本講義のトピック

そのA：展開図とそこから折れる凸立体の研究

- 展開図と立体のとても悩ましい関係：最大の未解決問題
- 与えられた「展開図」を折って作れる（凸）「立体」をどうやって計算するか？
  - 数学的な特徴づけ/アルゴリズム/計算パワー

そのB：「折り」のアルゴリズムと計算量の関係

- 折り紙の基本操作 ← 済
- 折り紙のアルゴリズムと計算量
  - １次元の紙における効率のよい折り方（アルゴリズムと計算量）
    - 高速に折るアルゴリズム（折る回数を減らせるか？）
    - 「良い折り畳み状態」を評価する指標のモデル
  - １次元の紙における計算不能性（計算の理論）
    - 計算モデル

そのB:「折り」のアルゴリズムと計算量の関係

5. 時間計算量
   - “Folding complexity”入門
     - 理論上、世界最速のジャバラ折りアルゴリズム

6. 領域計算量(?)
   - 切手折り問題
   - 折り目幅最小化問題
     - NP完全問題、FPTアルゴリズムなど
     - これも予想以上にコンピュータサイエンス!

7. (折り紙における決定不能問題)
   - 対角線論法と不完全性定理

A survey on computational complexity of finding good folded state with few crease width

Ryuhei Uehara (JAIST, Japan)

- Erik D. Demaine, David Eppstein, Adam Hesterberg, Hiro Ito, Anna Lubiw, Ryuhei Uehara and Yushi Uno: Folding a Paper Strip to Minimize Thickness, *The 9th Workshop on Algorithms and Computation (WALCOM 2015)*, Lecture Notes in Computer Science Vol. 8973, pp. 113-124, 2015/02/26-2015/02/28, Dhaka, Bangladesh.
- Takuya Umesato, Toshiki Saitoh, Ryuhei Uehara, Hiro Ito, and Yoshio Okamoto: Complexity of the stamp folding problem, Theoretical Computer Science, Vol. 497, pp. 13-19, 2012.

関連研究？

- 英語のイデオムで「紙を10回半分に折る」で「できないことの例え」になるらしい。

Folding Paper in Half 12 Times:
The story of an impossible challenge solved at the Historical Society office

Alice laughed: "There's no use trying," she said; "one can't believe impossible things." "I daresay you haven't had much practice," said the Queen.

Through the Looking Glass by L. Carroll

BRITNEY'S FOLDING RECORD STILL HOLDS

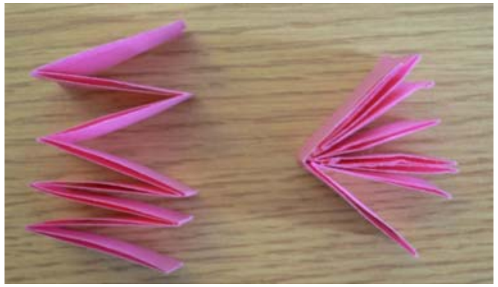
The long standing challenge was that a *single* piece of paper, no matter the size, cannot be *folded* in half more than 7 or 8 times. Recently, reports have been made that Britney's paper folding record of folding a piece of paper in half 12 times has been broken. These current attempts, though laudable and will eventually be

Photo of the 11th Fold, One More to go.

Minimization of Crease width

Good!

Bad!



Input: Paper of length $n+1$ and $s \in \{M, V\}^n$

Output: folded paper according to s

Goal: Find a *good* folded state with *few* *crease width*

- At each crease, the number of paper layers between the paper segments hinged at the crease is *crease width* at the crease
- Two minimization problems;
    - minimize maximum
    - minimize total (=average)

No!!

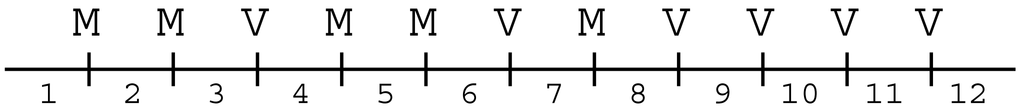
It seems simple, … so easy??

