

今日の予定

1. 展開図の基礎的な知識
2. 複数の箱が折れる共通の展開図
 - 2通りの箱が折れる共通の展開図
 - 3通りの箱が折れる共通の展開図そして....残された未解決問題たち
3. 正多面体の共通の展開図

このミステリー(?)の
中でメイントリックに使
われました!



Some nets are available at <http://www.jaist.ac.jp/~uehara/etc/origami/nets/index-e.html>

主な文献

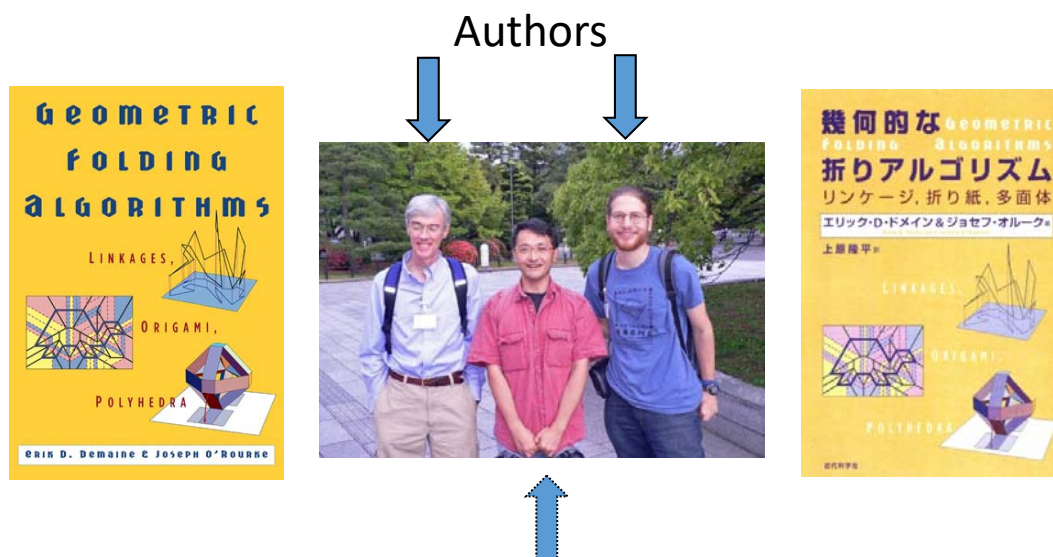
- Dawei Xu, Takashi Horiyama, Toshihiro Shirakawa, Ryuhei Uehara: Common Developments of Three Incongruent Boxes of Area 30, *COMPUTATIONAL GEOMETRY: Theory and Applications*, Vol. 64, pp. 1-17, August 2017.
- Toshihiro Shirakawa and Ryuhei Uehara: Common Developments of Three Incongruent Orthogonal Boxes, *International Journal of Computational Geometry and Applications*, Vol. 23, No. 1, pp. 65-71, 2013.
- Zachary Abel, Erik Demaine, Martin Demaine, Hiroaki Matsui, Guenter Rote and Ryuhei Uehara: Common Developments of Several Different Orthogonal Boxes, *Canadian Conference on Computational Geometry (CCCG' 11)*, pp. 77-82, 2011/8/10-12, Toronto, Canada.
- Jun Mitani and Ryuhei Uehara: Polygons Folding to Plural Incongruent Orthogonal Boxes, *Canadian Conference on Computational Geometry (CCCG 2008)*, pp. 39-42, 2008/8/13.

計算折り紙(Computational ORIGAMI)とは？

• 文献紹介

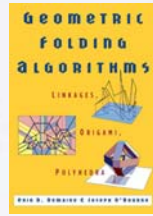
Geometric Folding Algorithms: Linkages, Origami, Polyhedra

by J. O'Rourke and E. D. Demaine, 2007.

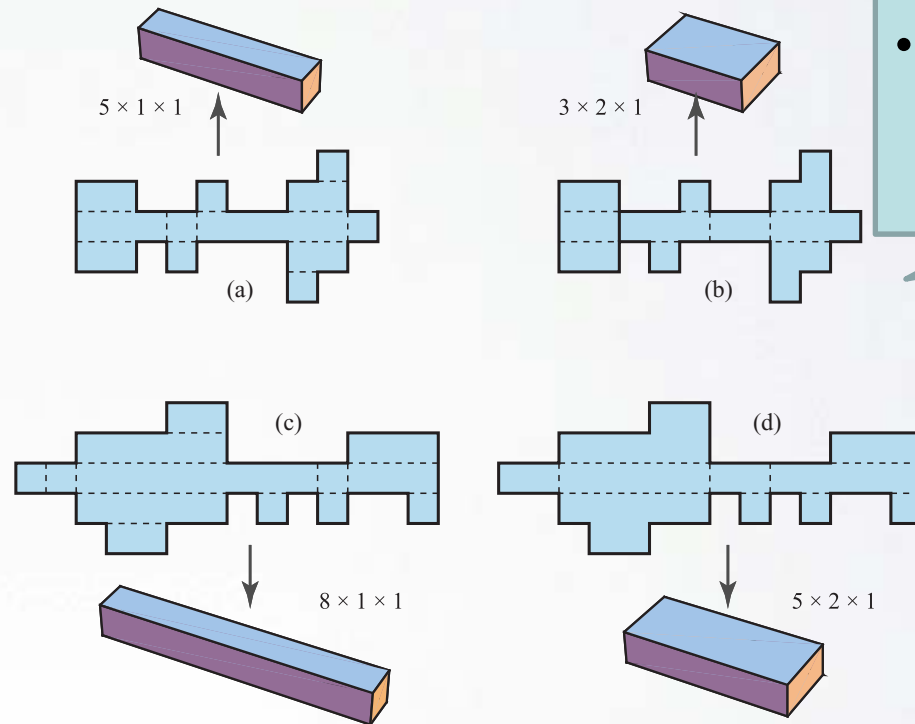


I, translated it to Japanese (2009).

In ,



There were two developments that fold to two boxes;



- Are they exceptional?
- Is there any development that fold to 3 or more boxes??

[Biedl, Chan, Demaine, Demaine, Lubiw, Munro, Shallit, 1999]

Developments of two boxes

In [Uehara, Mitani 2007], randomized algorithm that looks for such polygons by *brute force*;

Polygons folding into 2 boxes:

1. There are
many (~ 9000)
(by supercomputer (SGI Altix 4700))
2. Theoretically,
infinitely many



Note:

