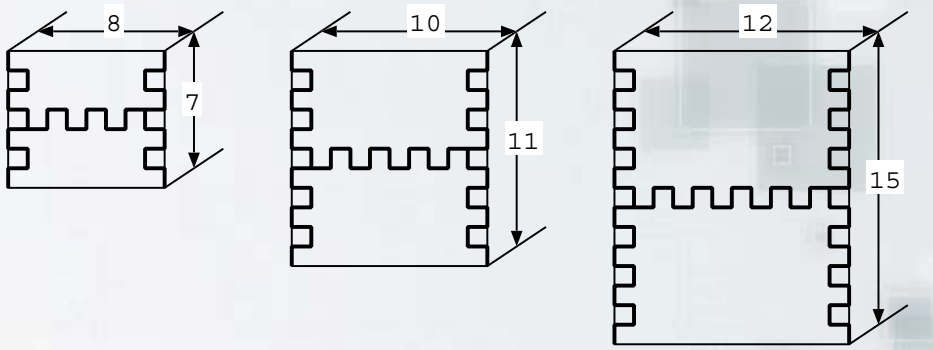

[Theorem]
There exist an infinite number of polygons that fold into 3 different boxes.

[Generalization]
- Basic box is flexible for the edge lengths.
- Zig-zag pattern can be extended.

I put it at
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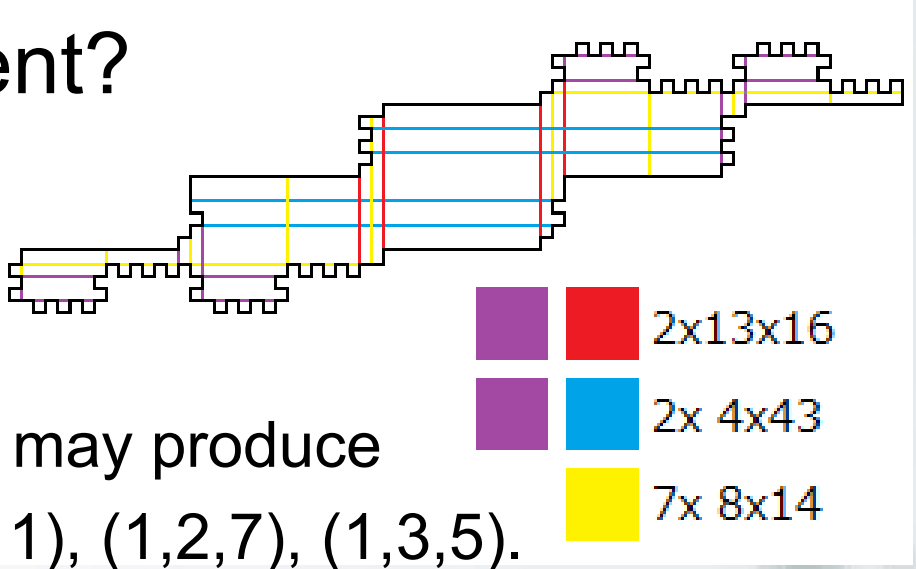
Future works

- Smallest development?

The current “smallest” development requires **532** squares.

>> the smallest area **46** that may produce three boxes of size (1,1,11), (1,2,7), (1,3,5).



(Remind:

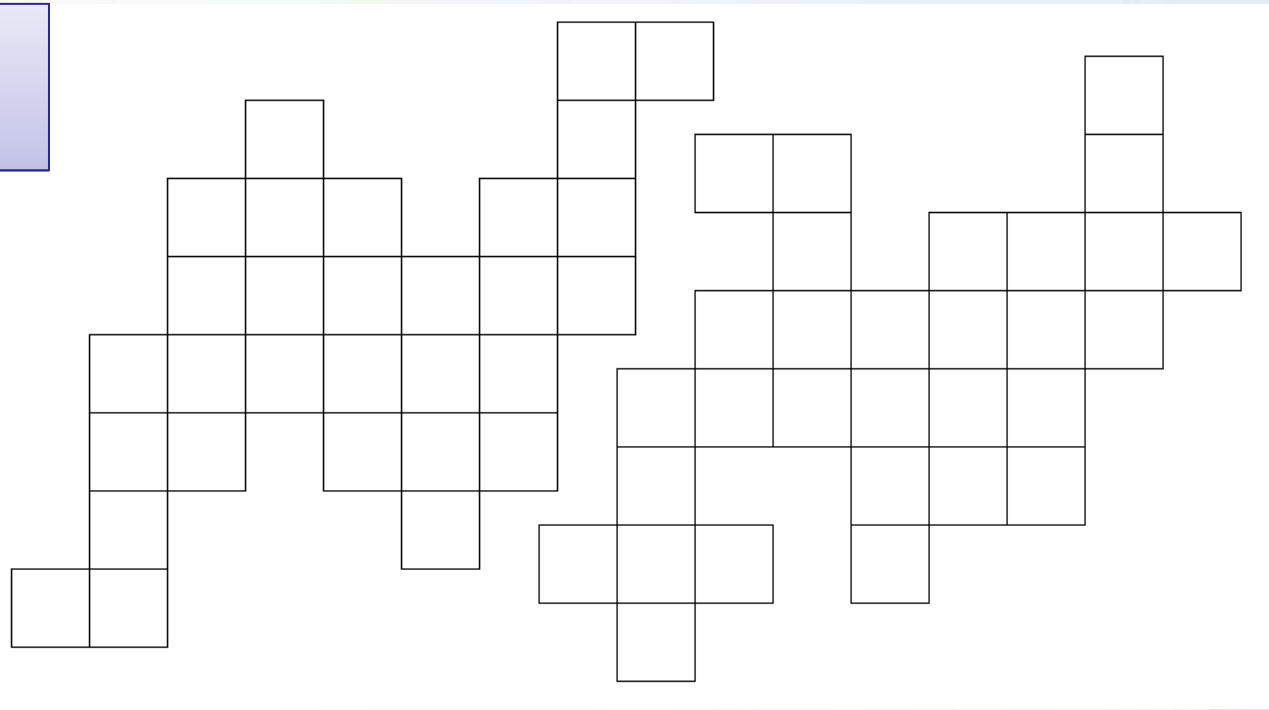
2263 polygons of area 22 folding to (1,1,5), (1,2,3))