Cf. I'd checked that by brute force...

Crease width problem

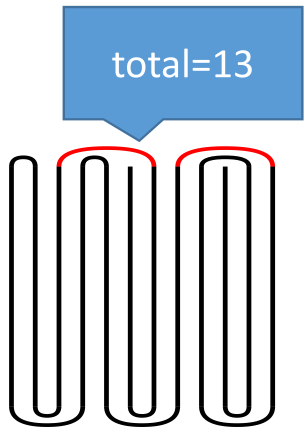
Simple non-trivial example (1)

Input: MMVMMVMVVVV

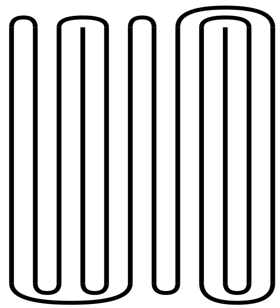


The number of feasible folded states : **100**

Goal: Find a *good* folded state with few *crease width*



- The **unique** solution having min. max. value 3
  [5|4|3|6|7|1|2|8|10|12|11|9]



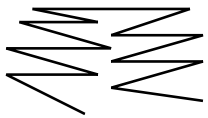
- The **unique** solution having min. total value 11
  [5|4|3|1|2|6|7|8|10|12|11|9]

Crease width problem

Simple non-trivial example (2)

A few facts;

- a pattern has a unique folded state iff it is pleats
- solutions of {min max} and {min total} are different depending on a crease pattern.
- there is a pattern having *exponential* combinations



This pattern is almost pleats, but it has exponentially many folded states….

Main Results

The crease width problem (unit interval)

- Min-Max problem: NP-complete

  (Reduction from 3-Partition)

- Min-Total problem:
  - Complexity is *still open*...
  - *Fixed Parameter Tractable algorithm*;

    it runs in $O((k+1)^k n)$ time, where k is the total crease width.

    the algorithm itself is natural, but analysis is bit tricky.

It is intractable even for small n...

It is solvable if k *is* quite small...

MinMax is NP-complete

Proof: Polynomial time reduction from 3-Partition.

<u>3-Partition</u>:

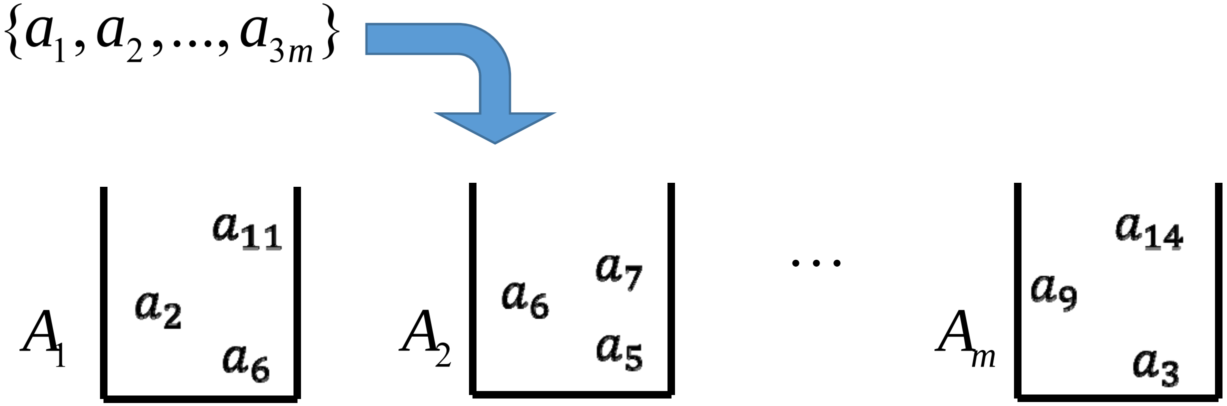
$(B/4 < a_j < B/2)$

Input: Set of integers $A = \{a_1, a_2, \ldots, a_{3m}\}$ and integer B

Question: Is there a partition of A to $A_1, \ldots, A_m$

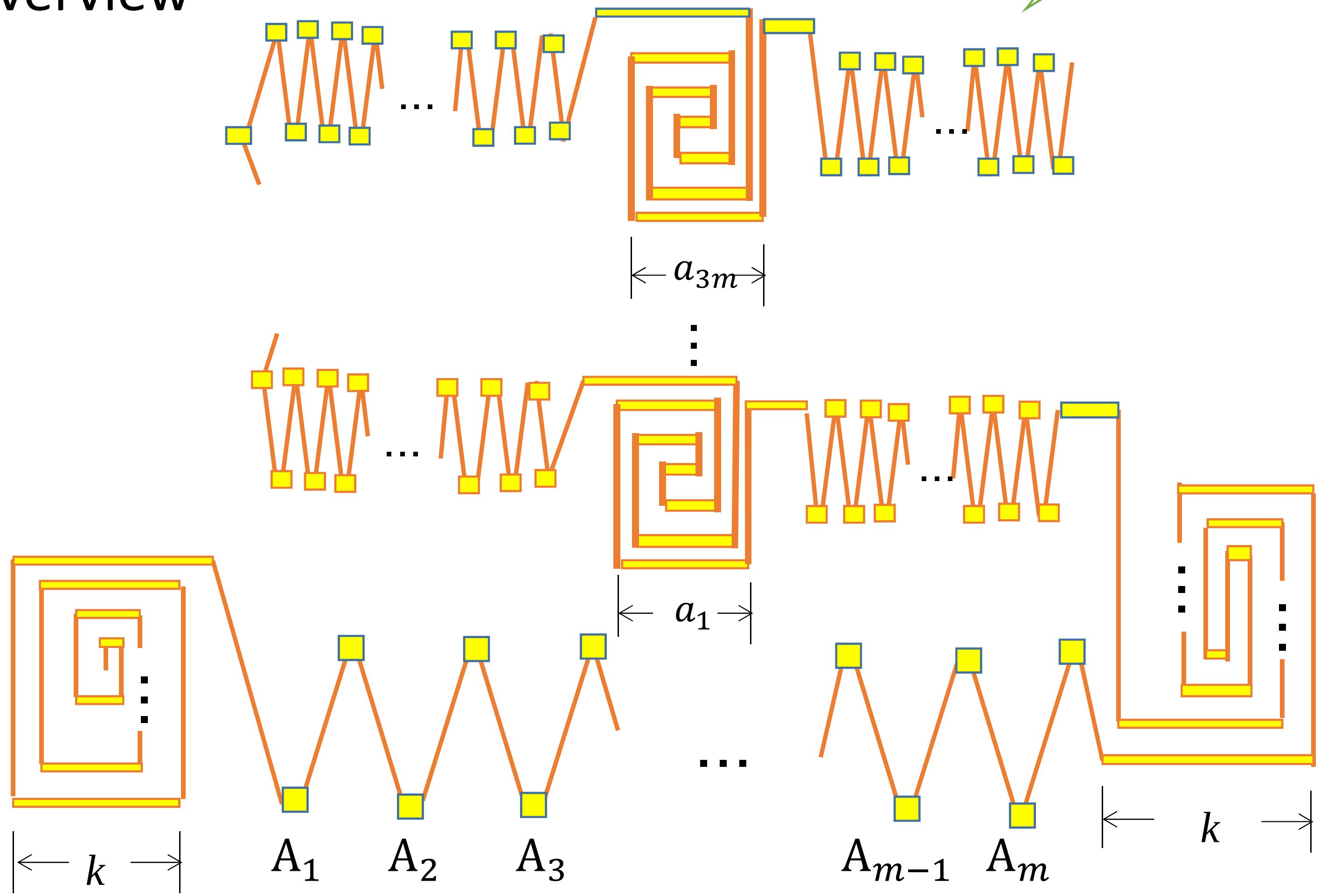
such that $|A_i| = 3$ and $\sum_{a_j \in A_i} a_j = B$

