計算折り紙の最前線

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今日の予定：最新の結果たち

8. Rep-cube=Rep-tile + 展開図
9. ペタル型の紙で折るピラミッド型
10. ジッパー辺展開可能性
11. まとめ

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Rep-cubes: Dissection of Unfolding of Cubes

参考文献

- Xu Dawei, Takashi Horiyama, and Ryuhei Uehara: Rep-cubes: Unfolding and Dissection of Cubes, *The 29th Canadian Conference on Computational Geometry (CCCG 2017)*, 2017/07/26-2016/07/28, Ottawa, Canada.
- Zach Abel, Brad Ballinger, Erik D. Demaine, Martin L. Demaine, Jeff Erickson, Adam Hesterberg, Hiro Ito, Irina Kostitsyna, Jayson Lynch, and Ryuhei Uehara: Unfolding and Dissection of Multiple Cubes, Tetrahedra, and Doubly Covered Squares, Journal of Information Processing, accepted, 2017.（JCDCG^3 2016 で発表）

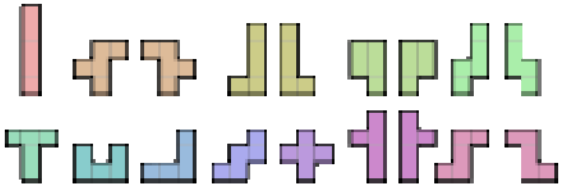
Rep-cubes: Dissection of Unfolding of Cubes

Started at 31st Bellairs Winter Workshop on Computational Geometry, Barbados, 2016

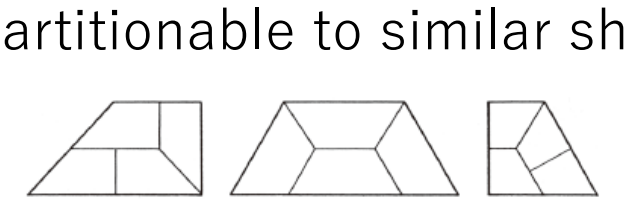
Solomon W. Golomb (1932-2016)

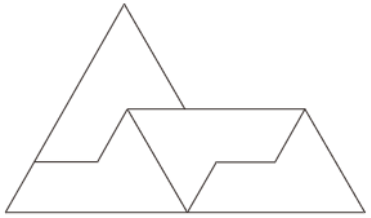
From the viewpoint of Recreational Mathematics, he invented

Polyominoes: shapes made by unit squares



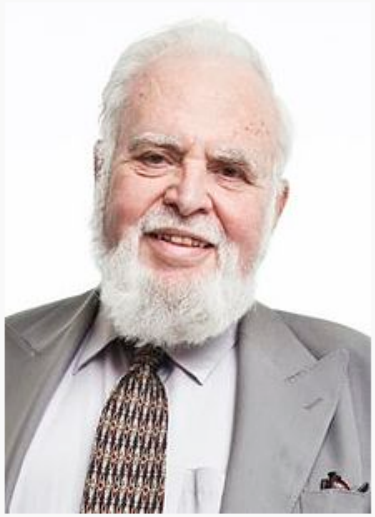
Rep-tiles: shapes partitionable to similar shape





Academic achievements [edit]