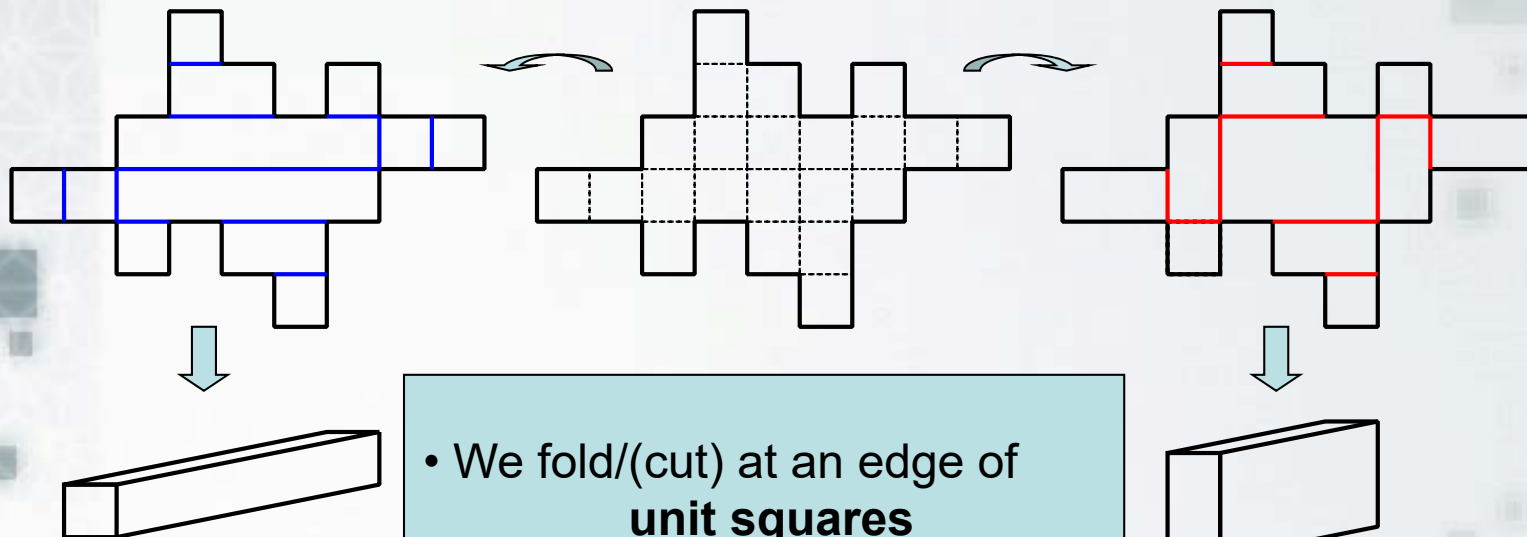
Polygons folding to 2 different orthogonal boxes

Example:

$$1 \times 1 + 1 \times 5 + 1 \times 5$$

$$= 1 \times 2 + 2 \times 3 + 1 \times 3$$

$$= 11 \text{ (surface area: 22)}$$



$$1 \times 1 \times 5 \\ = a \times b \times c$$

- We fold/(cut) at an edge of **unit squares**

- Surface area:

$$2(ab + bc + ca)$$

- Necessary condition:

$$1 \times 2 \times 3 \\ = a' \times b' \times c'$$

$$ab + bc + ca = a'b' + b'c' + c'a'$$

It seems to be better to have many combinations...

Note:

Surface areas;

If you try to find for three boxes,

If you try to find for four boxes,

Area	Trios	Area	Trios
<u>22</u>	(1, 1, 5), (1, 2, 3)	46	(1, 1, 11), (1, 2, 7), (1, 3, 5)
30	(1, 1, 7), (1, 3, 3)	70	(1, 1, 17), (1, 2, 11), (1, 3, 8), (1, 5, 5)
<u>34</u>	(1, 1, 8), (1, 2, 5)	94	(1, 1, 23), (1, 2, 15), (1, 3, 11), (1, 5, 7), (3, 4, 5)
38	(1, 1, 9), (1, 3, 4)	118	(1, 1, 29), (1, 2, 19), (1, 3, 14), (1, 4, 11), (1, 5, 9), (2, 5, 7)

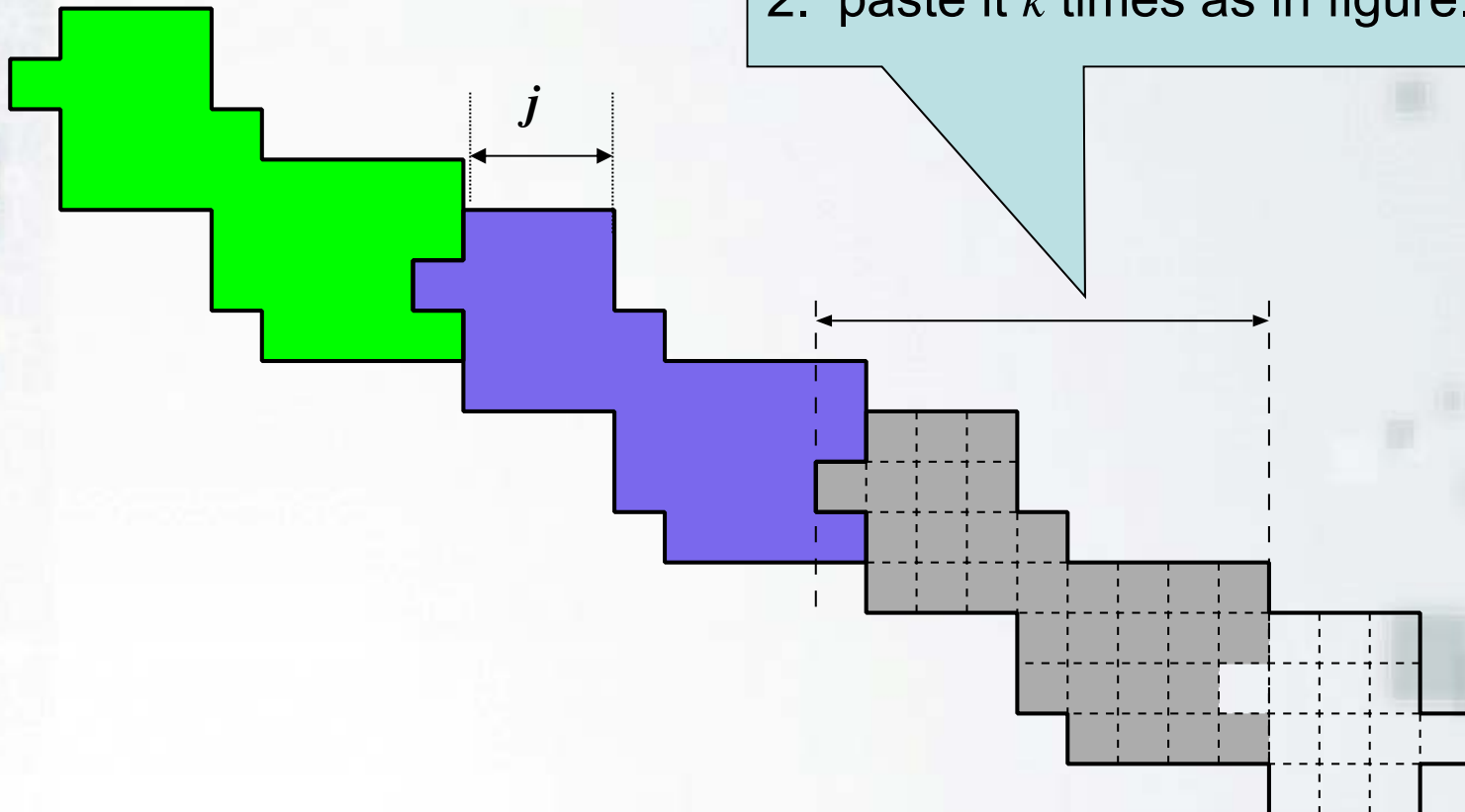
known results

Developments of two boxes

[Thm] There exist an infinitely many developments that fold to 2 boxes.