$A = \{a_1, a_2, \ldots, a_{3m}\}$



Construction for MinMax

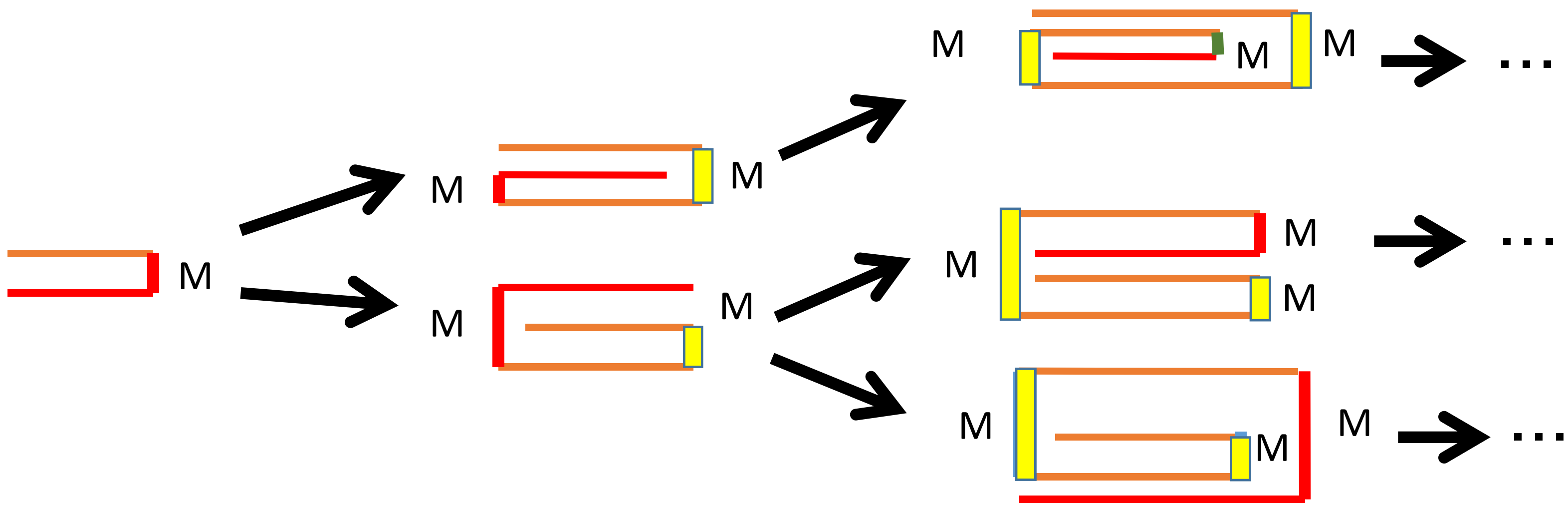
Overview



図がわかりにくいのであとで再放送。。。

(Poly-time) Algorithm for MinTotal

- Enumerate all folding ways with respect to the string s up to total crease width k.
- Each folded state is generated incrementally.



- Check the total crease width at each increment.

Running time

- The algorithm for given fixed total crease width k runs in $O(n^{2+k})$ time.

  at each crease, the sequence of c.w. is

  - $c_1, c_2, \ldots, c_i$, ... with $\sum_{i=1,2,\ldots} c_i \leq k$

  that is a partition of $\leqq k$
- With more careful (and complex) analysis shows that the algorithm runs in $O((k+1)^k\, n)$ time!!

That is, this is fixed parameter tractable algorithm!

Known results

最近ここで進展がありました。

Possible extensions in 2012;

1. Non-unit intervals between creases

   How can you measure the thickness of pile of various lengths?