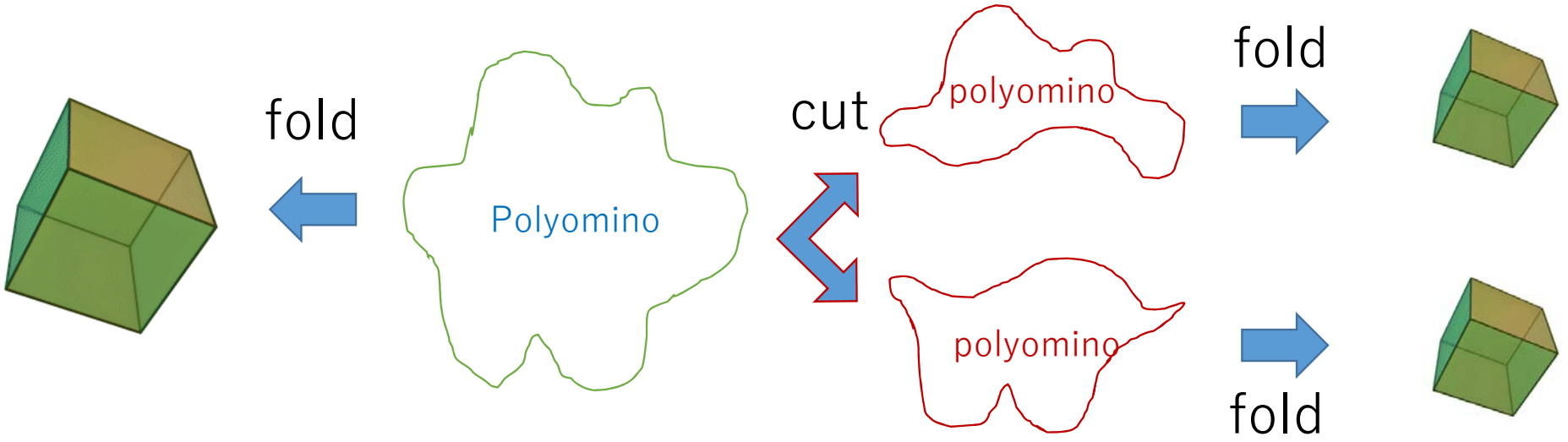
Solomon W. Golomb



2014 studio portrait

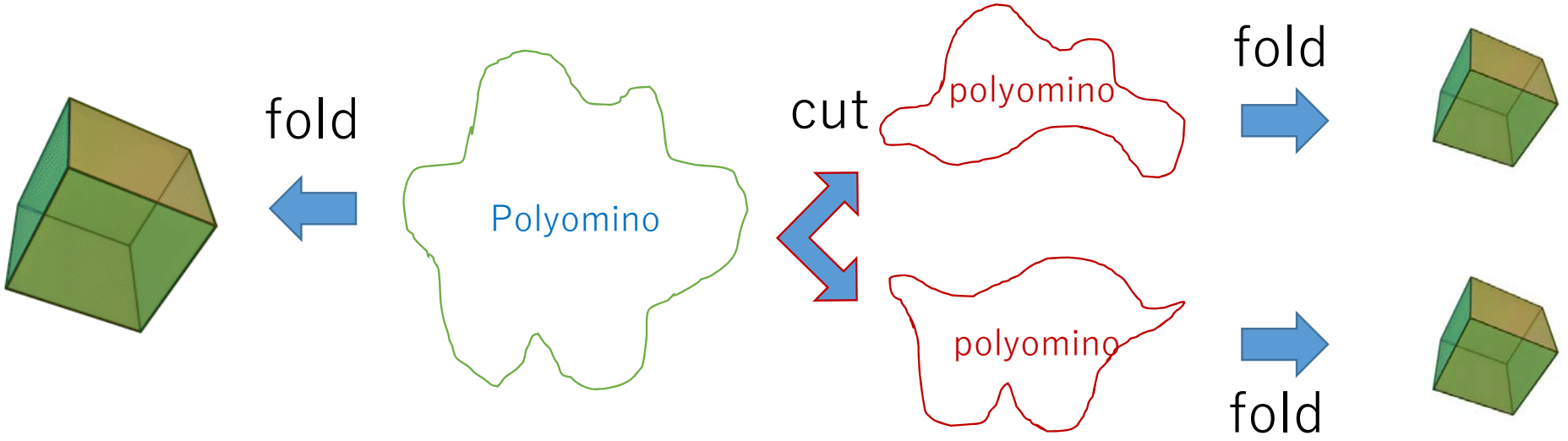
Extension to Folding problem

Natural Question: Is there any polyomino that folds to a cube and partitioned into some polyominoes s.t. each of which admits to fold a small cube?



New notion: “Rep-cube”

- A polyomino is “rep-cube” of order k
  ⇔ it folds to a cube and it can be cut into k pieces s.t. each of them folds to a cube
- A rep-cube is regular ⇔ k parts have the same size (area)



First main results:

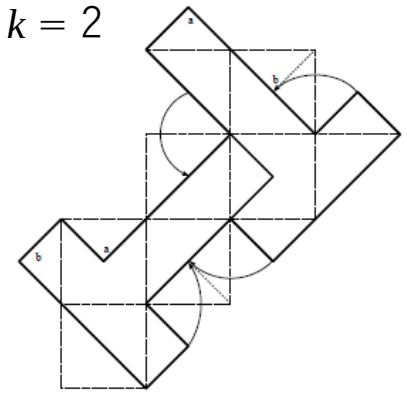
Thm 1 *There exists a regular rep-cube of order* k *for* $k = 2, 4, 5, 8, 9, 36, 50, 64$.

