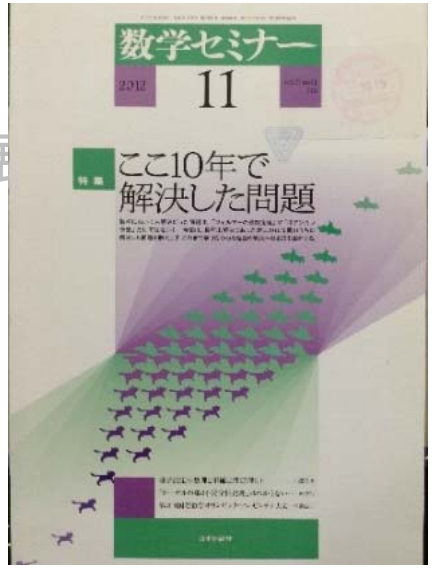
A. 展開図とそこから折れる凸立体の研究

1. 複数の箱が折れる共通の展開図
   - 2通りの箱が折れる共通の展開図
   - 3通りの箱が折れる共通の展開図
     そして….残された未解決問題たち
2. 正多面体の共通の展開図
3. 正多面体に近い立体と正4面体の共通の展

この雑誌に載ってます！

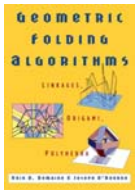


このミステリー（？）の中でメイントリックに使われました！

主な文献

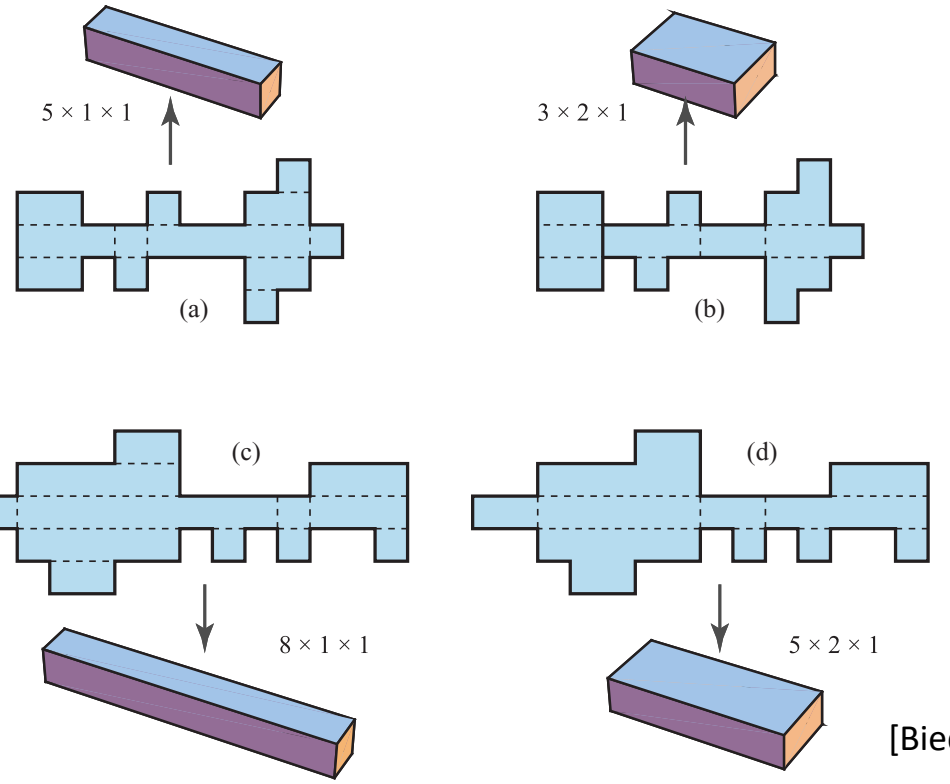
- Dawei Xu, Takashi Horiyama, Toshihiro Shirakawa, Ryuhei Uehara: Common Developments of Three Incongruent Boxes of Area 30, *COMPUTATIONAL GEOMETRY: Theory and Applications*, Vol. 64, pp. 1-17, August 2017.
- Toshihiro Shirakawa and Ryuhei Uehara: Common Developments of Three Incongruent Orthogonal Boxes, *International Journal of Computational Geometry and Applications*, Vol. 23, No. 1, pp. 65-71, 2013.
- Zachary Abel, Erik Demaine, Martin Demaine, Hiroaki Matsui, Guenter Rote and Ryuhei Uehara: Common Developments of Several Different Orthogonal Boxes, *Canadian Conference on Computational Geometry* (CCCG' 11), pp. 77-82, 2011/8/10-12, Toronto, Canada.
- Jun Mitani and Ryuhei Uehara: Polygons Folding to Plural Incongruent Orthogonal Boxes, *Canadian Conference on Computational Geometry* (CCCG 2008), pp. 39-42, 2008/8/13.

いくつかの展開図は以下で公開:
http://www.jaist.ac.jp/~uehara/etc/origami/nets/index-e.html

In 

,

- There were two developments that fold to two boxes;



- Are they exceptional?
- Is there any development that fold to 3 or more boxes??

[Biedl, Chan, Demaine, Demaine, Lubiw, Munro, Shallit, 1999]

Developments of two boxes

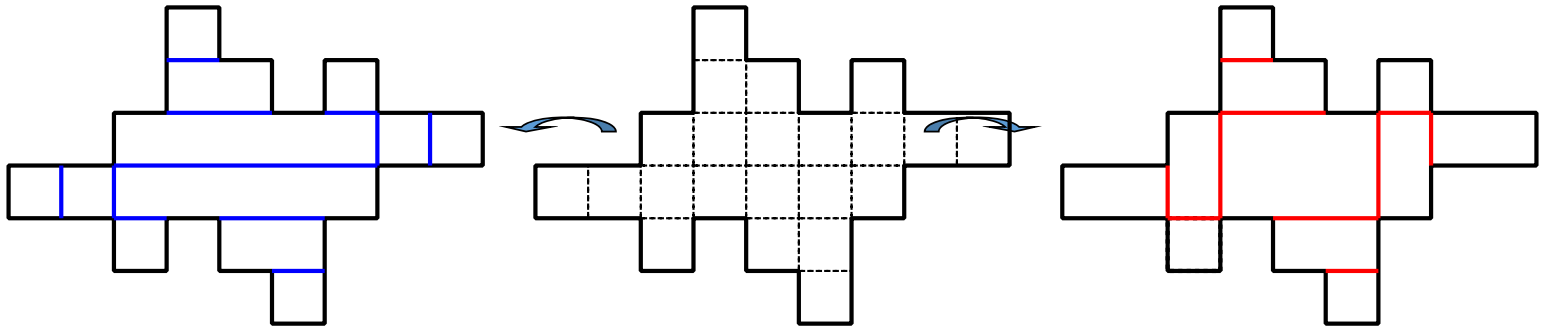
In [Uehara, Mitani 2007], randomized algorithm that looks for such polygons by *brute force*;

- Polygons folding into 2 boxes:
  1. There are
     many (～9000)
     (by supercomputer (SGI Altix 4700))
  2. Theoretically,
     infinitely many

Note:

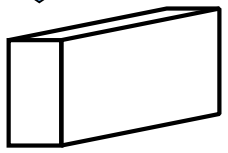
Example:
1×1+1×5+1×5
=1×2+2×3+1×3
=11（surface area: 22)

- Polygons folding to 2 different orthogonal boxes



1×1×5
= a × b × c

- We fold/(cut) at an edge of **unit squares**
- Surface area:
$$2(ab+bc+ca)$$
- Necessary condition:



1×2×3
= a' × b' × c'

$$ab+bc+ca=a'b'+b'c'+c'a'$$

It seems to be better to have many combinations…

Note:

Surface areas;