2. 2-Dimenaional (…related to Map-folding?)

   How can you measure the crease-width in 2D?

3. Different Criteria for "space complexity"

   We have few ideas…

   (by Prof. Iwama: you can fold left $<k$ and right$>k$ creases in [1..n])

   (Area to fold for long-pipe folding)

Now we turn to･･･

- Quite simple model
  1. 1 dimensional
  2. ~~Unit length between~~ creases

Non-unit intervals!!

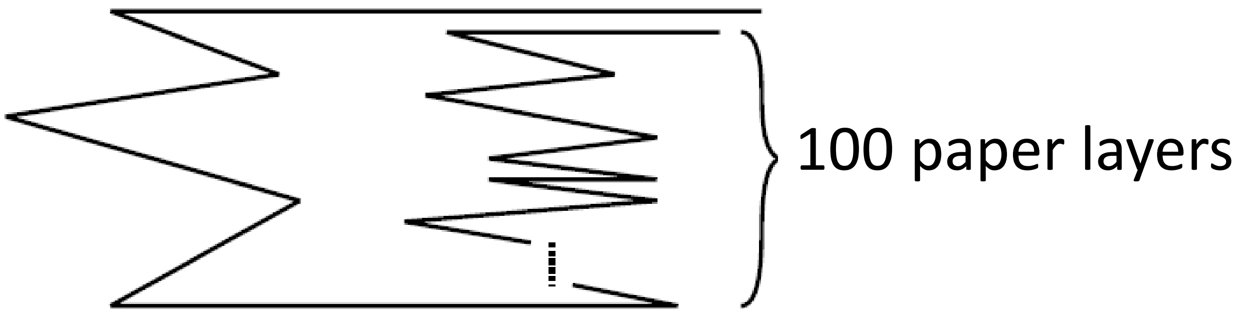
Not only M/V, but also lengths between creases are given

But,,, wait,,,

Erik D. Demaine, David Eppstein, Adam Hesterberg, Hiro Ito, Anna Lubiw, Ryuhei Uehara and Yushi Uno: Folding a Paper Strip to Minimize Thickness, *The 9th Workshop on Algorithms and Computation (WALCOM 2015)*

For non-unit interval creases…

Crease width = the number of paper layers at a crease?



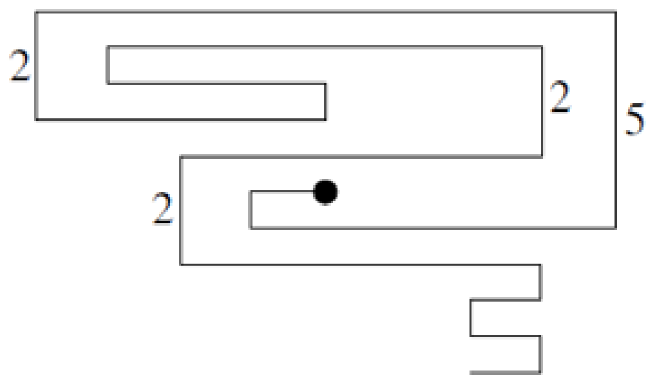
How can we count the paper layers?

For non-unit interval creases…

Crease width = the number of paper layers at a crease?

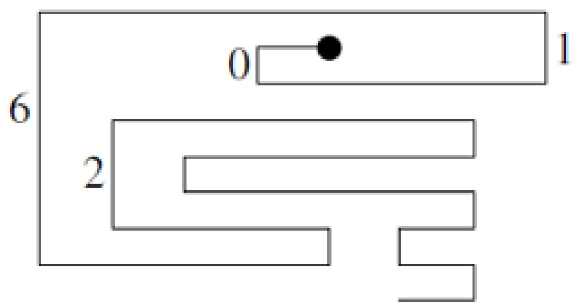
- We introduce three new "widths" of a folded state:
  - Two are natural extensions of Max-CW and Total-CW; one is totally new!
- For VMVMVVMMMM, e.g., we have;