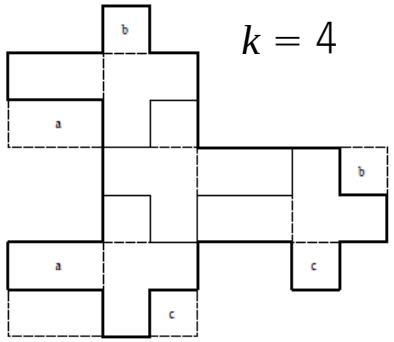
Thm 2 *There exists a regular rep-cube of order* $36gk'^2$ *for any positive integer* k' *and an integer* g *in* $\{2, 4, 5, 8, 10, 50\}$. *I.e., there exists an infinite number of regular rep-cubes.*

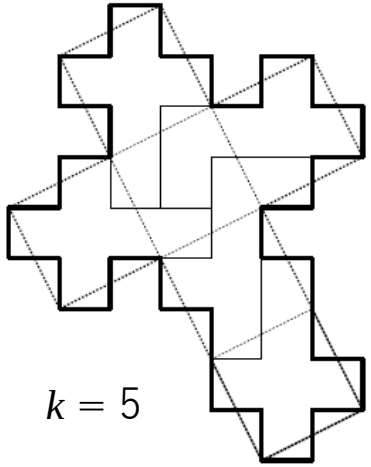
Thm 3 *There exists a non-regular rep-cube of order* k *for* $k = 2, 10$.

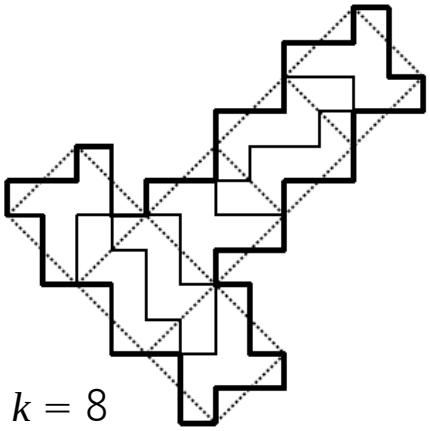
Thm 1 *There exists a regular rep-cube of order* k *for* $k = 2, 4, 5, 8, 9, 36, 50, 64.$

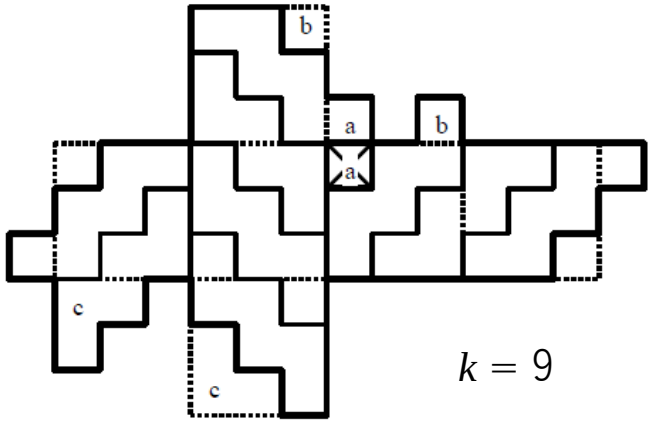
技法:
試行錯誤





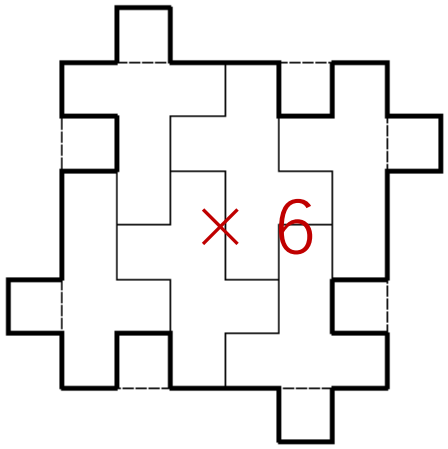


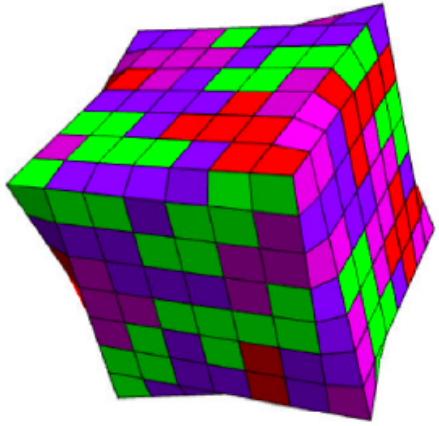




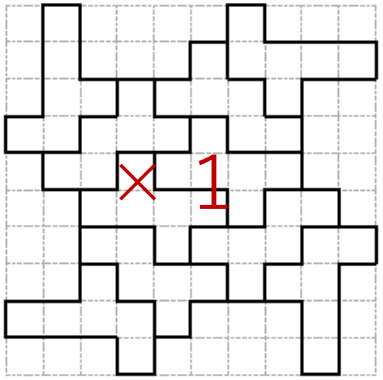
Thm 1 *There exists a regular rep-cube of order* k *for* $k = 2, 4, 5, 8, 9, 36, 50, 64$.