If you try to find for three boxes,

If you try to find for four boxes,

Area	Trios	Area	Trios
22	(1,1,5),(1,2,3)	46	(1,1,11),(1,2,7),(1,3,5)
30	(1,1,7),(1,3,3)	70	(1,1,17),(1,2,11),(1,3,8),(1,5,5)
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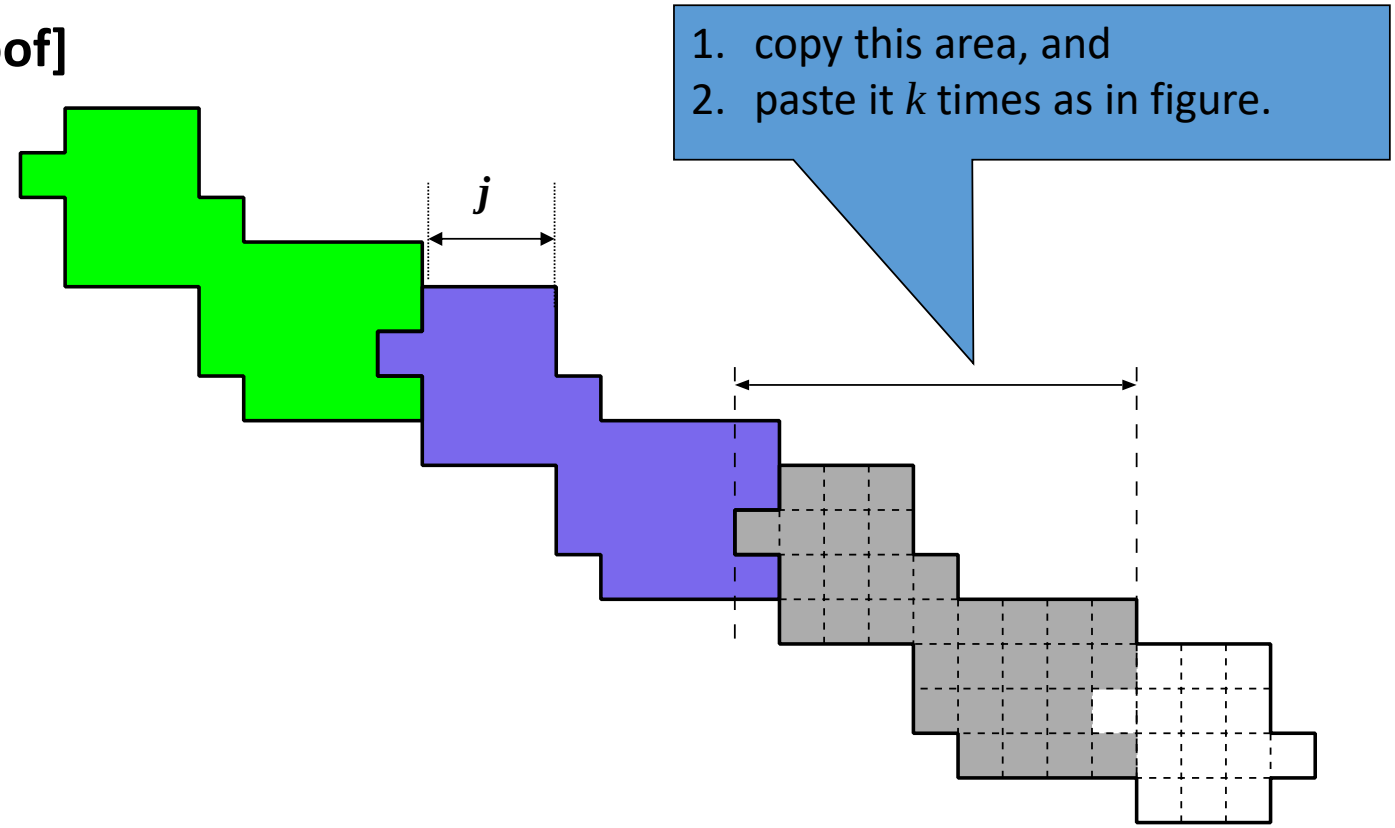
known results

Developments of two boxes

[Thm] There exists an infinitely many developments that fold to 2 boxes.

[Proof]



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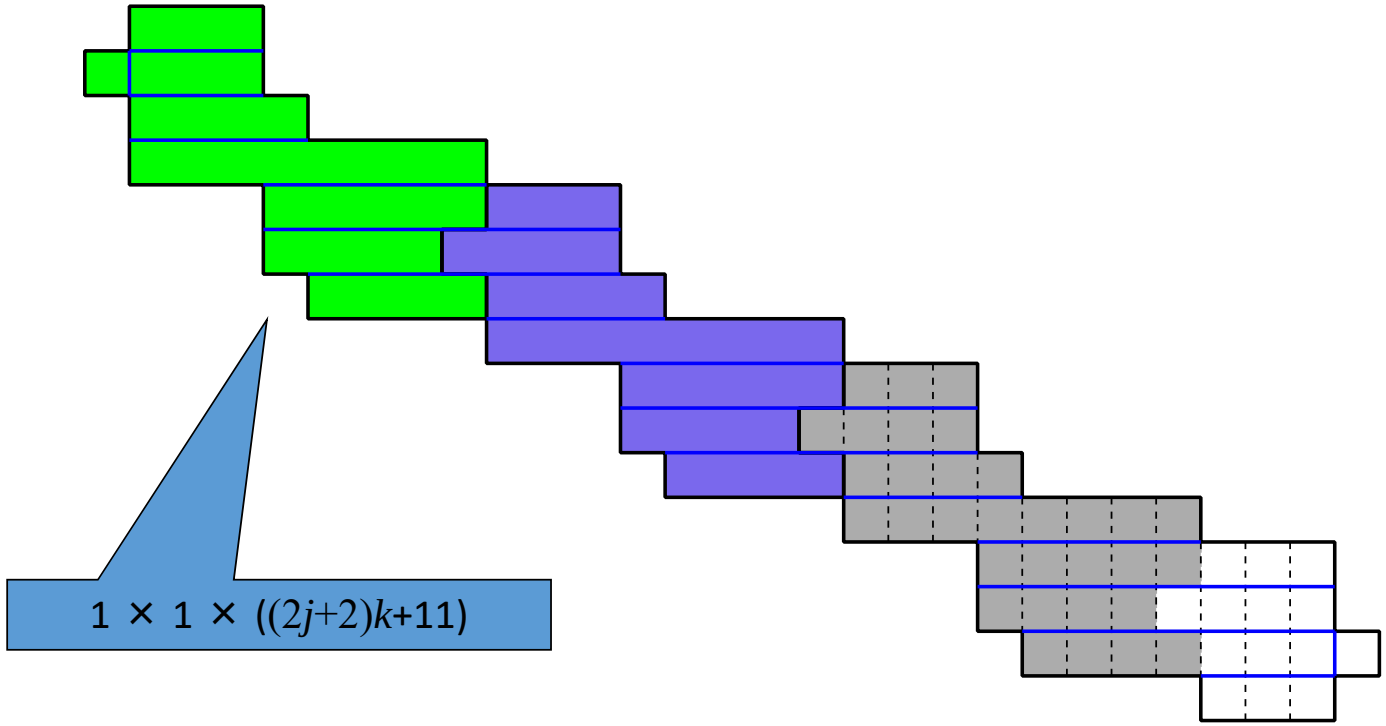