Minimum *max crease width*



$h = 11, m = 5, t = 11$

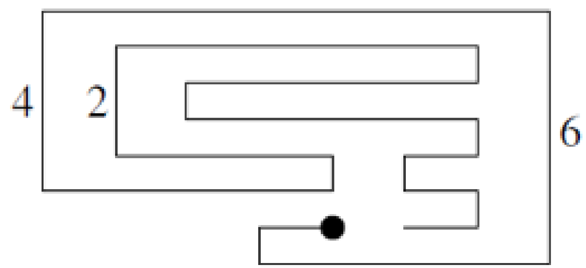
Minimum *total crease width*



$h = 9, m = 6, t = 9$

New

Minimum **height**



$h = 8, m = 6, t = 12$

Main results in (DEHILUU 2015)

Summary

	Unit interval model in [USUIO2012]	General model in (DEHILUU 2015)
max crease width	NP-complete	⇒ NP-complete
total crease width	open	NP-complete [DEHILUU 2015]
height	trivial	NP-complete [DEHILUU 2015]

Proof Idea

Minimize height is NP-complete

Proof: Polynomial time reduction from 3-Partition.

<u>3-Partition</u>:

$(B/4 < a_j < B/2)$

Input: Set of integers $A = \{a_1, a_2, \dots, a_{3m}\}$ and integer B

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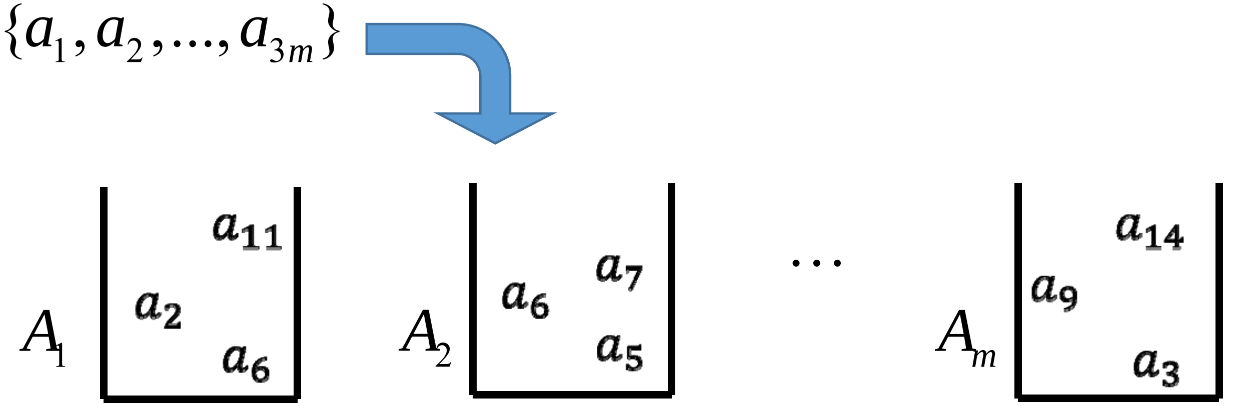
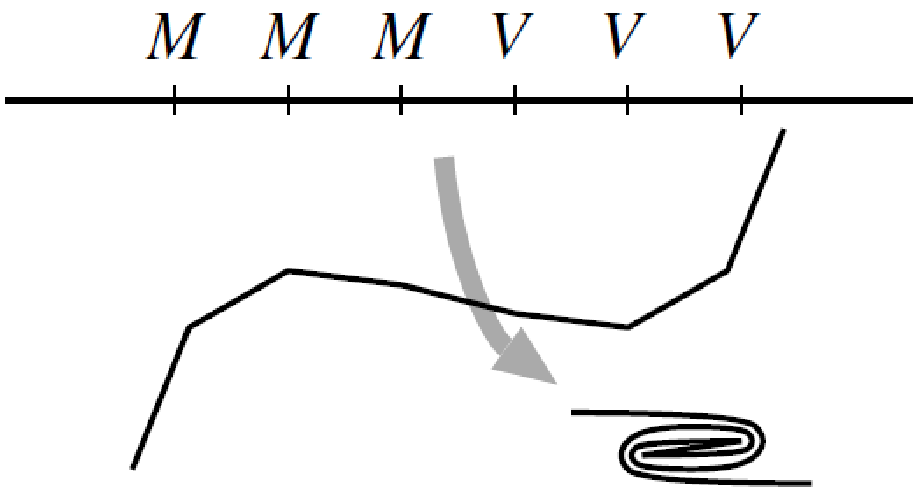