技法:
試行錯誤と
Mathematica

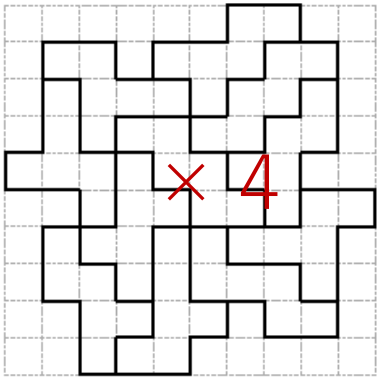


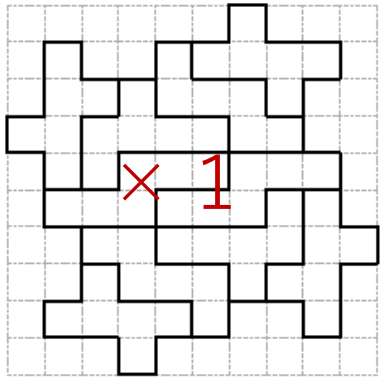


$k = 36$



$k = 50$

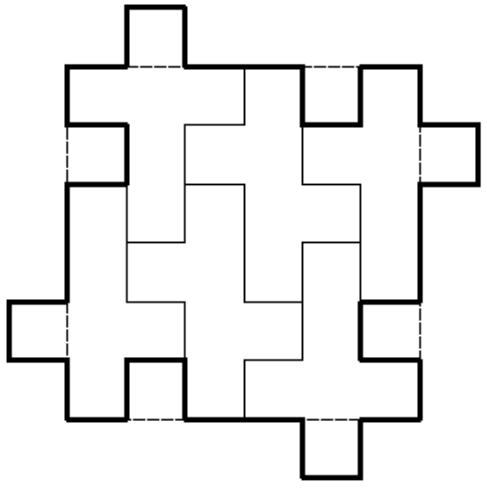




$k = 64$

Thm 2 *There exists a regular rep-cube of order* $36gk'^2$ *for any positive integer* k' *and an integer* g *in* $\{2, 4, 5, 8, 10, 50\}$.
I.e., there exists an infinite number of regular rep-cubes.

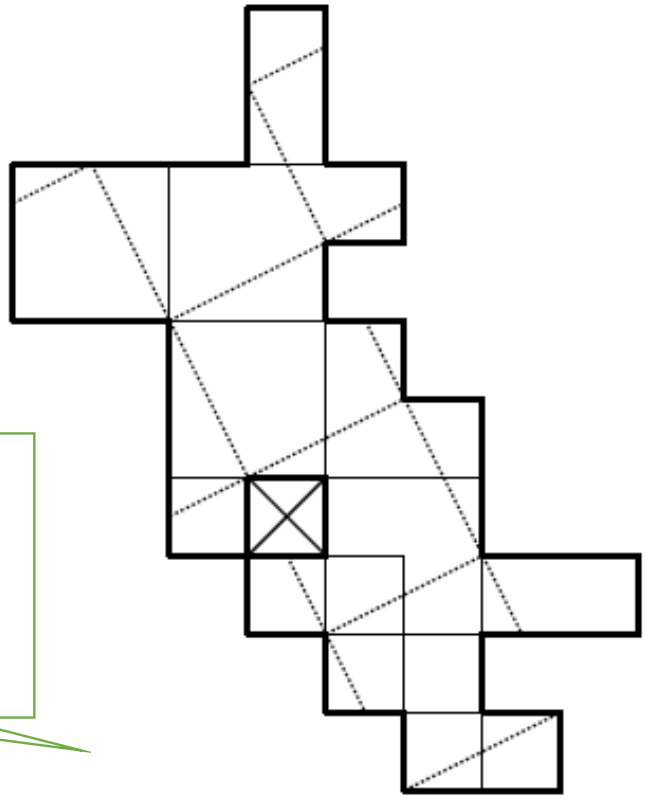
Proof Take any pattern in Thm 1.
Then replace each unit square by the right pattern for $k=36$ in Thm 1. We can repeat it recursively any times.