[Proof]

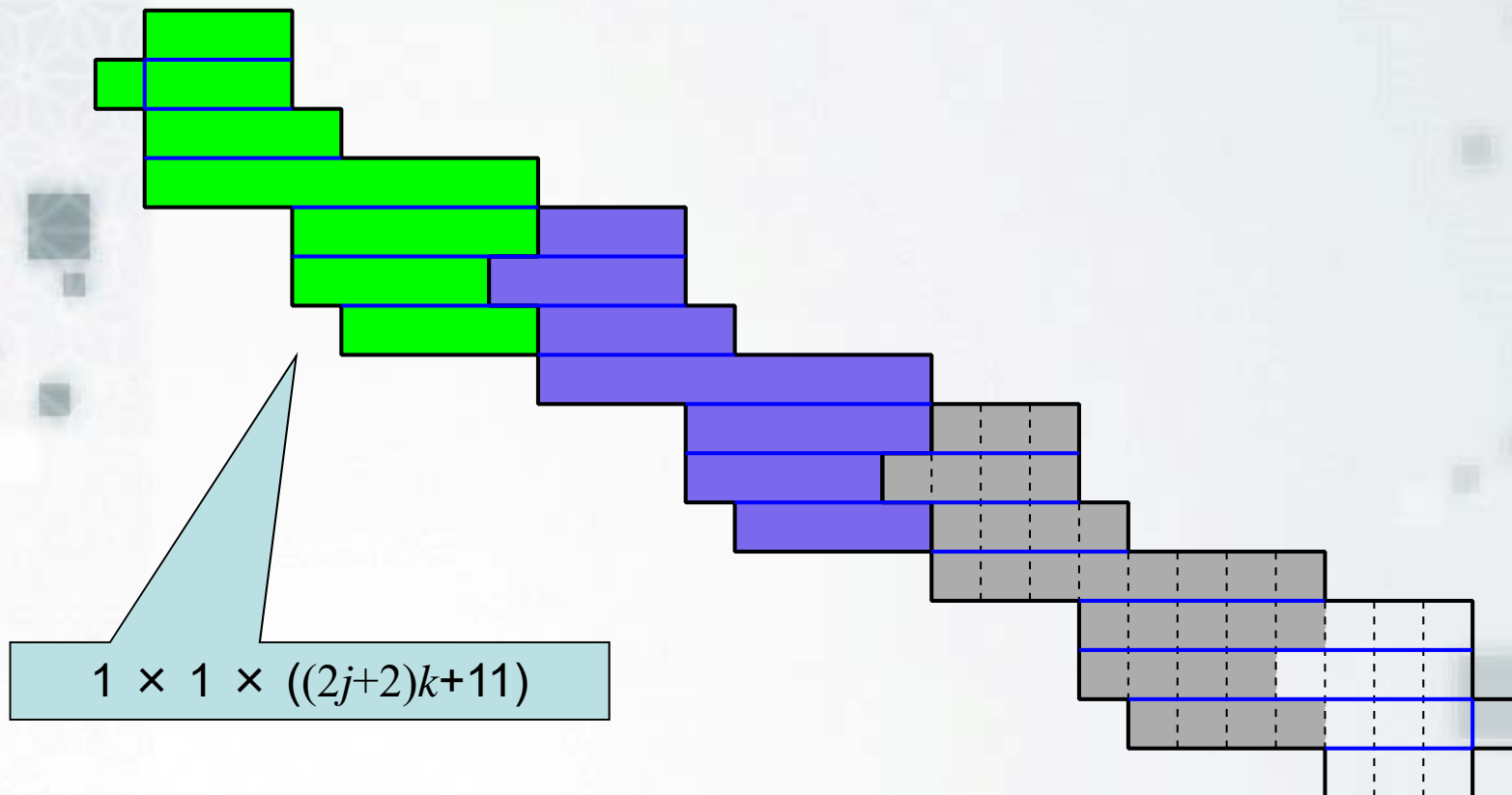
1. copy this area, and
2. paste it k times as in figure.



Developments of two boxes

[Thm] There exists an infinitely many polygons...

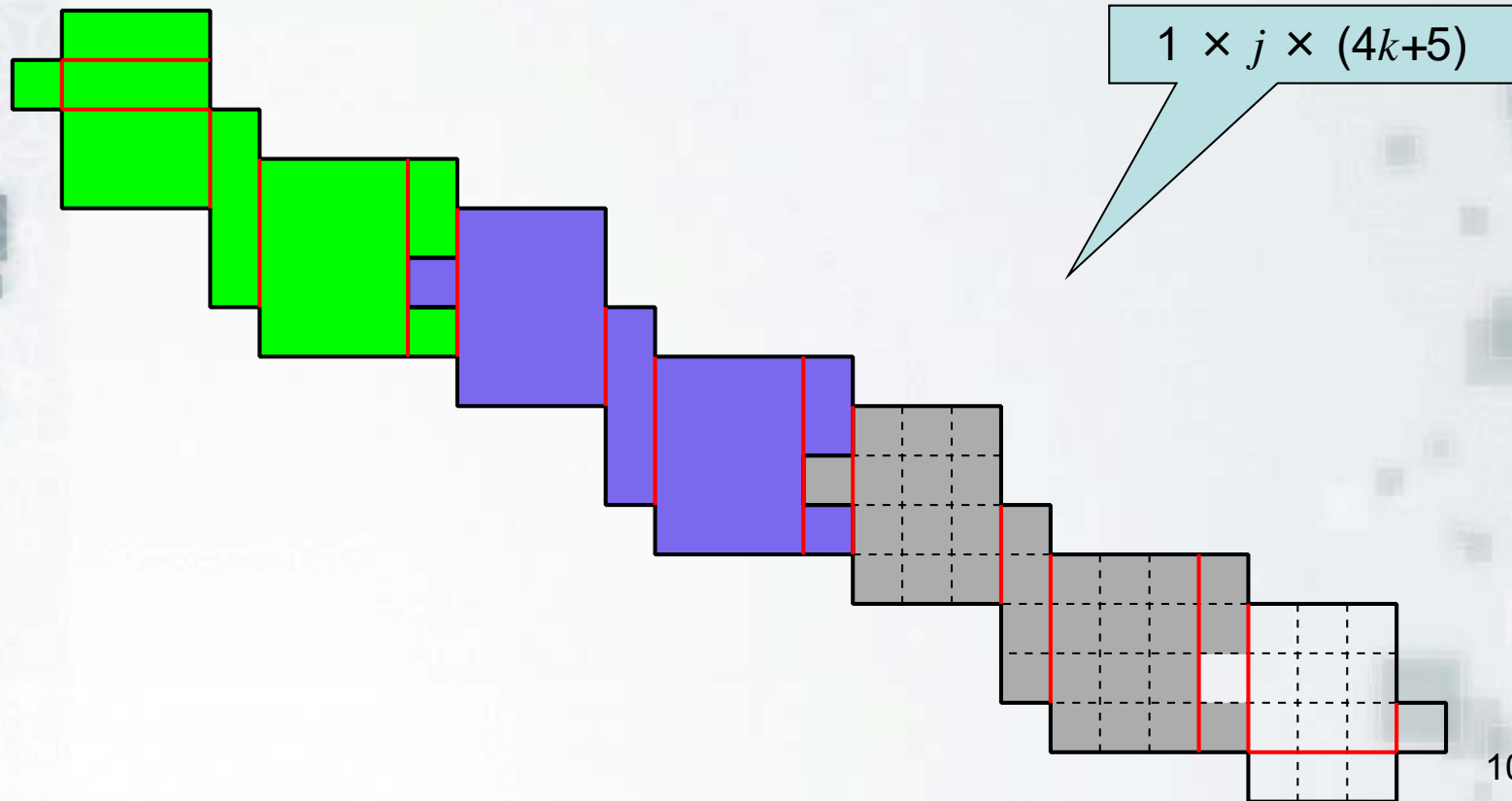
[Proof]