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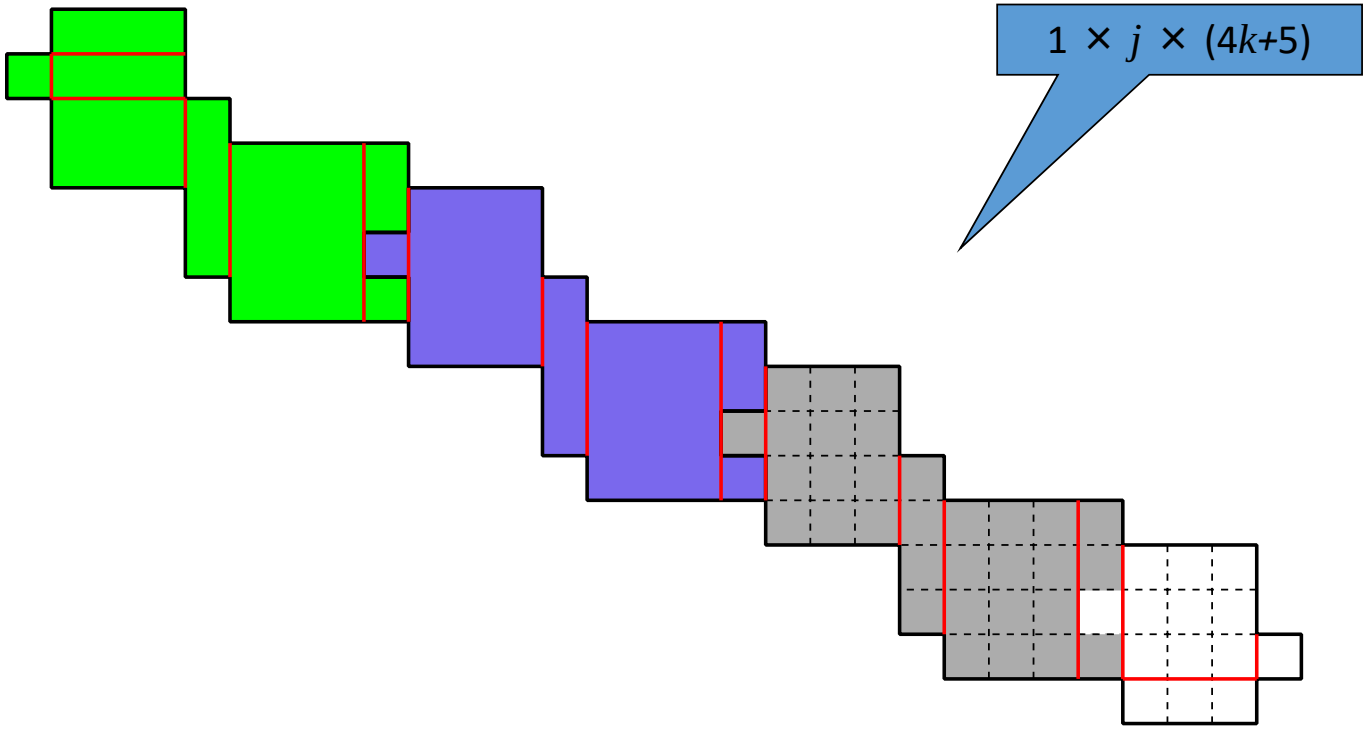
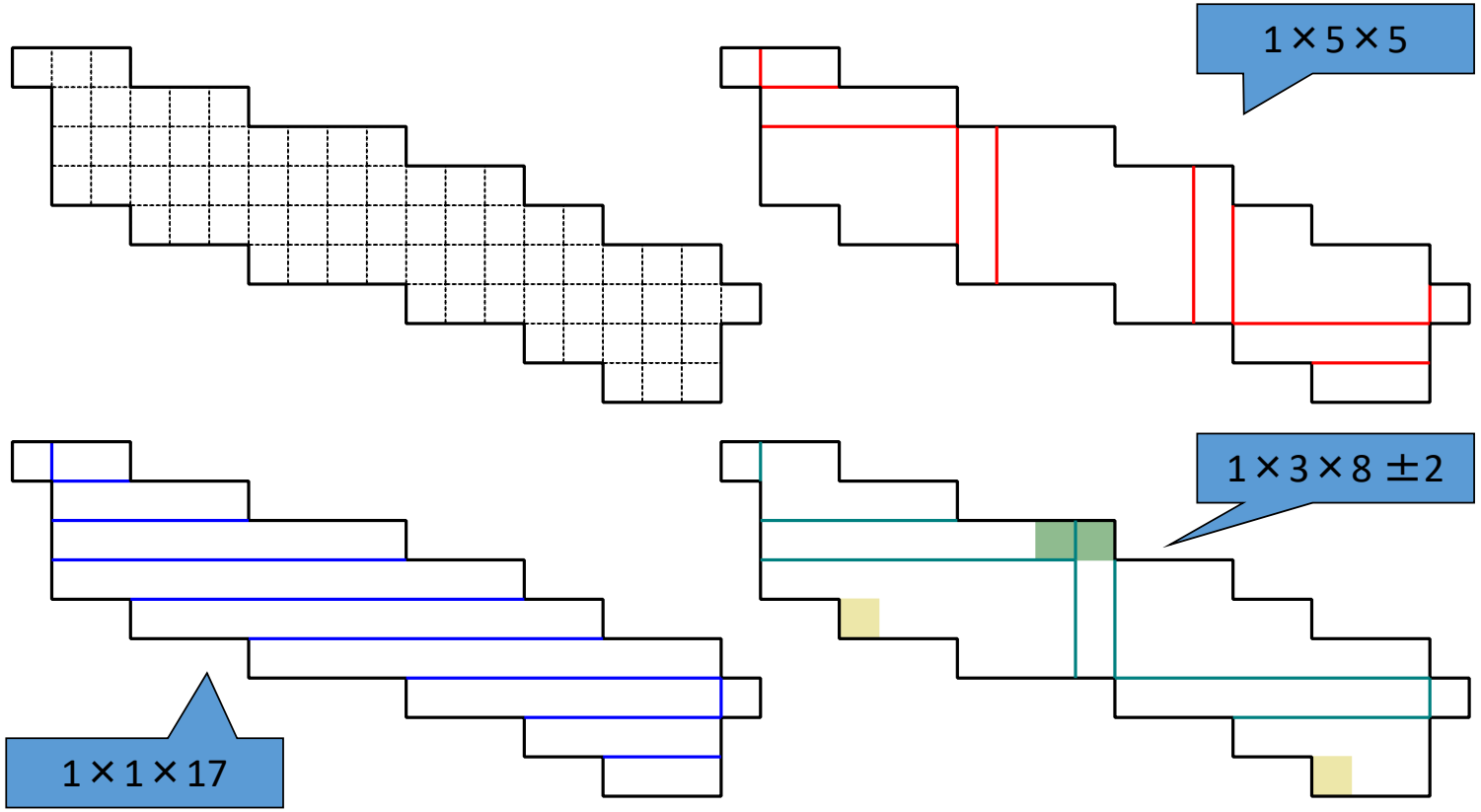


Developments of *three* boxes(?)

- A polygon that can fold to **three** distinct boxes…?
  - close solution…



Developments of *three* boxes(?)

- In [Abel, Demaine, Demaine, Matsui, Rote, Uehara 2011],
  - The number of developments that fold to $1 \times 1 \times 5$ box and $1 \times 2 \times 3$ box is 2263.

    the latest algorithm runs in around 10 hrs.
  - Among them, there is only one pearl development…

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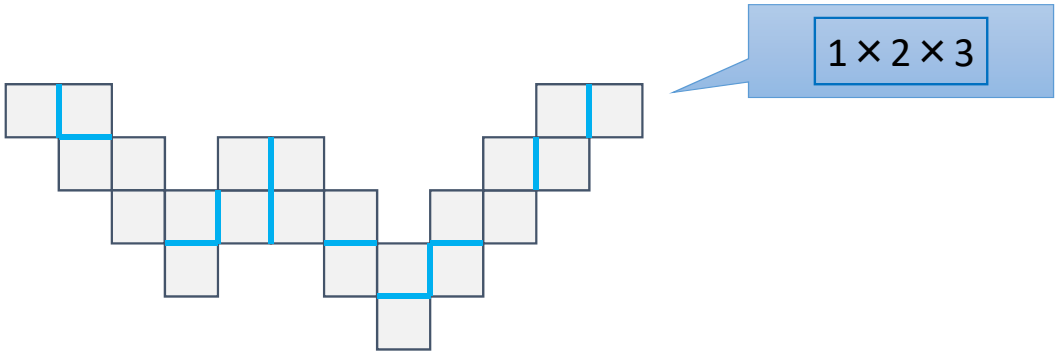

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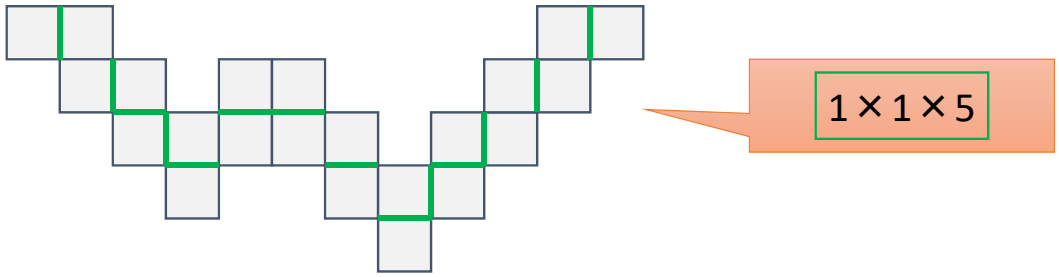

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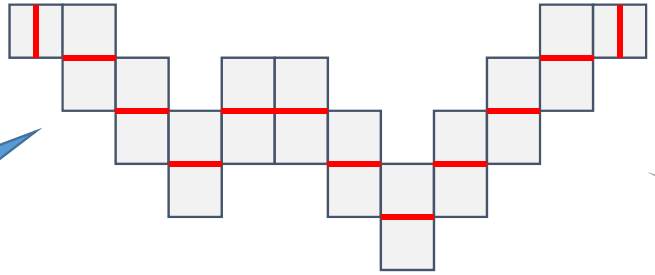
おまけ問題:
3通り折ってみよう

Is the "box" *cheat* having volume 0?