Minimize height is NP-complete

Proof: Polynomial time reduction from 3-Partition.

<u>Basic gadget</u>

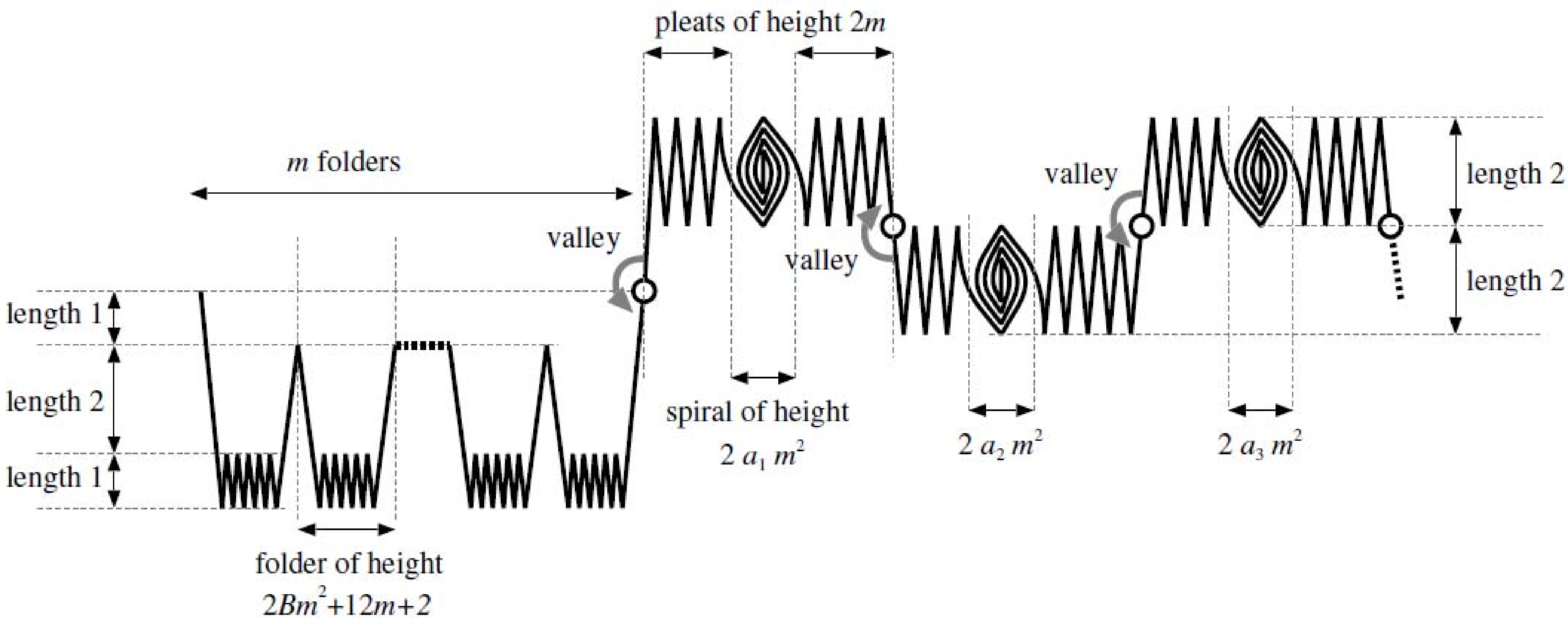


The way of folding is unique by bit longer end-edges

Minimize height is NP-complete

Proof: Polynomial time reduction from 3-Partition.