Developments of two boxes

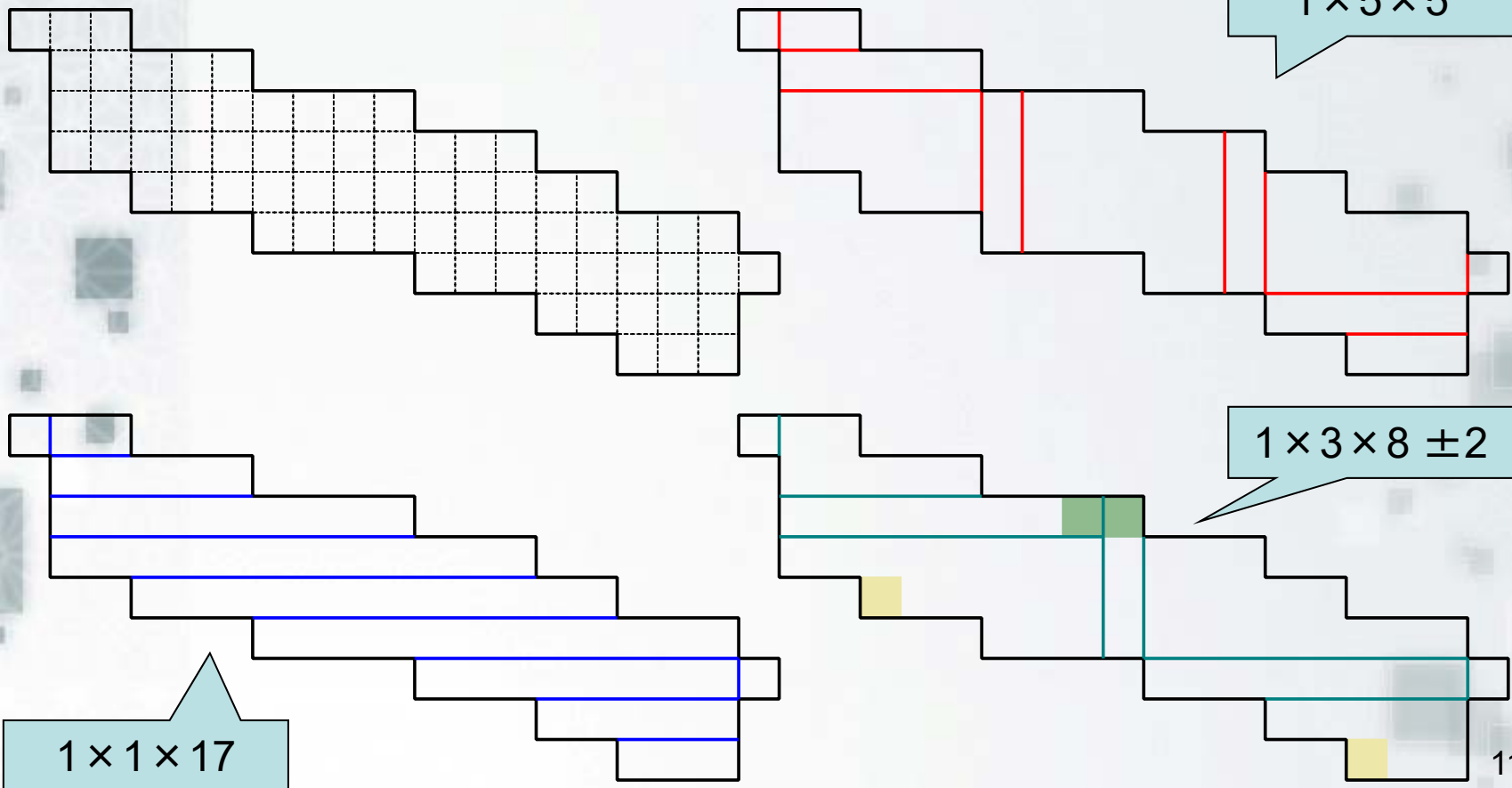
[Thm] There exists an infinitely many polygons...

[Proof]



Developments of *three* boxes(?)

- A polygon that can fold to three distinct boxes...?
 - close solution...



Developments of *three* boxes(?)

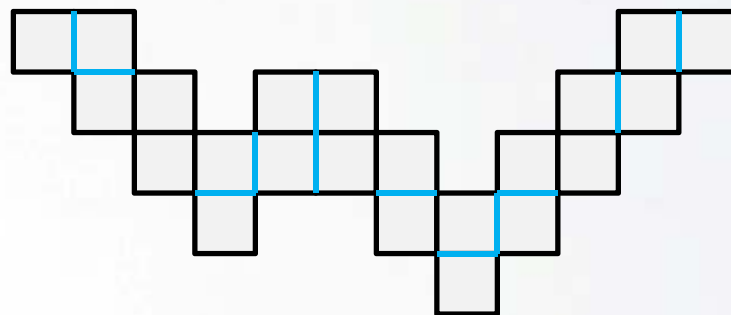
In [Abel, Demaine, Demaine, Matsui, Rote, Uehara 2011],

- The number of developments that fold to $1 \times 1 \times 5$ box and $1 \times 2 \times 3$ box is 2263.
- ◆ the latest algorithm runs in around 10 hrs.
- Among them, there is only one **pearl** development...

Developments of *three* boxes(?)

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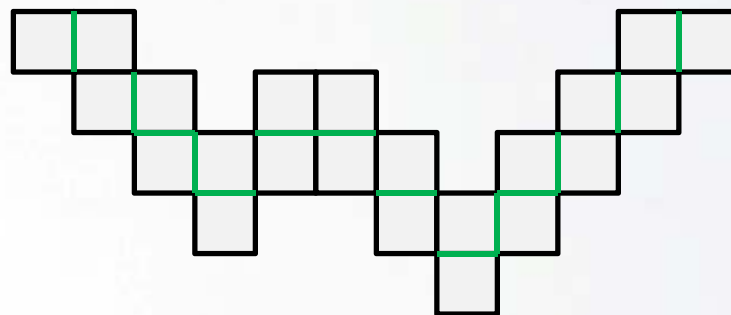


$1 \times 2 \times 3$

Developments of *three* boxes(?)

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$1 \times 1 \times 5$

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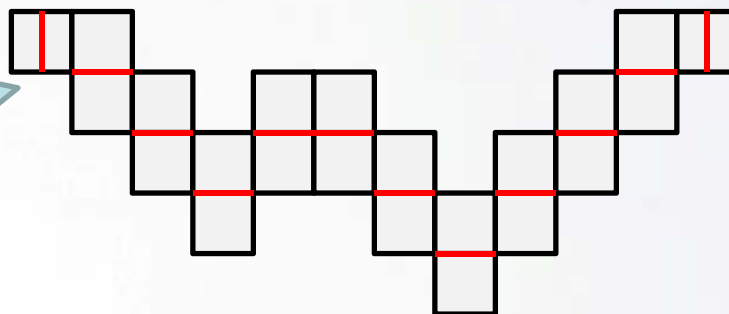
the latest algorithm runs in around 10 hrs.

Among them, there is only one **pearl** development...

おまけ問題:
3通り折ってみよう

Is the “box” *cheat*
having volume 0?

If you don't like $\frac{1}{2}$,
refine each square
into 4 squares



Since each column
has height 2
except both sides

$1 \times 11 \times 0$

!