If you don't like ½, refine each square into 4 squares

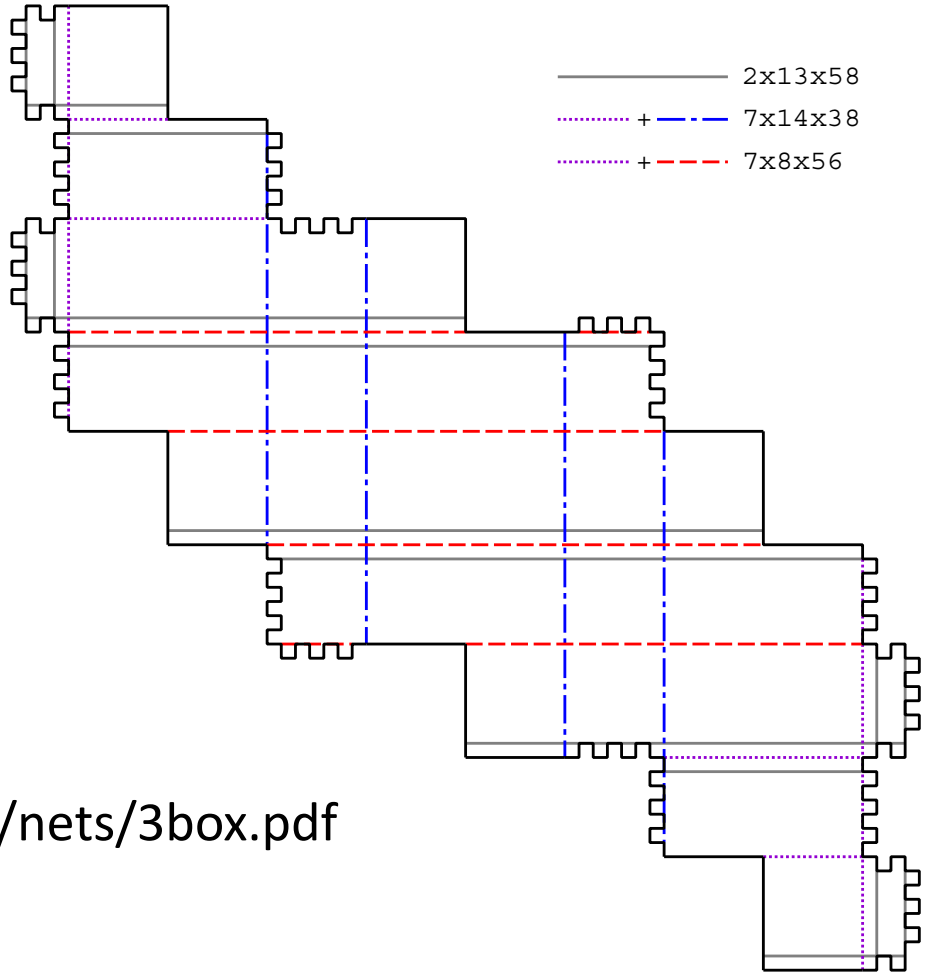


Since each column has height 2 except both sides

1 × 11 × 0 !

Developments of *three* boxes(!)

- In February 2012, Shirakawa
  (and I) finally found
  a polygon that folds to 3 boxes!!



[Basic Idea] From a development of
2 boxes, we make **one more** box.

I put it at
http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf

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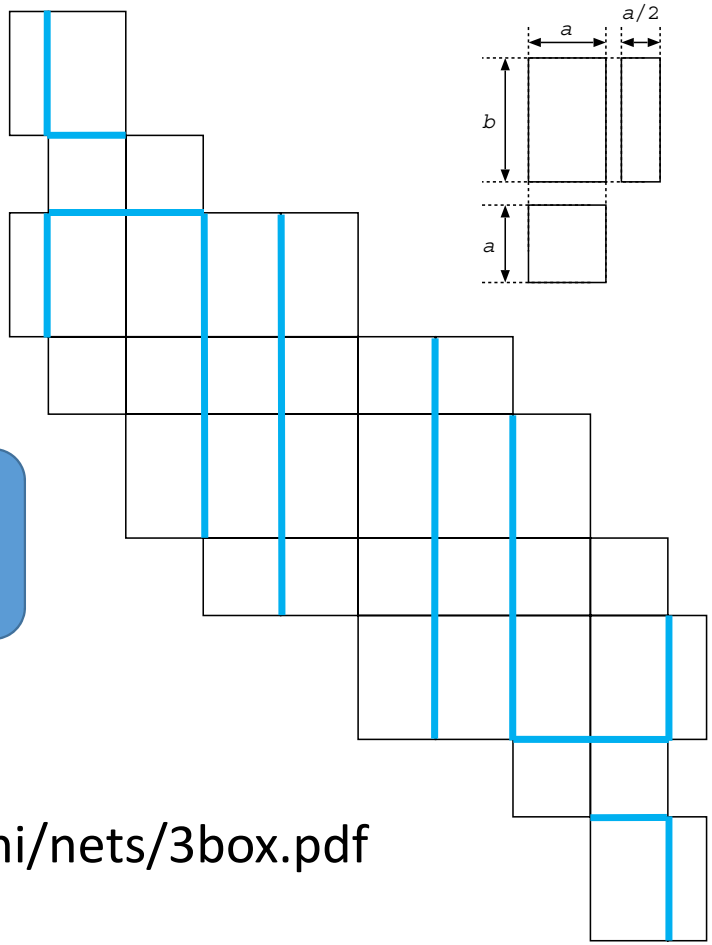

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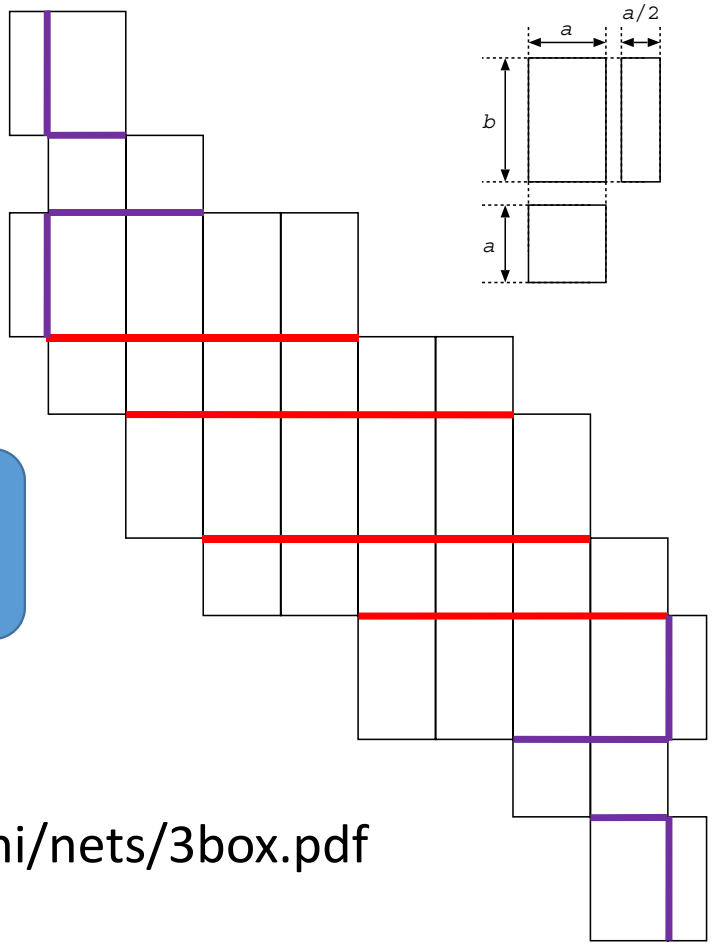

