Geographical Construction of Scale-free Networks with Both Shortest Path Lengths and Hops

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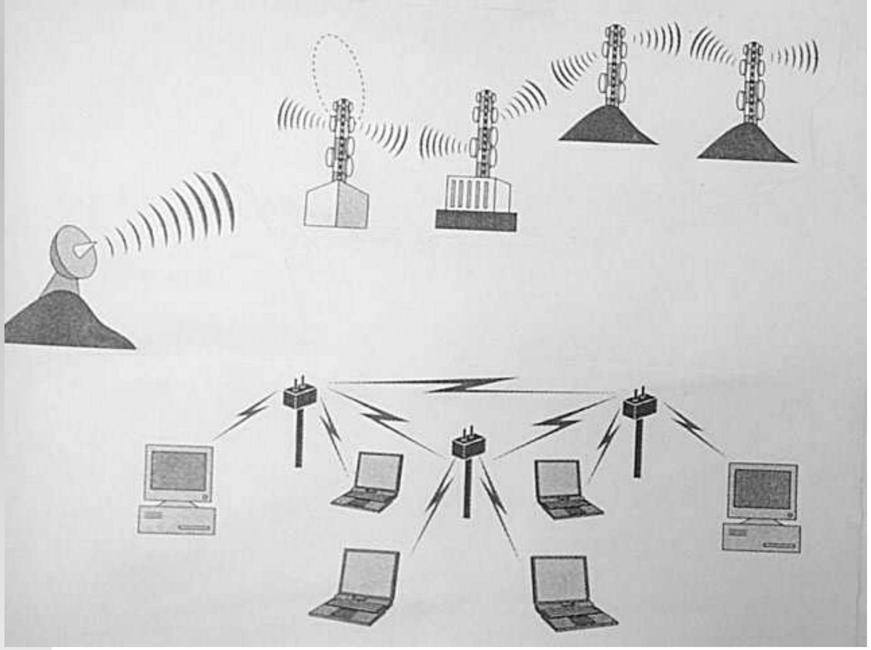
1. Introduction

Dynamic configuration of (backbone) networks

Problems for efficient communication such as in distributed sensor networks or P2P systems



- **dissipation** of wireless beam-power or wired line-cost for long-range links
- **interference** by crossing of links (non-planar)