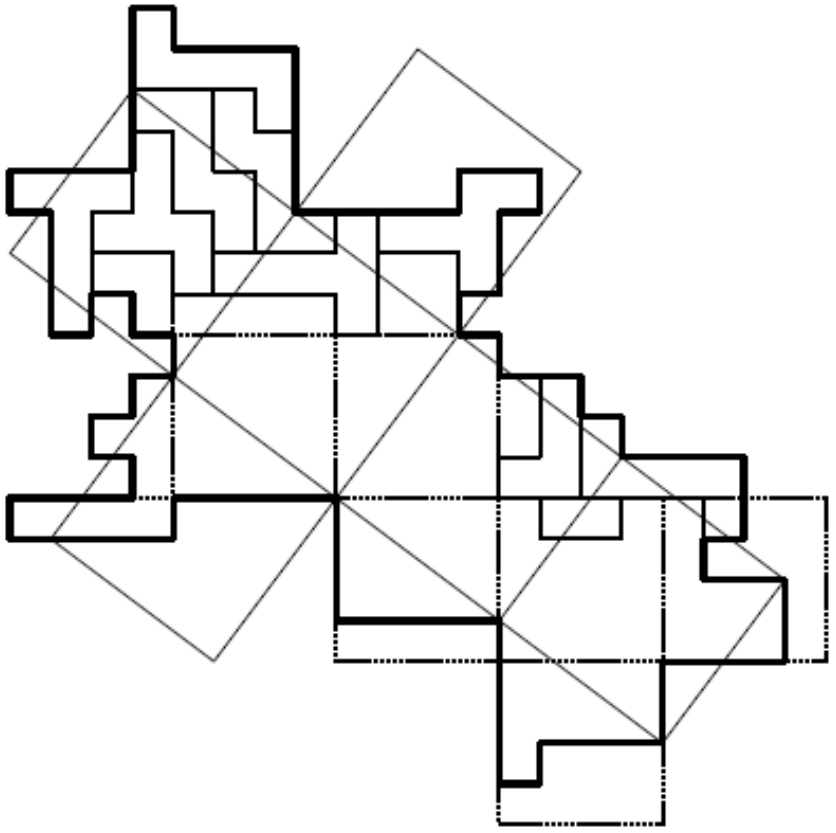
$k = 36$

Thm 3 *There exists a non-regular rep-cube of order* k *for* $k = 2, 10$.