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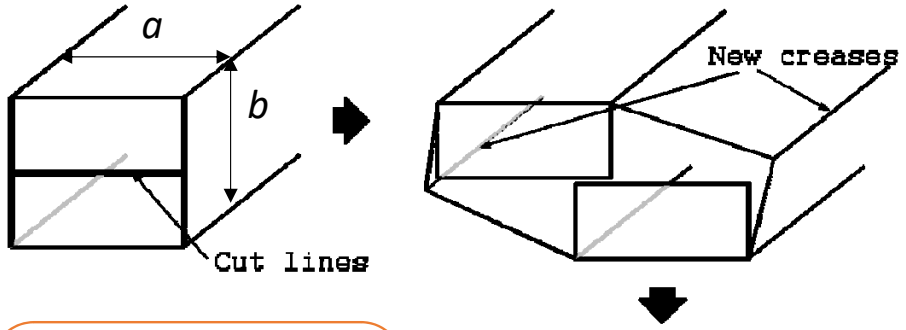

[Basic Idea] From a development of 2 boxes, we make **one more** box.

One more box is obtained by this *squashing!?*

[No!!]
This works iff $a=2b$, i.e.,
from 1×2 square to 2×1 square

I put it at
http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf

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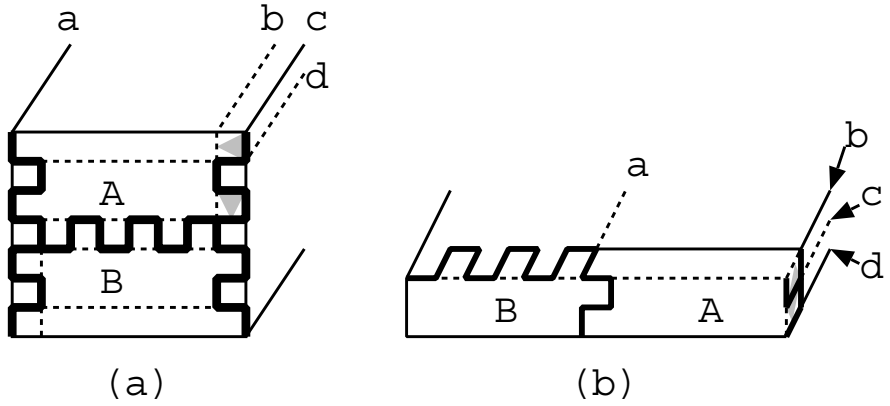

[Basic Idea] From a development of 2 boxes, we make **one more** box.

[Yes… with a trick!]
This idea works;
move a part of
the lid to 4 *sides*!

I put it at
http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf

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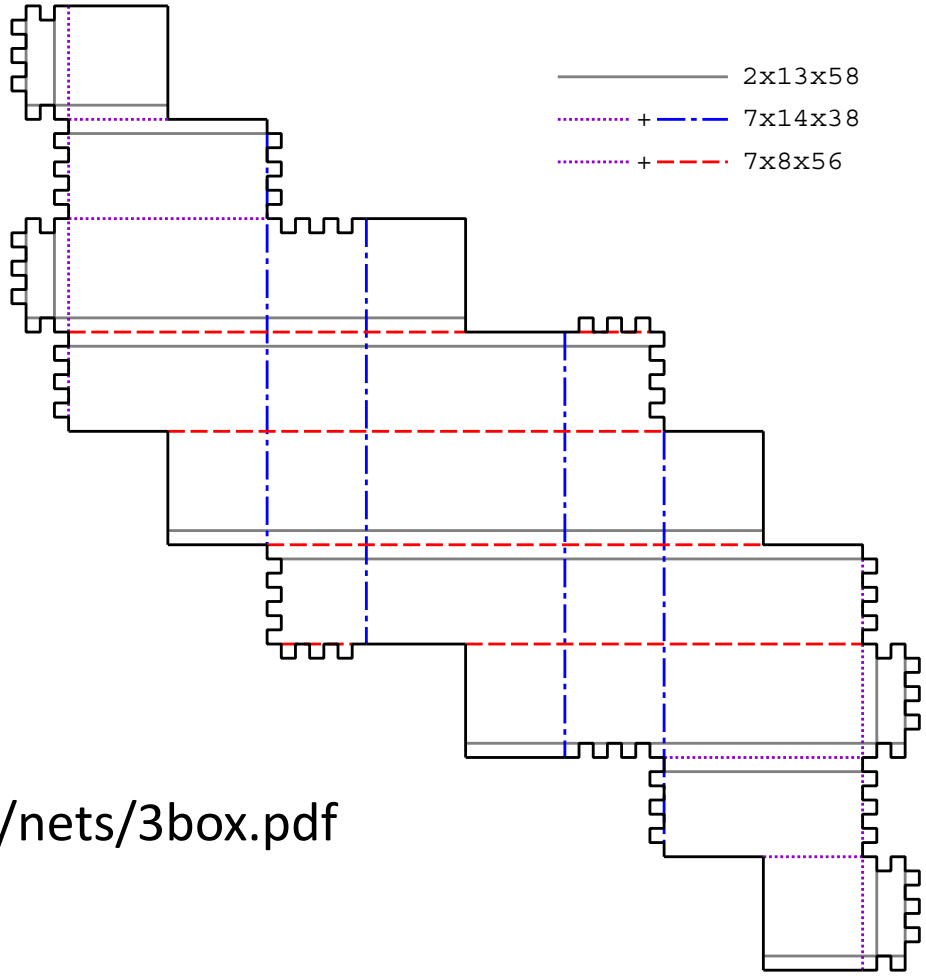

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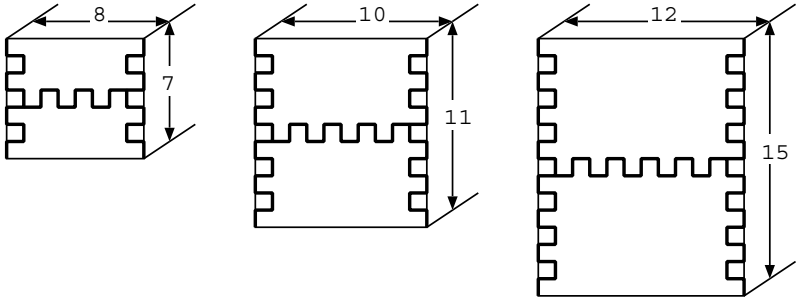


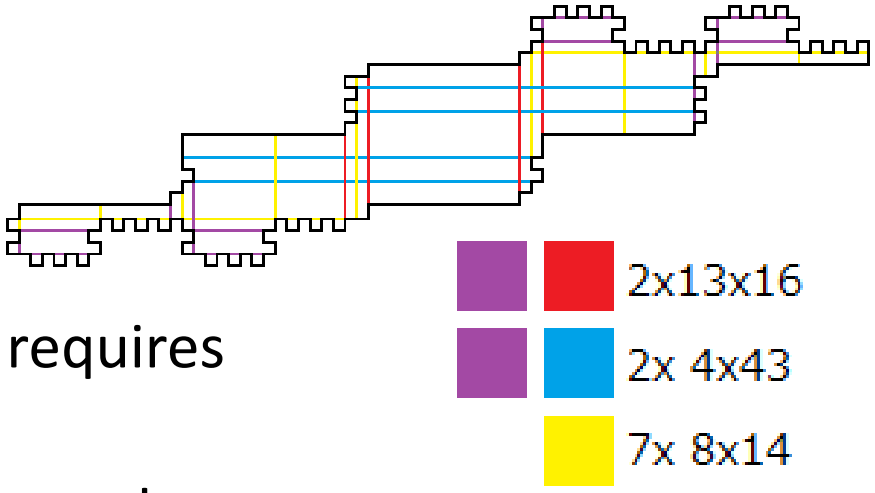
[Basic Idea] From a development of 2 boxes, we make **one more** box.