2-1. Geographical Nets

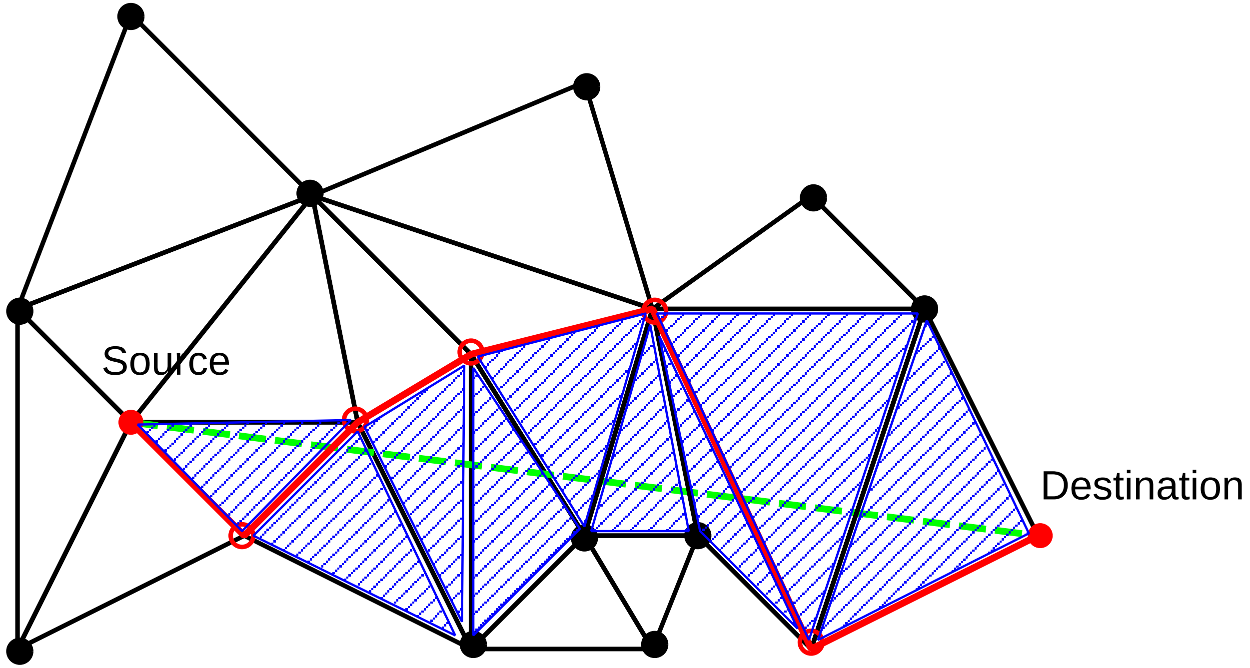
We assume that

- a network evolves with a new node
- the position of each node is fixed
- a few nodes have sufficient power to be hubs
- various transmission ranges and the orientation are controllable (for wireless links)
- It is better that only local information is used because of the topological change

These are reasonable in the current technology, but we don’t care about the details.

2-2. Efficient Routing

Planar triangulation: reasonable math. abstraction of ad hoc net. (each triangle forms a service region) Moreover, a memoryless, no defeat, and competitive online routing algorithm has been developed for such networks taking into account the face.



Bose and Morin, SIAM J. of Comp. 33(4), 2004

2-3. Objective

To make a good design of geo. nets, we investigate

the communication efficiency measured by the shortest path lengths or the min. hops

the tolerance of connectivity to random failures and targeted attacks on hubs

in typical planar network models:

- random Apollonian network in complex network science,
- Delaunay triangulation in computer science,
- our proposed models to bridge them.

3-1. Scale-Free Nets with Hubs

Existing a surprisingly common structure: SF net. the degree dist. follows $P(k) \sim k^{-\gamma}$, $2 < \gamma < 3$, in many social, technological, and biological nets.

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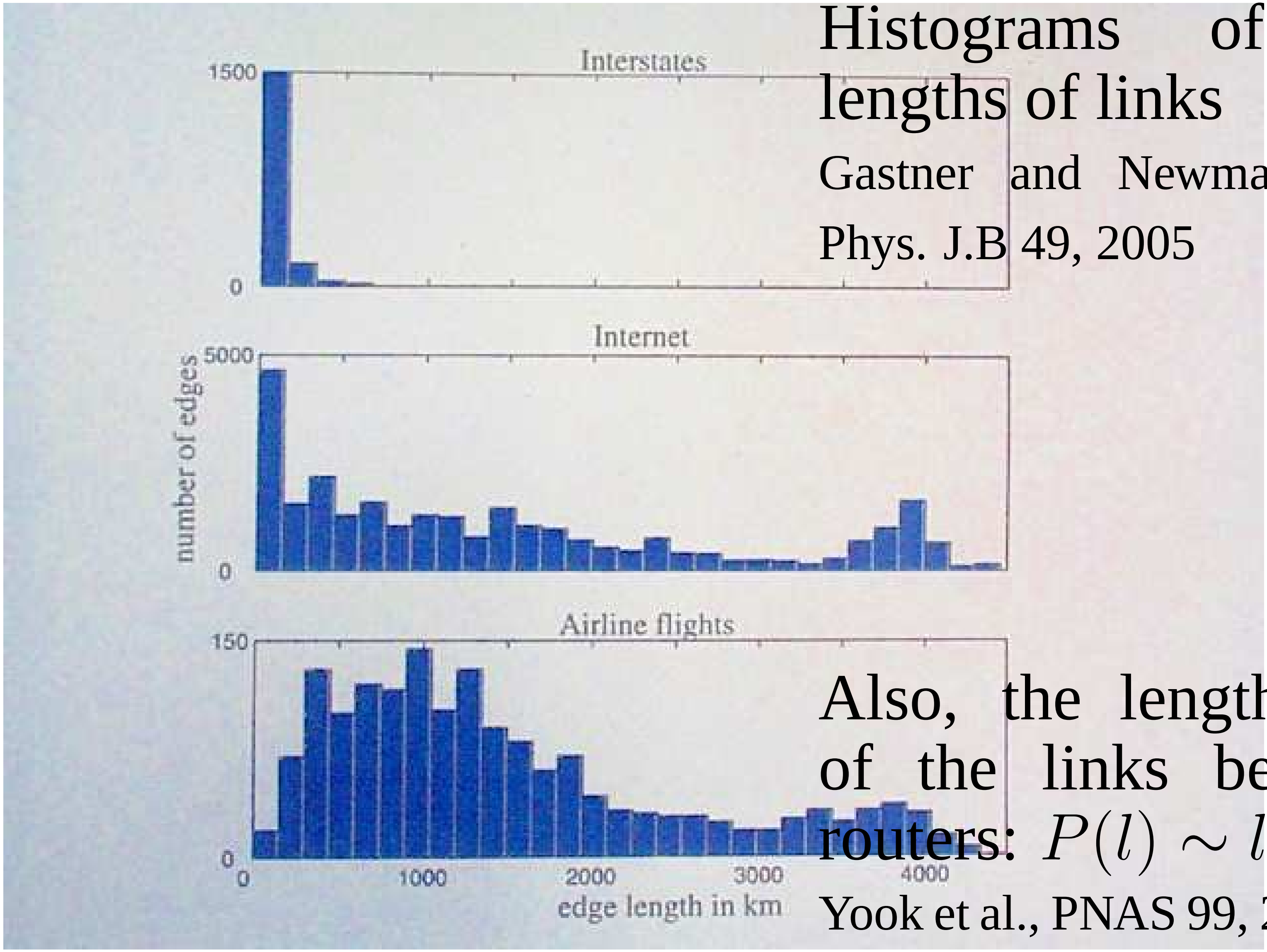
⇒ However, most of SF net models were irrelevant to a geographical space.