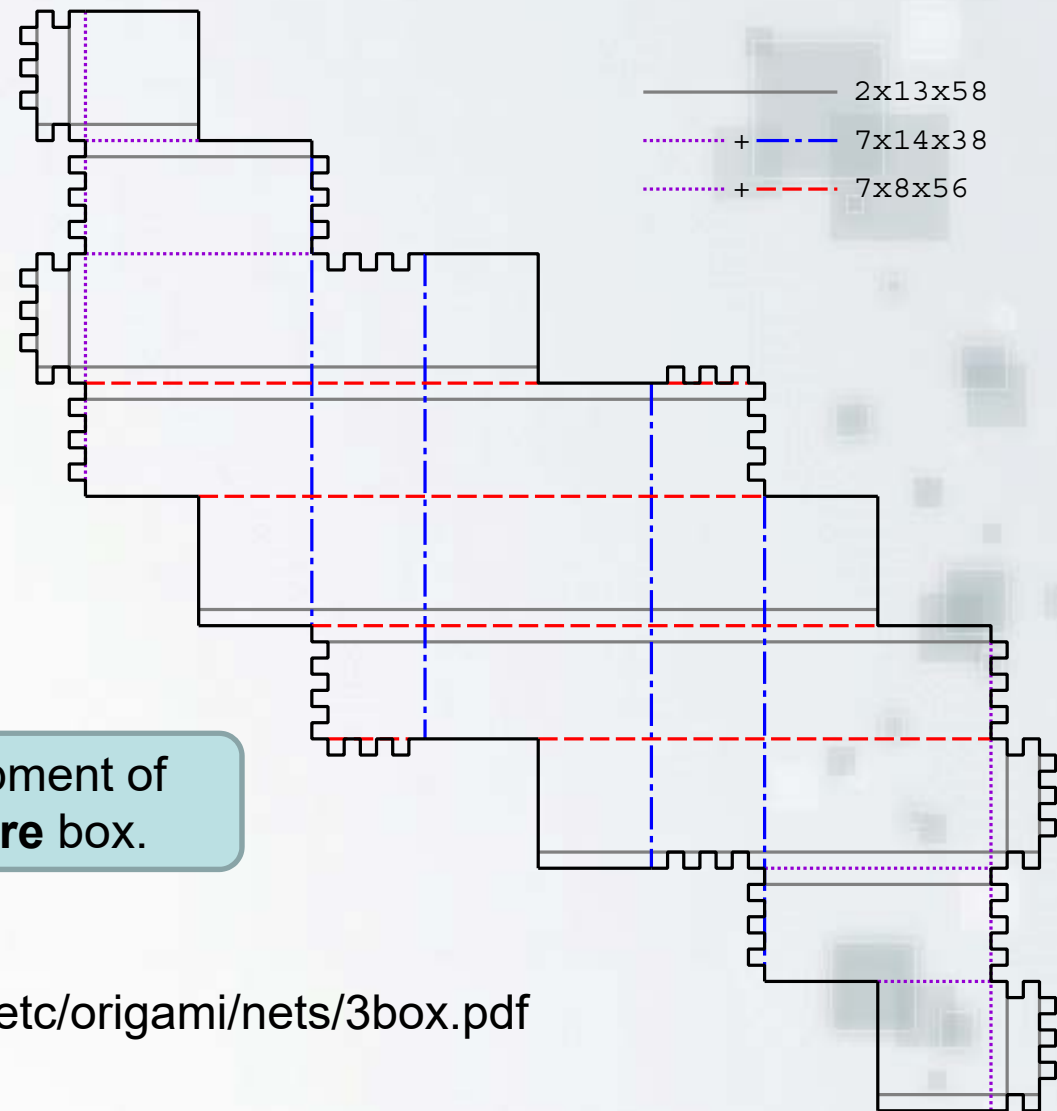
Developments of three boxes(!)

In February 2012,
Shirakawa (and I)
finally found that:

There exists
a polygon that
folds to **3 boxes!!**

[Basic Idea] From a development of
2 boxes, we make **one more** box.

I put it at
<http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf>



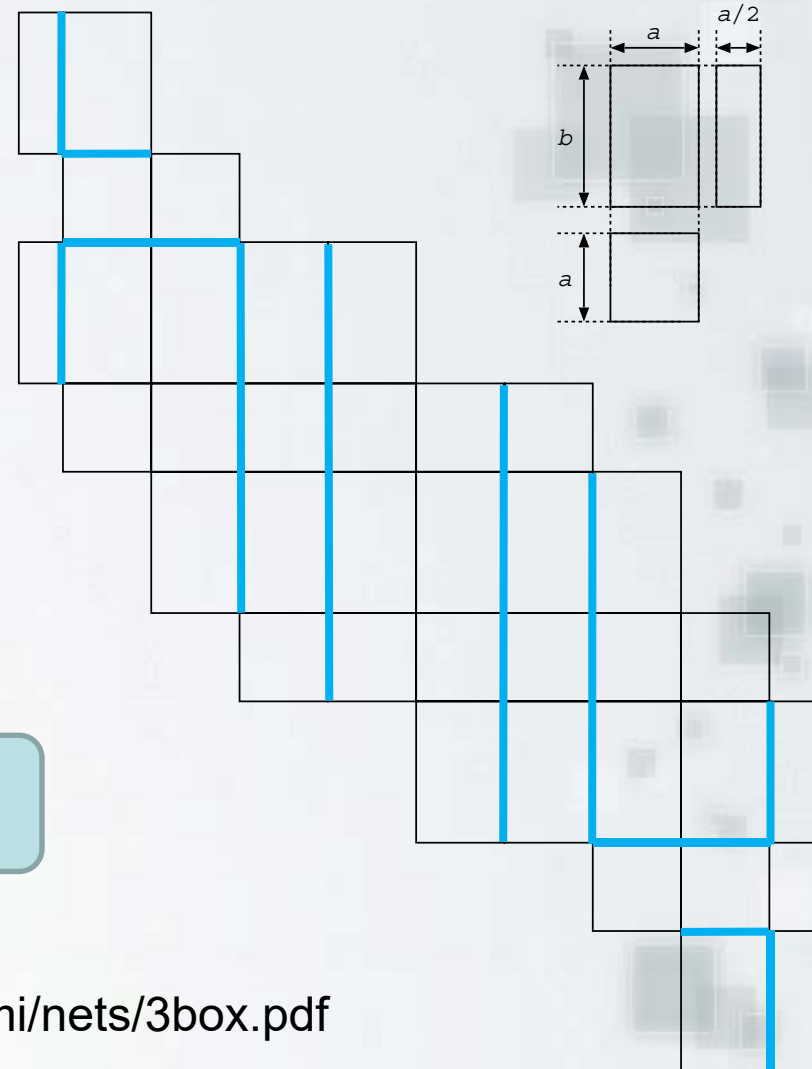
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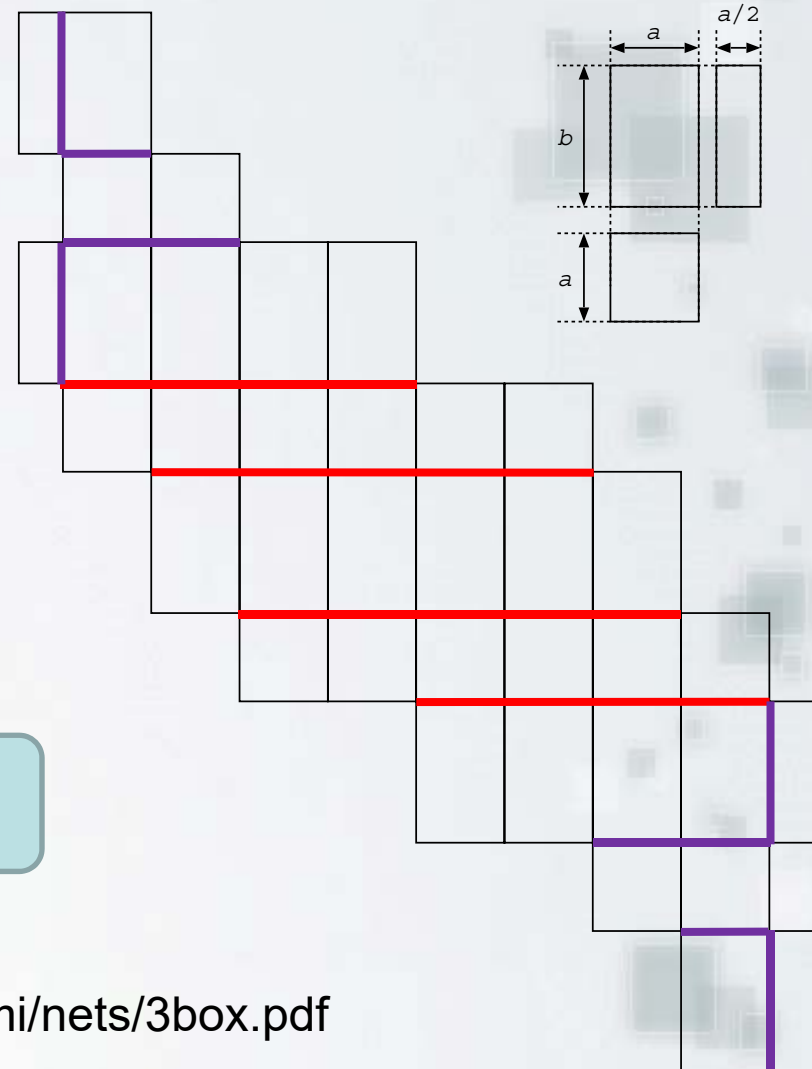
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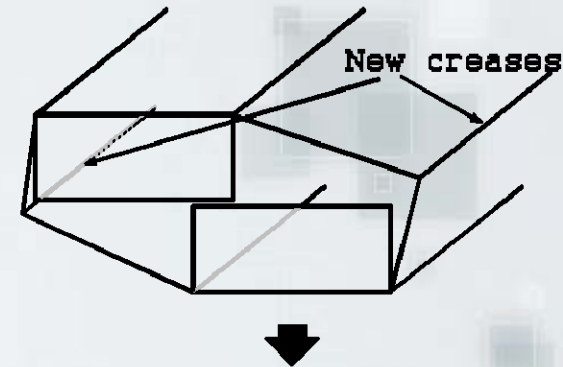
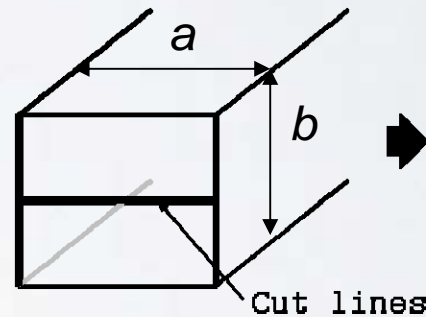
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One more box
is obtained by
this *squashing*!?