技法:
試行錯誤



$k = 2$



$k = 10$

2次元への一般化（？）

基本アイデア

もはやポリオミノではありませんが。。。

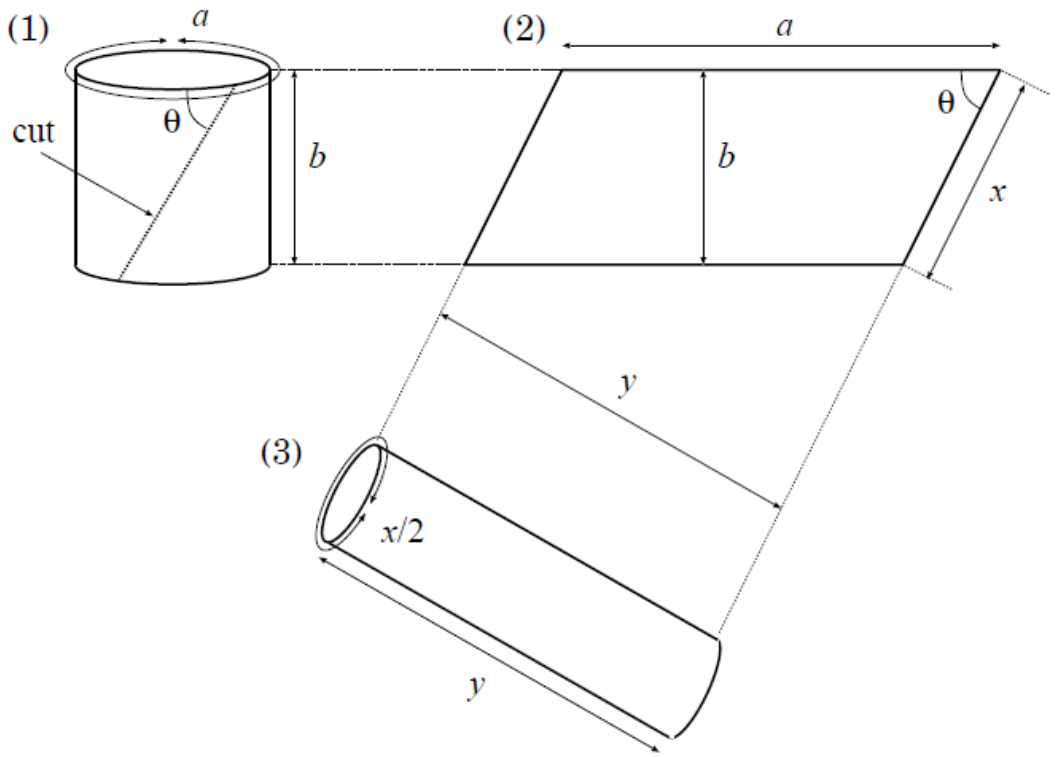
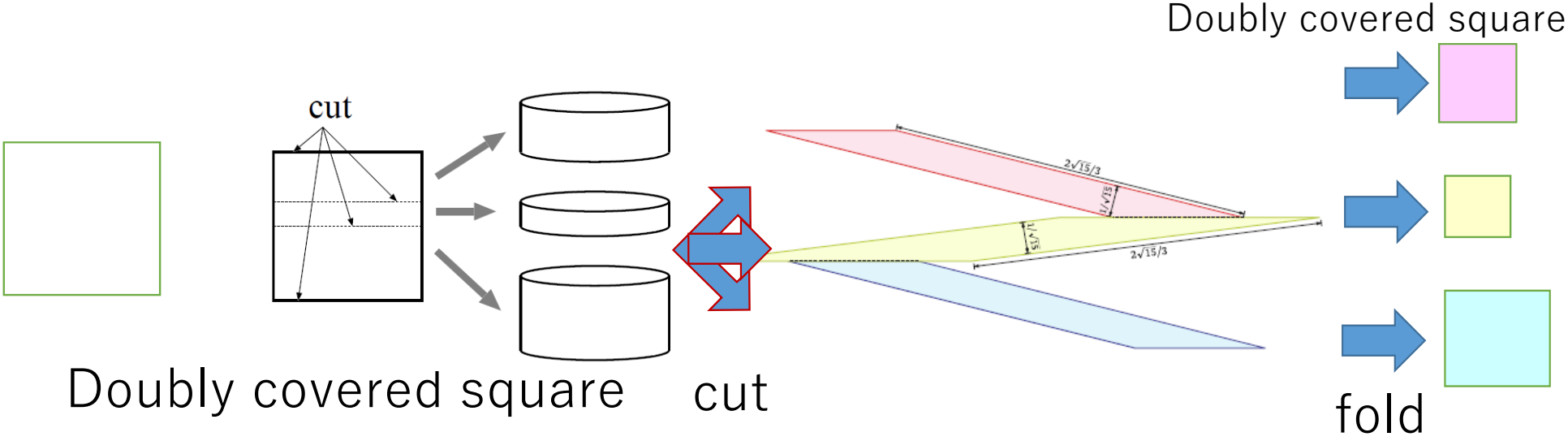


Fig. 6 (1) A cylinder of circumference a and height b, (2) a common development of two cylinders, (3) the other cylinder of circumference $x/2$ and height y.