[Theorem]
There exist an infinite number of polygons that fold into 3 different boxes.

[Generalization!]
- Basic box is flexible for the edge lengths.
- Zig-zag pattern can be extended.

I put it at
http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf

Future works



- Smallest development?

The current “smallest” development requires

532 squares in this method.

>> the smallest area **46** that may produce

three boxes of size (1,1,11), (1,2,7), (1,3,5).

(Remind:

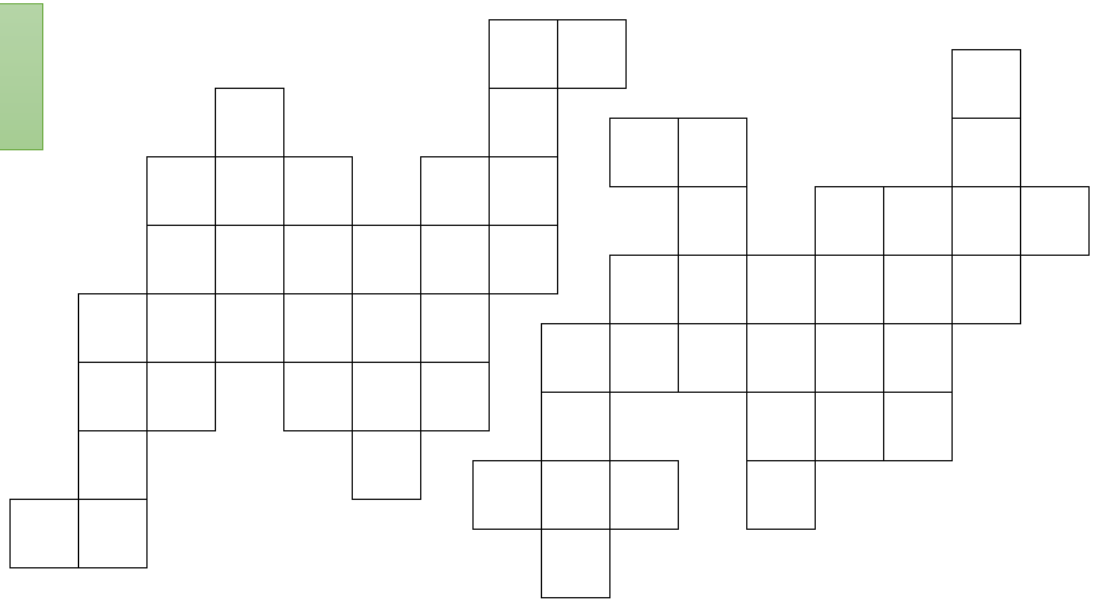
2263 polygons of area **22** folding to (1,1,5), (1,2,3))

Is there a polygon that folds to 4 or more boxes?

2012年10月23日 : 白川さんからのメール

「面積30で、1x1x7と√5x√5x√5の2通りの箱を折れる展開図を見つけました。この面積は1x3x3も作れるので、斜めを許した場合3通りの箱が折れる最小のポリオミノになる可能性があります。」

おまけ問題:
2通り折ってみよう



Note:

- Surface areas;

If you try to find for three boxes,

If you try to find for four boxes,

Area	Trios	Area	Trios
22	(1,1,5),(1,2,3)	46	(1,1,11),(1,2,7),(1,3,5)
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known results

面積30は微妙な数字…

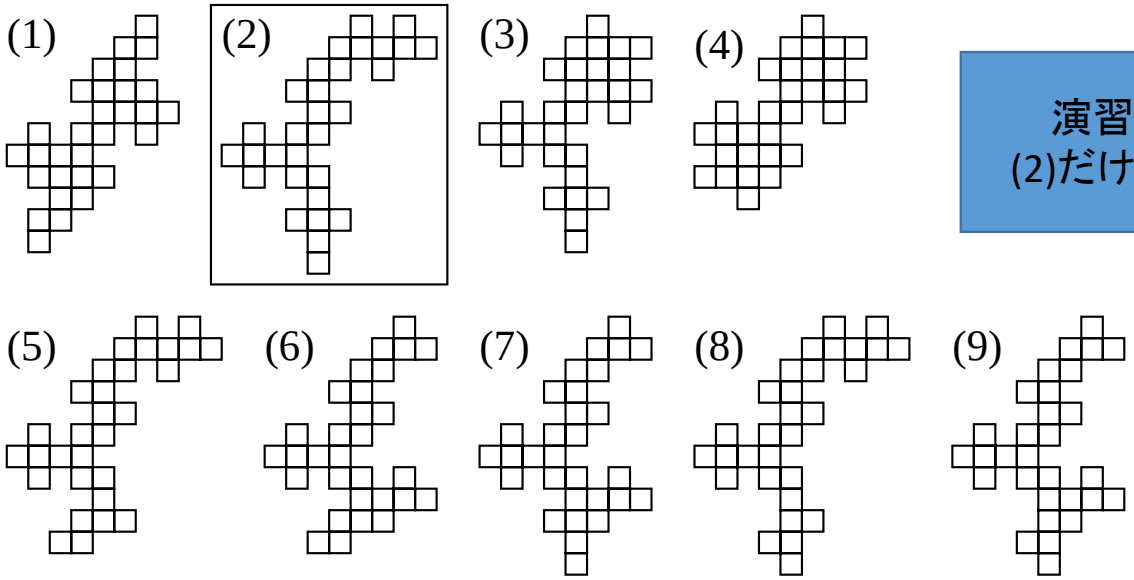
2011年当時の松井君のプログラム:

- 面積22の展開図を全探索:
  - 1×1×5と1×2×3の箱を折る2263個の展開図
- 実行時間はパソコンで10時間

Dawei君の結果(2014年6月)

- 面積30の展開図の全探索に成功！[Xu, Horiyama, Shirakawa, Uehara 2015]
- 大雑把な結果
  - JAISTのスパコン（Cray XC 30）で二ヶ月
  - 1×1×7の箱と1×3×3の箱が折れる共通の展開図は1080個
  - そのうち、√5×√5×√5の三つ目の箱が折れる展開図は9個

追記：その後BDDの使用により[PCで10日]というレベルの驚愕の高速化に成功.