[Basic Idea] From a development of
2 boxes, we make **one more** box.

[No!!]
This works iff $a=2b$, i.e.,
from 1×2 square to 2×1 square

I put it at

<http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf>

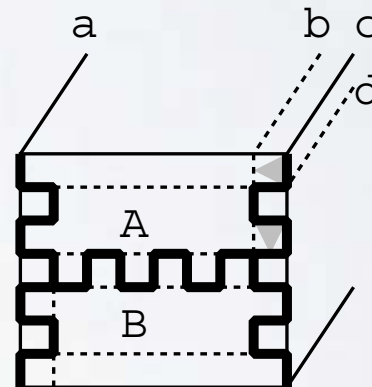
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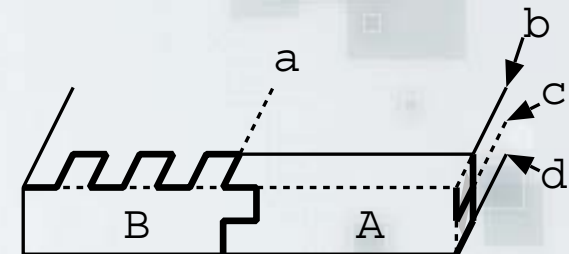
There exists
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[Basic Idea] From a development of
2 boxes, we make **one more** box.

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(a)



(b)

[Yes... with a trick!]
This idea works;
move a part of
the lid to 4 *sides*!

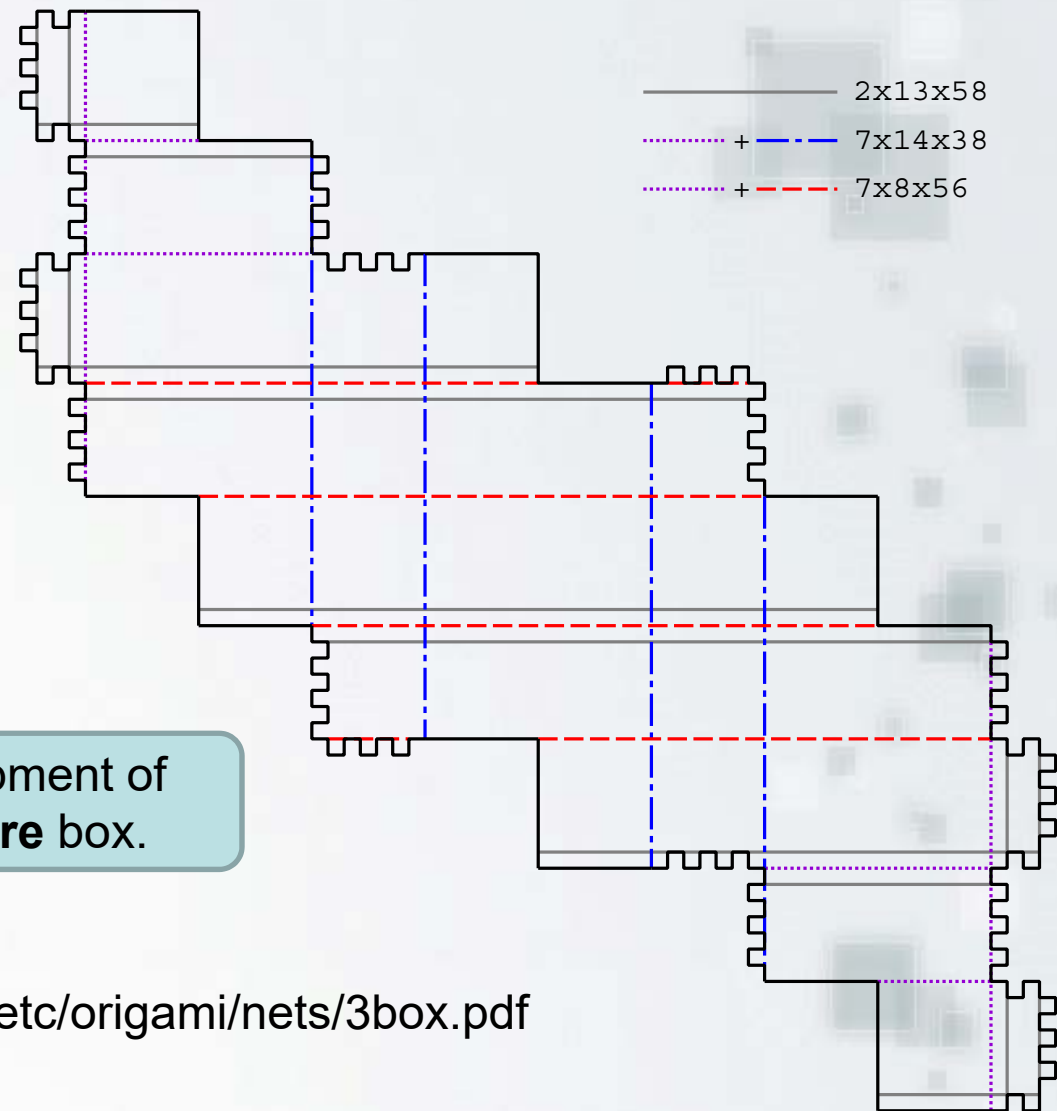
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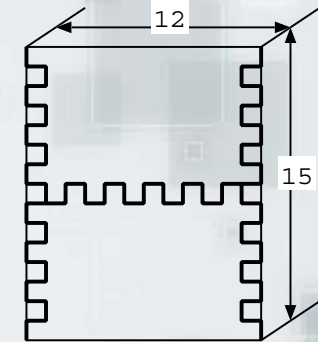
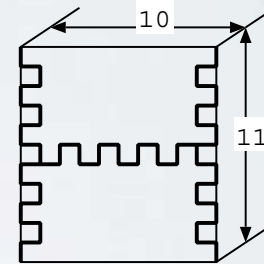
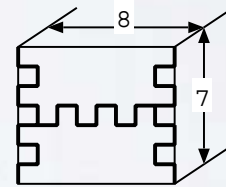
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Developments of *three boxes*(!)