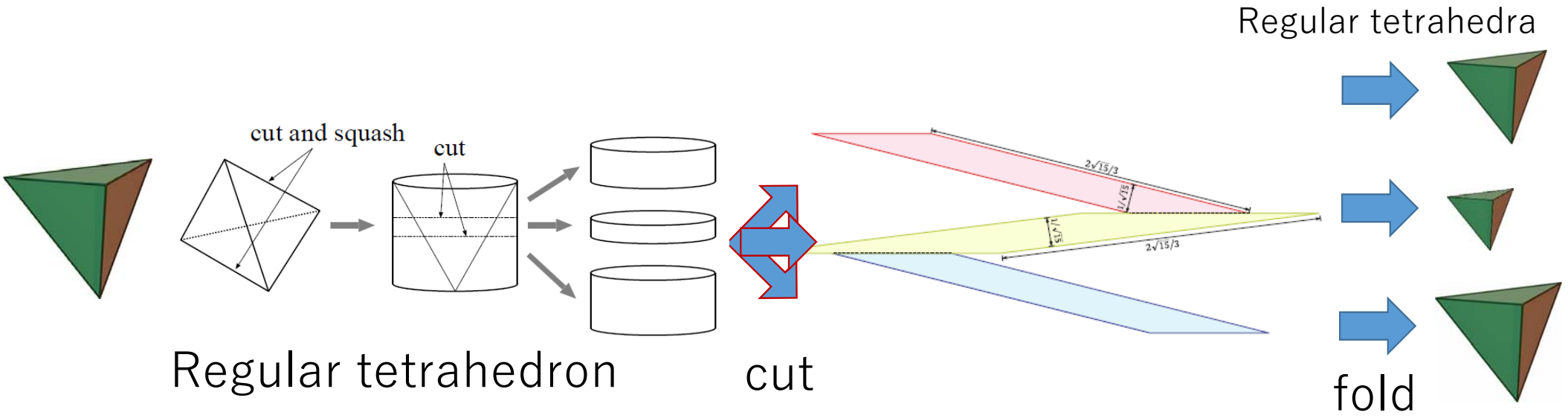
2次元への一般化：2重被覆正方形

Thm 4 *For any positive real numbers* $A, a_1, a_2, \ldots, a_k$ *such that* $\Sigma_i a_i = A$, *there is a net of a doubly-covered square with area* A *that can be cut into* k *polygons with areas* $a_1, a_2, \ldots, a_k$, *each of which can be folded into a doubly-covered square.*



3次元立体への回帰：正4面体

Thm 5 *For any positive real numbers* $A, a_1, a_2, \ldots, a_k$ *such that* $\Sigma_i a_i = A$, *there is a net of a regular tetrahedron with area* A *that can be cut into* k *polygons with areas* $a_1, a_2, \ldots, a_k$, *each of which can be folded into a regular tetrahedron.*



