GRADIENT FLOWS ON A SPACE OF POWER-FUNCTIONS FOR MINIMIZING A CONVEX FUNCTIONAL

YUKIO HAYASHI*
Japan Advanced Institute of Science and Technology, Hokuriku
Tatsunokuchi 923-1292, Ishikawa, Japan
E-mail:`yhayashi@jaist.ac.jp`

Keywords: optimization algorithm, self-concordant barrier function, information geometry, gradient system.

2000 Mathematics Subject Classification: *Primary: 90C25; Secondary: 53C22, 90C51.*

We study a class of nonlinear dynamical systems to develop efficient algorithms. As an efficient algorithm, interior point method based on Newton's method is well known for solving convex programming problems which include linear, quadratic, semidefinite, l_p-programming problems [2] [3]. On the other hand, the geodesic of information geometry [1] is represented by a continuous Newton's method for minimizing a convex functional called divergence. Thus, we discuss a relation between information geometry and convex programming in a related family of continuous Newton's method. In particular, as the optimization of parameter values, we consider the α-projection problem from a given data onto an information geometric submanifold spanned with power-functions such as a weighted l_p-norm.

First, we present there exists a structural similarity between the α-projection and semidefinite programming problems [4]. The geometric structure is based on the autoparallelisms [1] or linear property in the function space over finite discrete variables or the space of positive definite matrices, respectively. The property is used to derive an approximation algorithm as predictor-corrector method for solving the problem through the geodesic. The maximum step-length is determined by geometric quantities with respect to the Riemannian metric and the dual-connection. We also show that the proposed method is effective in simulation.

Next, we reformulate the α-projection problem into a form of convex programming as a l_p-programming and the related ones. From the reformulated problems, we derive self-concordant barrier functions [2] [3] according to the real values of α. Thus, it means the existence of a polynomial time interior-point algorithm for our problem. Furthermore, we present the coincidence with the gradient directions on the geodesic for the divergence and on the affine-scaling trajectory for a modified barrier function. These results connect part of nonlinear and algorithmic analyses with the discreteness of variables.

References

[1] S. Amari and H. Nagaoka (2000). *Methods of Information Geometry*, Oxford Univ. Press, Translations of Mathematical Monographs, Vol.191.

[2] D. den Hertog (1994). *Interior Point Approach to Linear, Quadratic and Convex Programming*, Kluwer Academic Publishers.

[3] Y. Nesterov and A. Nemirovskii (1994). *Interior-Point Polynomial Algorithms in Convex Programming*, SIAM Studies in Applied Mathematics.

[4] A. Ohara (1997). *Information Geometric Analysis of a Interior-Point Method for Semidefinite Programming*, RIMS Kokyuroku, pp. 71-89.