In February 2012,
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[Theorem]

There exist an infinite number of polygons
that fold into 3 different boxes.

[Generalization]

- Basic box is flexible for the edge lengths.
- Zig-zag pattern can be extended.

I put it at

<http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf>

Future works

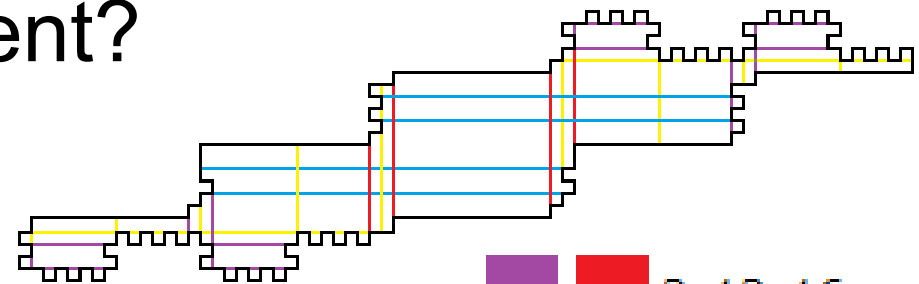
➤ Smallest development?

The current “smallest” development requires **532** squares.

>> the smallest area **46** that may produce three boxes of size $(1,1,11)$, $(1,2,7)$, $(1,3,5)$.

(Remind:

2263 polygons of area **22** folding to $(1,1,5)$, $(1,2,3)$)



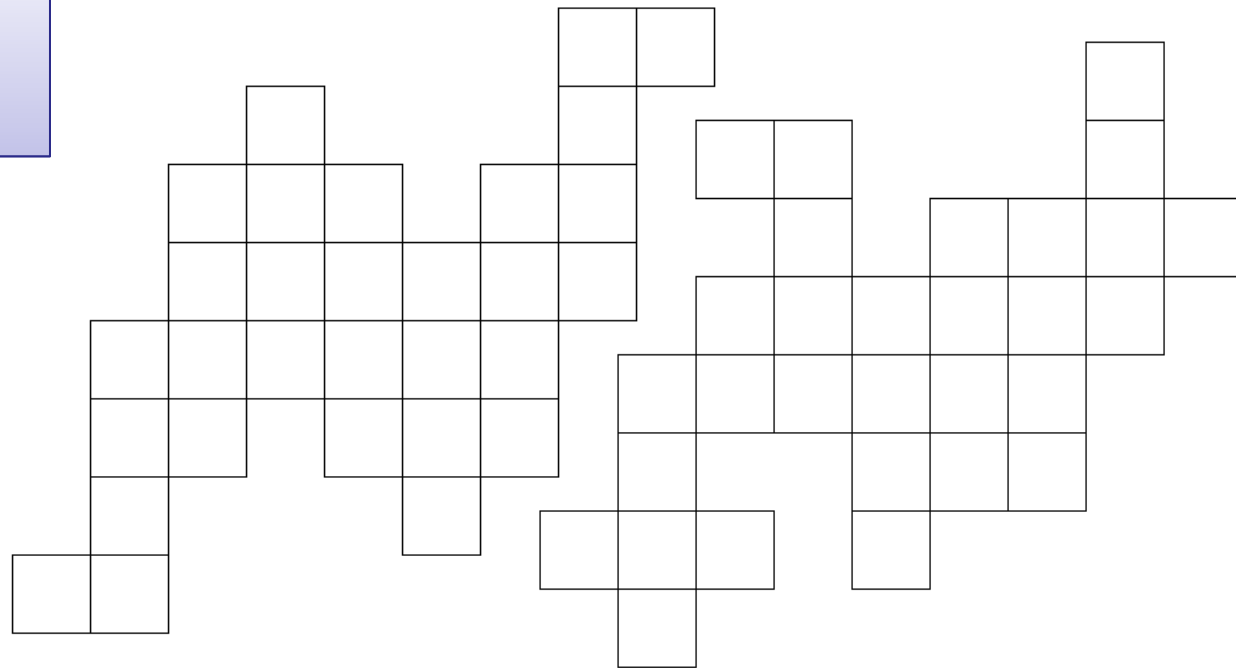
		2x13x16
		2x 4x43
		7x 8x14

Is there a polygon that folds to 4 or more boxes?

2012年10月23日：白川さんからのメール

「面積30で、 $1 \times 1 \times 7$ と $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$ の2通りの箱を折れる展開図を見つけました。この面積は $1 \times 3 \times 3$ も作れるので、斜めを許した場合3通りの箱が折れる最小のポリオミノになる可能性があります。」

おまけ問題：
2通り折ってみよう



Note:

Surface areas;

If you try to find for three boxes,

If you try to find for four boxes,

Area	Trios	Area	Trios
22	(1,1,5),(1,2,3)	46	(1,1,11),(1,2,7),(1,3,5)
30	(1,1,7),(1,3,3)	70	(1,1,17),(1,2,11),(1,3,8),(1,5,5)
34	(1,1,8),(1,2,5)	94	(1,1,23),(1,2,15),(1,3,11), (1,5,7),(3,4,5)
38	(1,1,9),(1,3,4)	118	(1,1,29),(1,2,19),(1,3,14), (1,4,11),(1,5,9),(2,5,7)

known results

2011年当時の松井君のプログラム:

- **面積22**の展開図を全探索:
 - $1 \times 1 \times 5$ と $1 \times 2 \times 3$ の箱を折る2263個の展開図
- 実行時間はパソコンで**10時間**

面積30は微妙な数字...