演習問題(？)
(2)だけ特別です。

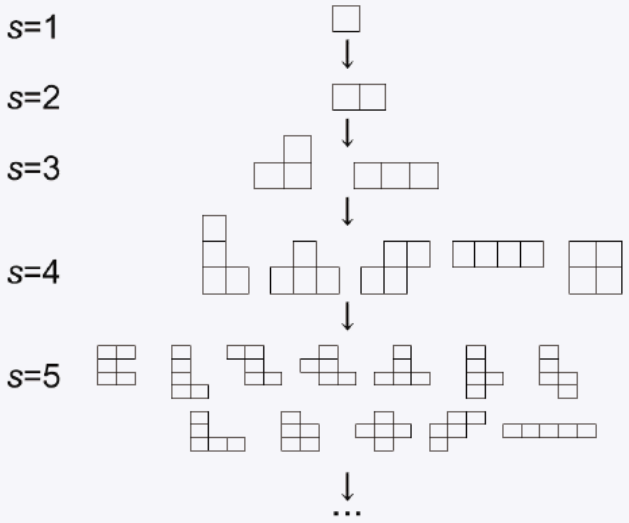
Dawei君の博士号公聴会（2017年6月15日）のスライド原稿より

どうやって見つけるのか？

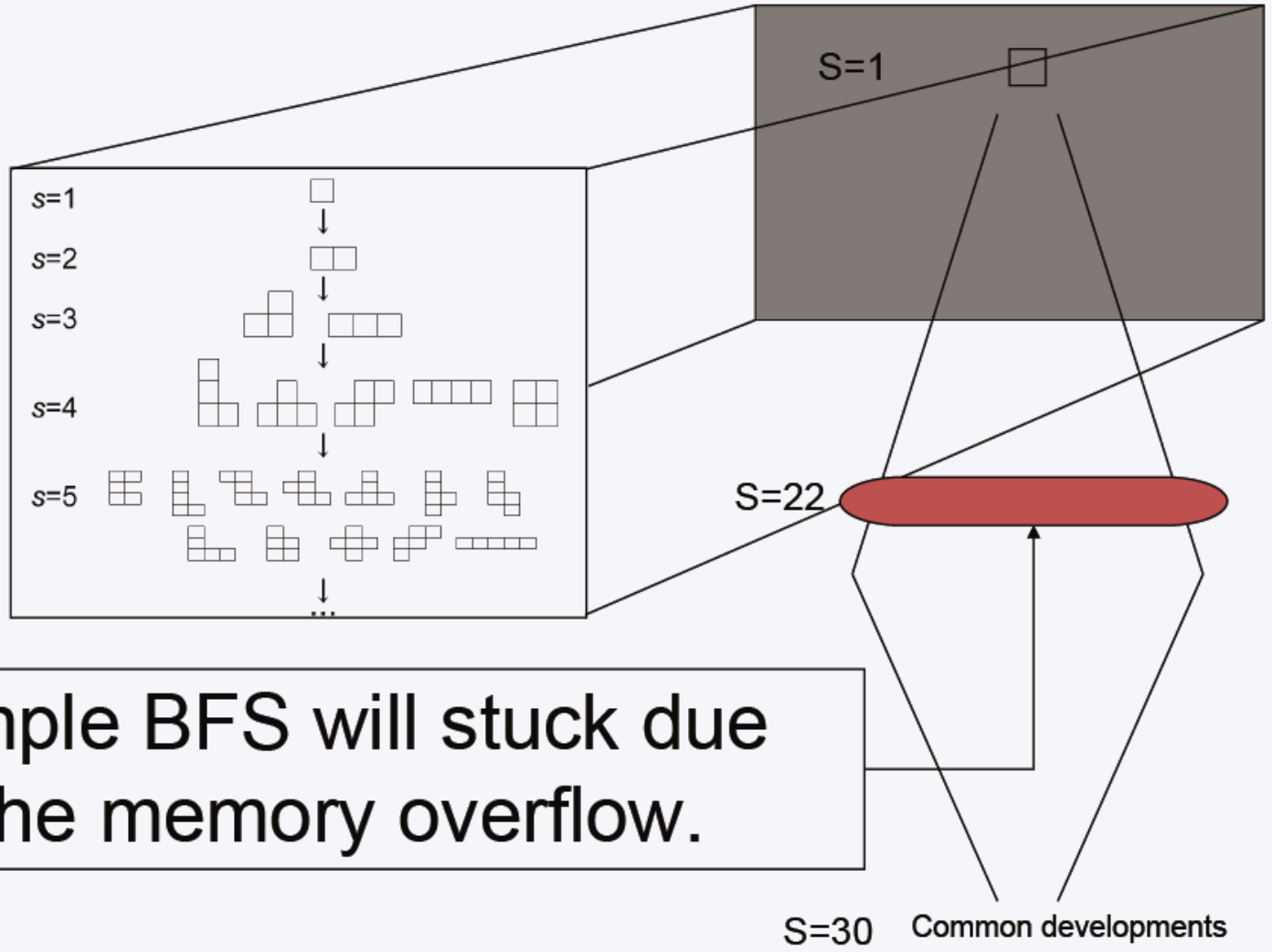
The enumerate approach

JAIST

- The basic idea is similar to finding two boxes of size 1×1×5 and 1×2×3 [6].
- We start from a single 1 square, then add another square adjacent to it, and extend the set of partial developments, repeat this step, untill 30 squares.

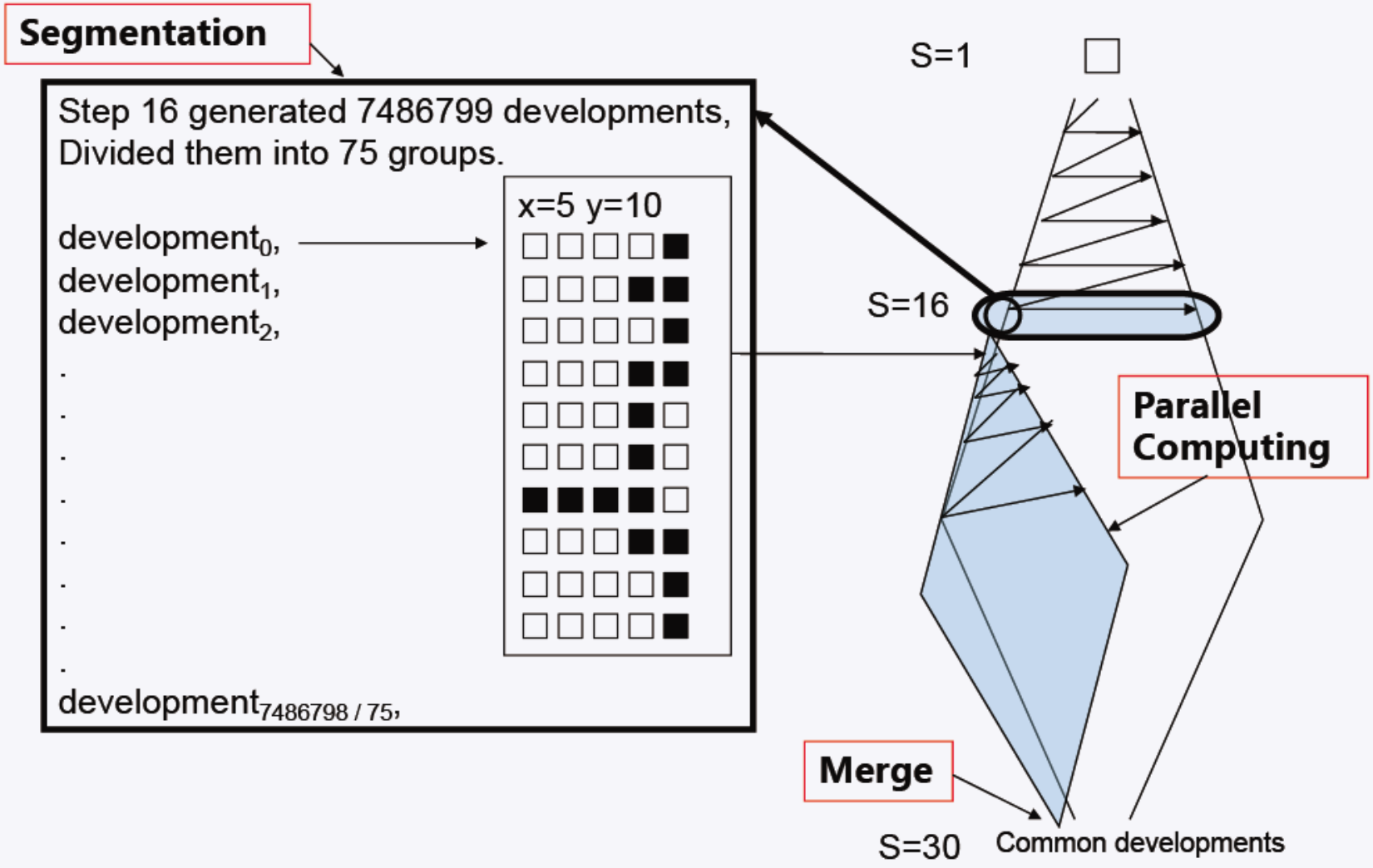


The simple BFS gets stuck



Simple BFS will stuck due to the memory overflow.