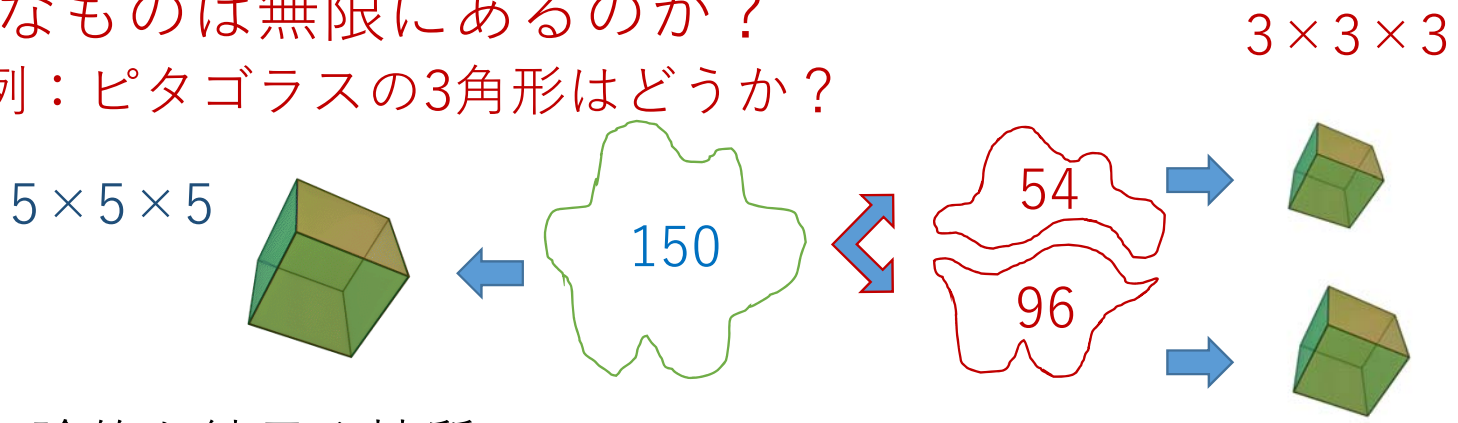
2016年時点での貢献と未解決問題

貢献：

- 新しい概念 rep-cube を導入し、多くの例を示した
- Rep-cube が無限に存在することを示した

未解決問題：

- Irregular なものは無限にあるのか？
  - 特殊な例：ピタゴラスの3角形はどうか？

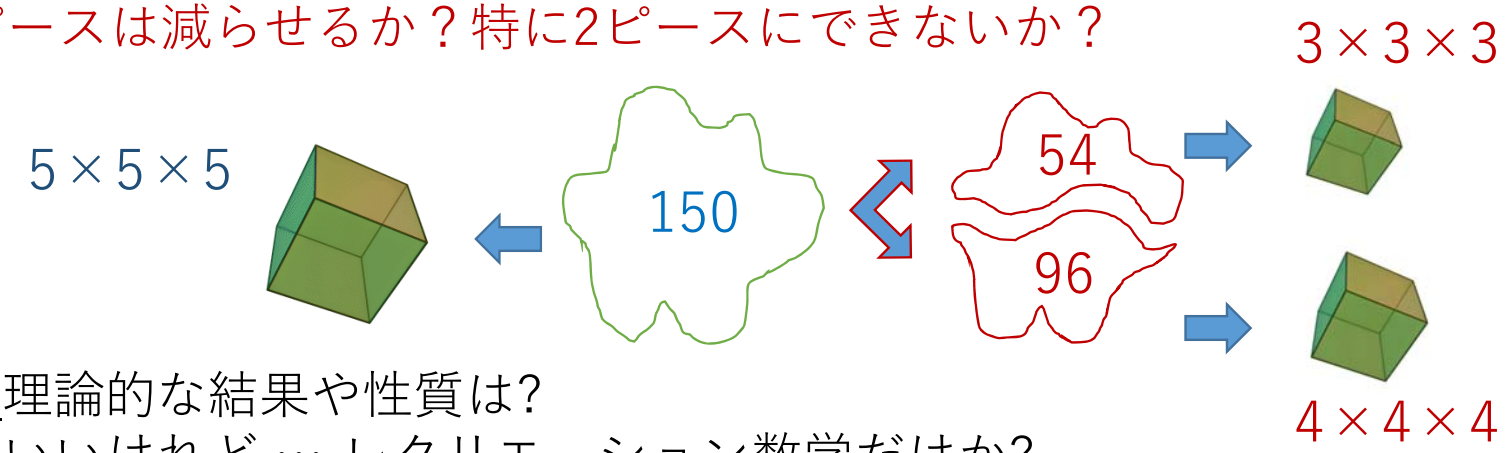


発展課題

- もっと理論的な結果や性質は?
- 応用があるといいけれど … レクリエーション数学だけか?

今日時点での未解決問題

- RegularなRep-cubeがある自然数とない自然数の分類
  1. 「ない」もの: 3, 6, … (個々に議論することはできるが…?)
  2. 「ある」もの: すでに現物を見つけたもの（試行錯誤で増やせるけど…）
  3. 「あってもよい」もの：1,2に該当しないもの
- “Uniform”なRep-cubeの特徴づけなど
- Irregular なものは無限にあるのか？
  - 特殊な例：ピタゴラスの3角形はどうか？
    5ピースは減らせるか？特に2ピースにできないか？



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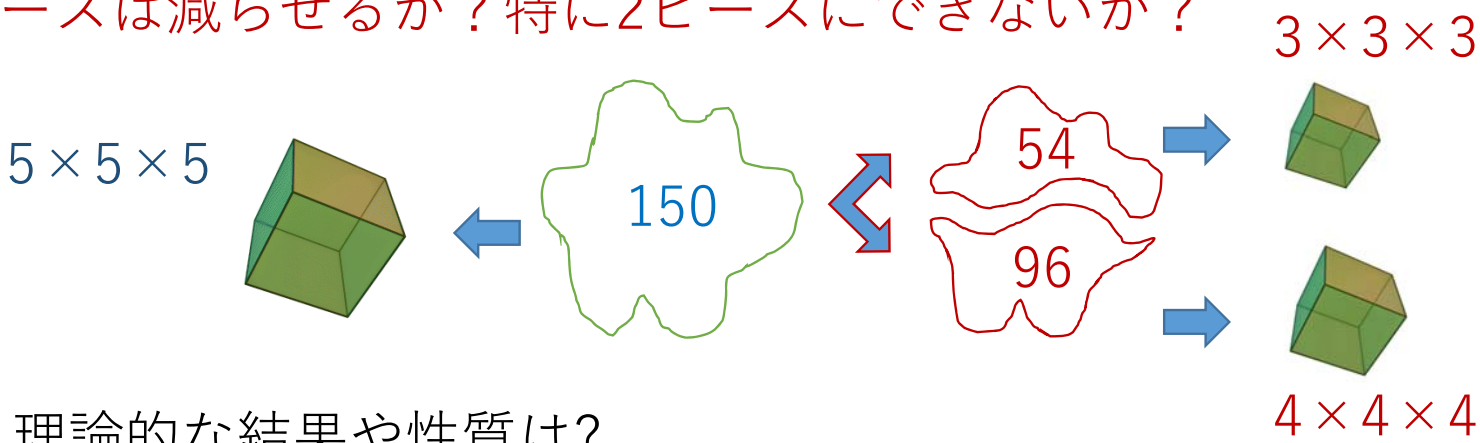
演習問題

- RegularなRep-cubeがある自然数とない自然数の分類
  1. 「ない」ものを増やす: 3, 6, … (個々に議論してみる)
  2. 「ある」ものを増やす:現物を見つける（試行錯誤＋α）
- Irregular なものの現物をもっと探す：まだ２つしかないけど
  - 特殊な例：ピタゴラスの3角形はどうか？
    5ピースは減らせるか？特に2ピースにできないか？



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