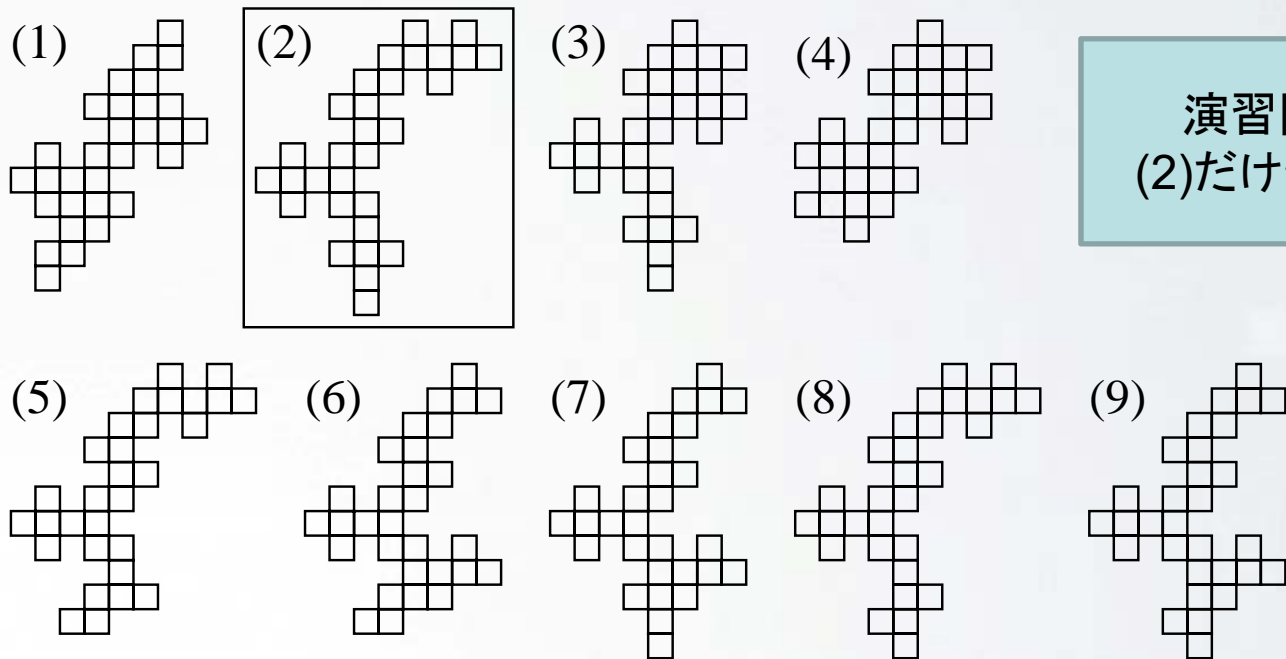
Dawei君の結果(2014年6月)

- 面積30の展開図の全探索に成功！ [Xu, Horiyama, Shirakawa, Uehara 2015]

追記:その後BDDの使用により[PCで10日]というレベルの驚愕の高速化に成功.

- 大雑把な結果

- JAISTのスパコン(Cray XC 30)で二ヶ月
- $1 \times 1 \times 7$ の箱と $1 \times 3 \times 3$ の箱が折れる共通の展開図は1080個
- そのうち、 $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$ の三つ目の箱が折れる展開図は9個



演習問題(?)
(2)だけ特別です。

まとめと課題

Surface areas;

If you try to find for three boxes,

If you try to find for four boxes,

Area	Trios	Area	Trios
22	(1,1,5),(1,2,3)	46	(1,1,11),(1,2,7),(1,3,5)
30	(1,1,7),(1,3,3)	70	(1,1,17),(1,2,11),(1,3,8),(1,5,5)
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known results

2011年, 面積22の展開図の全探索はPCで10時間.

2014年, 面積30の展開図の全探索はスパコンで2ヶ月.

BDDを使うとPCで10日間.

...この調子で46まで行くのは難しいと言わざるをえないが...?27

未解決問題

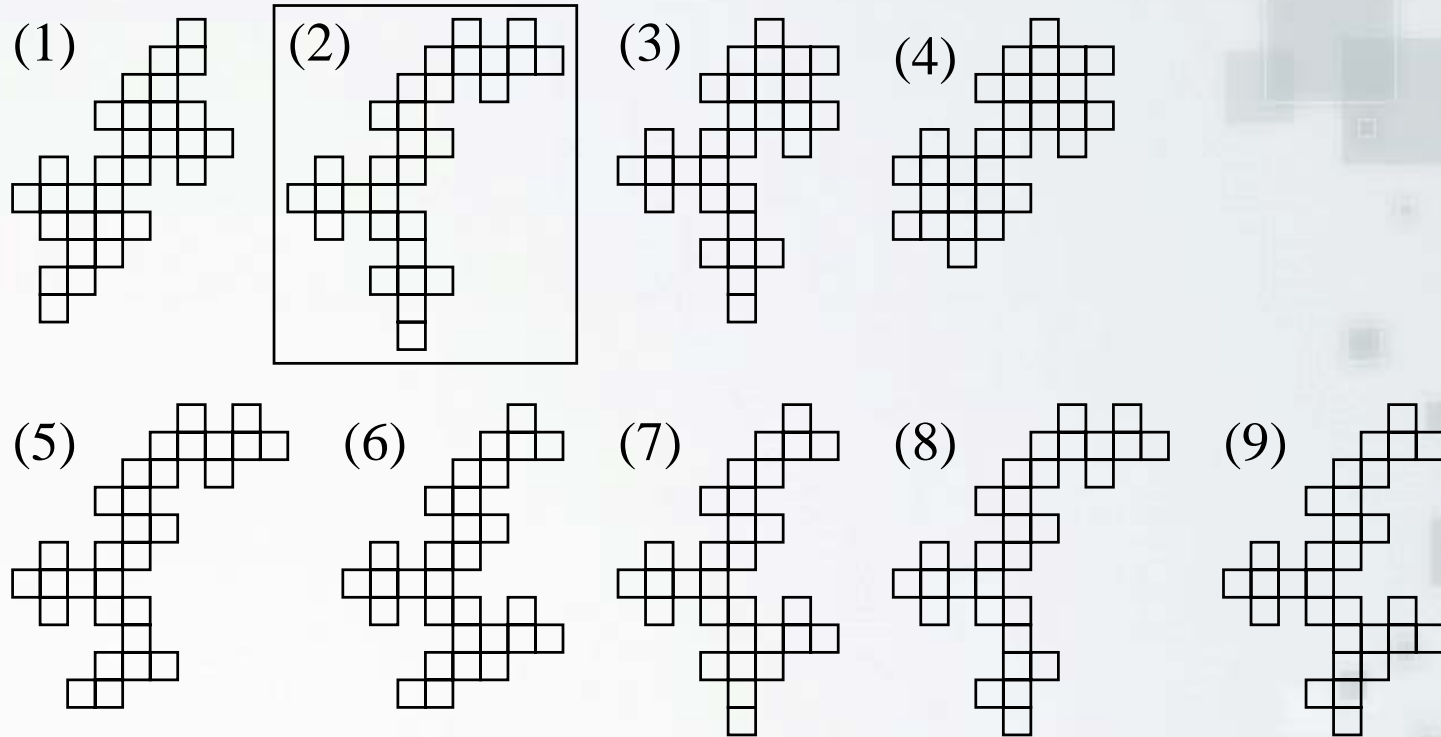
- 面積46や54で、3つの箱が折れる展開図はあるのか？
- 4つ以上の箱が折れる展開図は存在するのか？
- [k個以上の箱は折れない]という有限の上界はあるのか？(例えば10000通りの箱が折れる展開図などは想像しがたいけれど。。。)

おまけ情報:

- 「異なるポリオミノの数」は面積45まで既知(by 白川さん on OEIS)

演習問題

➤ (2)だけ特別なことを確かめてみよう



➤ ここで紹介した以外のアルゴリズムを考案してみよう(BDDなど)

発展問題

➤ ここで紹介なかった問題へのアルゴリズムを考案してみよう:

Q: 多角形 P が与えられたとき、そこから箱 Q が折れるか?

既知(?)の結果:

- 一般の多角形 P と凸多面体 Q の擬多項式時間アルゴリズムがあることはある
 - $O(n^{456.5})$ 時間アルゴリズム! (Kane, et al, 2009)
- Q が「大きさ $a \times b \times c$ の箱」で P は n 多角形
 - $O((n+m)\log n)$ 時間アルゴリズム
 - 変数 m は「一つの辺が m 個にばらけている」という未知のパラメータ(Horiyama, Mizunashi 2017)
- a, b, c が不明のときはどうか?