Our solution



まとめと課題

- Surface areas;

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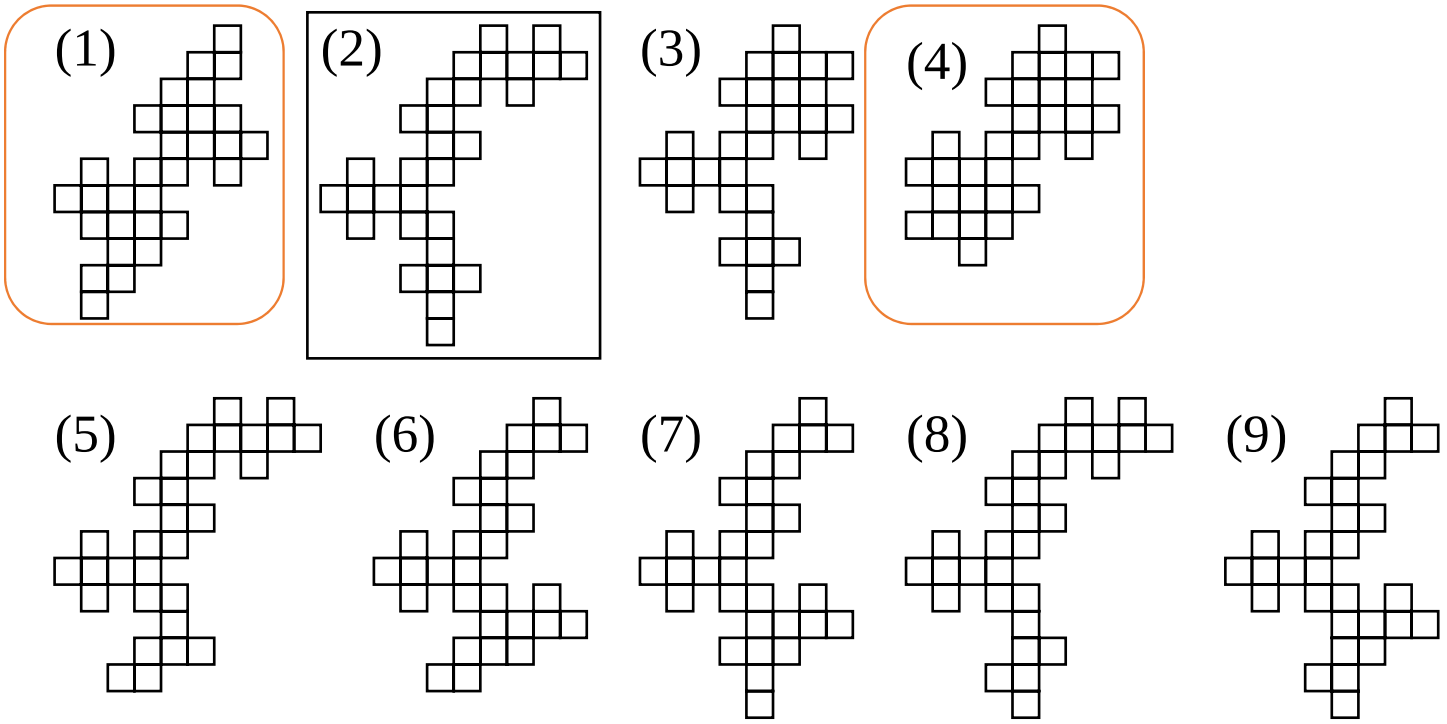
known results

2011年，面積22の展開図の全探索はPCで10時間.
2014年，面積30の展開図の全探索はスパコンで2ヶ月.
BDDを使うとPCで10日間.
…この調子で46まで行くのは難しいと言わざるをえないが…?

後日談

- Dawei君の博士論文の一部より
  - 回転対称な展開図に限定すれば、、、



後日談

- Dawei君の博士論文の一部より
  - 回転対称な展開図に限定すれば、、、
  1. 対称形に限定することで、探索対象を大幅に減らせる
  2. 展開図は半分サイズだけ覚えればよい
  3. 面積は2ずつ増やせる

（残念な）結論：面積46、54ではダメでした。

- 46: 1×1×11, 1×2×7, 1×3×5のうち、どの2つも折れる展開図があるけれど。。。
- 54: 1×1×13, 1×3×6, 3×3×3も同様。

未解決問題

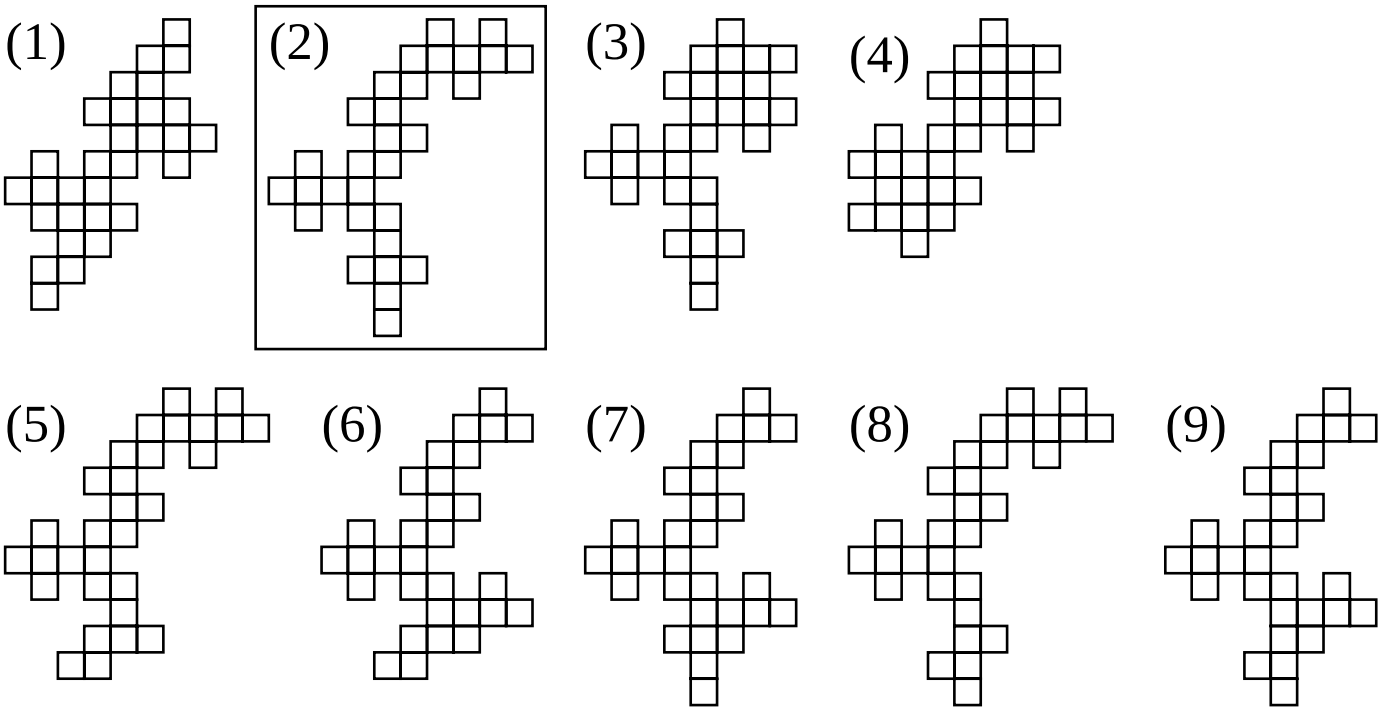
- 面積46や54で、3つの箱が折れる展開図はあるのか？
- ４つ以上の箱が折れる展開図は存在するのか？
- [k個以上の箱は折れない]という有限の上界はあるのか？(例えば10000通りの箱が折れる展開図などは想像しがたいけれど。。。)

おまけ情報:

- 「異なるポリオミノの数」は面積45まで既知(by 白川さん on OEIS)

演習問題

- （２）だけ特別なことを確かめてみよう



- ここで紹介した以外のアルゴリズムを考案してみよう（BDDなど）

発展問題

- ここで紹介しなかった問題へのアルゴリズムを考案してみよう:
  問題:多角形*P*が与えられたとき、そこから箱*Q*が折れるか？

既知(?)の結果:

- 一般の多角形Pと凸多面体Qの擬多項式時間アルゴリズムがあることはある
  - $O(n^{456.5})$時間アルゴリズム！(Kane, et al, 2009)
- Qが「大きさa×b×cの箱」でPはn多角形で糊付けも与えられたとき
  - O((n+m)log n) 時間アルゴリズム
  - 変数mは「一つの辺がm個にばらけている」という未知のパラメータ(Horiyama, Mizunashi 2017)
- a,b,cが不明のときはどうか？