Is there a polygon that folds to 4 or more boxes?

2012年10月23日：白川さんからのメール

「面積30で、1x1x7と√5x√5x√5の2通りの箱を折れる展開図を見つけました。この面積は1x3x3も作れるので、斜めを許した場合3通りの箱が折れる最小のポリオミノになる可能性があります。」

おまけ問題：
2通り折ってみよう



Note:

If you try to find for three boxes,

If you try to find for four boxes,

Surface areas;

Area	Trios	Area	Trios
22	(1,1,5),(1,2,3)	46	(1,1,11),(1,2,7),(1,3,5)
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known results

面積30は微妙な数字…

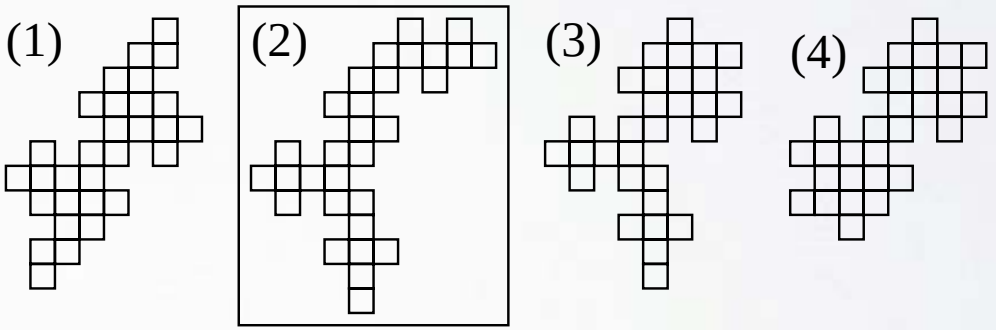
2011年当時の松井君のプログラム:

- 面積22の展開図を全探索:
  - 1×1×5と1×2×3の箱を折る2263個の展開図
- 実行時間はパソコンで10時間

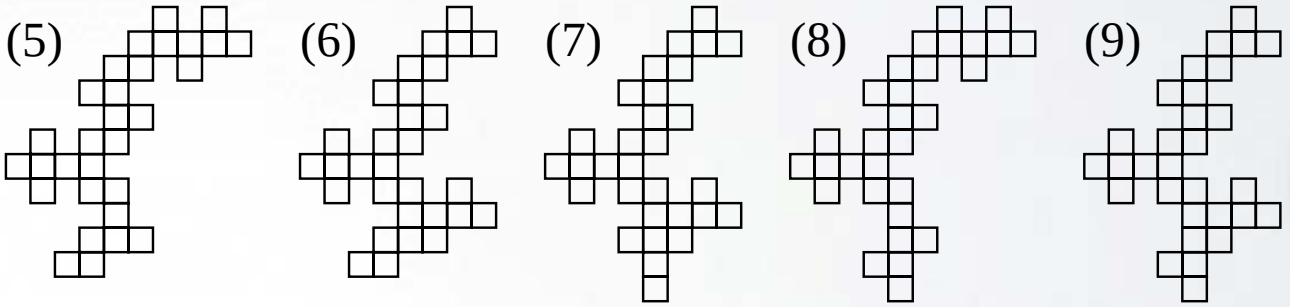
Dawei君の結果(2014年6月)

- 面積30の展開図の全探索に成功！[Xu, Horiyama, Shirakawa, Uehara 2015]
- 大雑把な結果
  - JAISTのスパコン（Cray XC 30）で二ヶ月
  - 1×1×7の箱と1×3×3の箱が折れる共通の展開図は1080個
  - そのうち、√5×√5×√5の三つ目の箱が折れる展開図は9個

追記：その後BDDの使用により[PCで10日]というレベルの驚愕の高速化に成功.



演習問題(？)
(2)だけ特別です。



まとめと課題

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2011年，面積22の展開図の全探索はPCで10時間.
2014年，面積30の展開図の全探索はスパコンで2ヶ月.
BDDを使うとPCで10日間.
…この調子で46まで行くのは難しいと言わざるをえないが…?