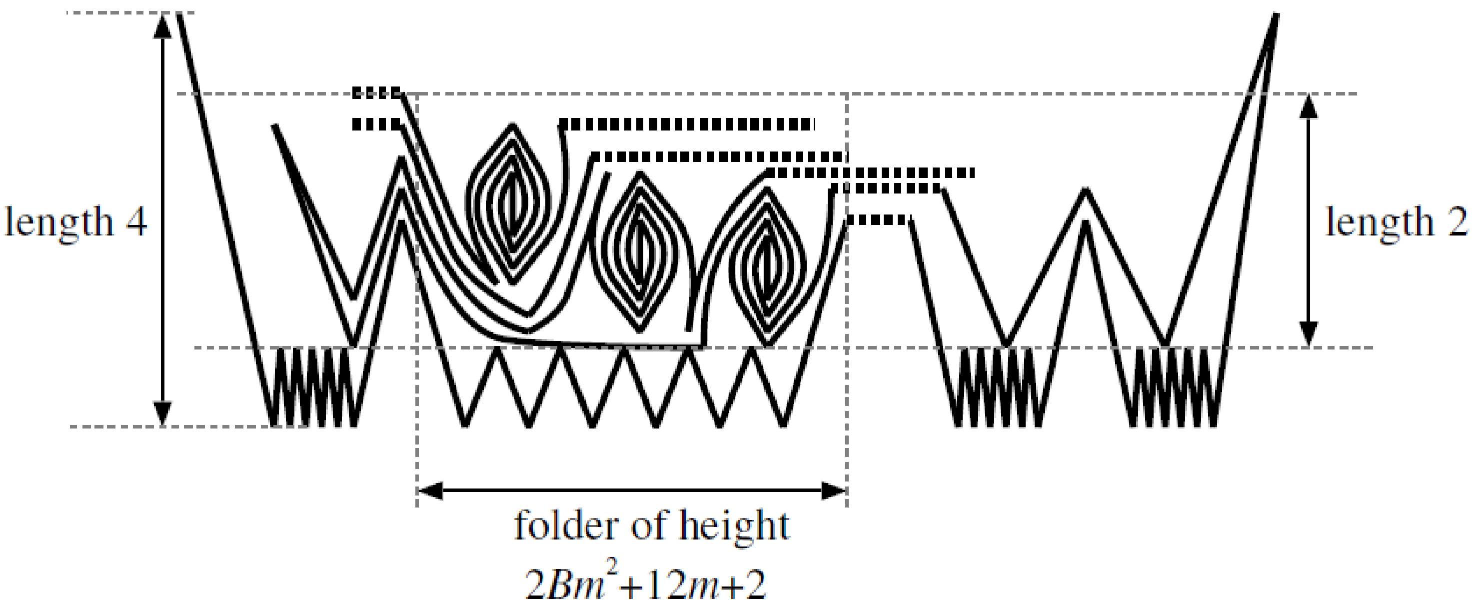
Overview



Minimize height is NP-complete

Proof: Polynomial time reduction from 3-Partition.

<u>Overview</u>



Summary & Future work…

Origami is interesting even in 1 dimension!!

	Unit interval model in [USUIO2012]	General model in (DEHILUU 2015)
max crease width	NP-complete	⇒ NP-complete
total crease width	open	NP-complete [DEHILUU 2015]
height	trivial	NP-complete [DEHILUU 2015]

Future work:

- Replace "open" into ???
- Extension to 2 dimension
  - Different measures of "thickness"?
- Estimation of the way of folding (~time complexity)
- Nicer model for "*Time-space trade off*" for Origami