3-2. Rare Long-range Links



Histograms of the lengths of links

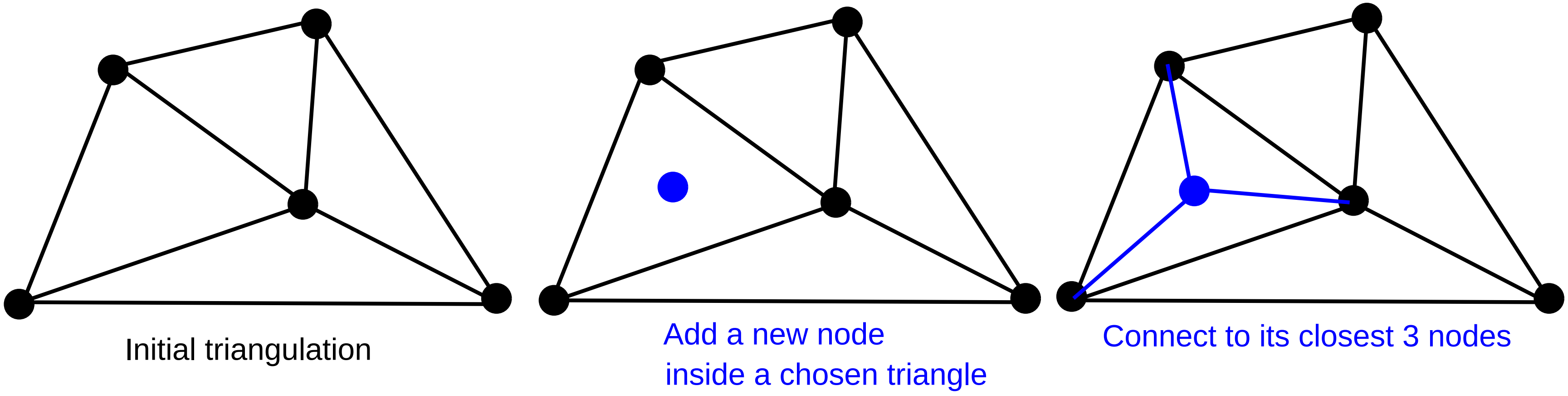
Gastner and Newman, Eur. Phys. J.B 49, 2005

Also, the length dist. of the links between routers: $P(l) \sim l^{-1}$

Yook et al., PNAS 99, 21, 2002

4-1. Random Apollonian Net

Configuration: iterative subdivision of a randomly chosen triangle from an initial triangulation

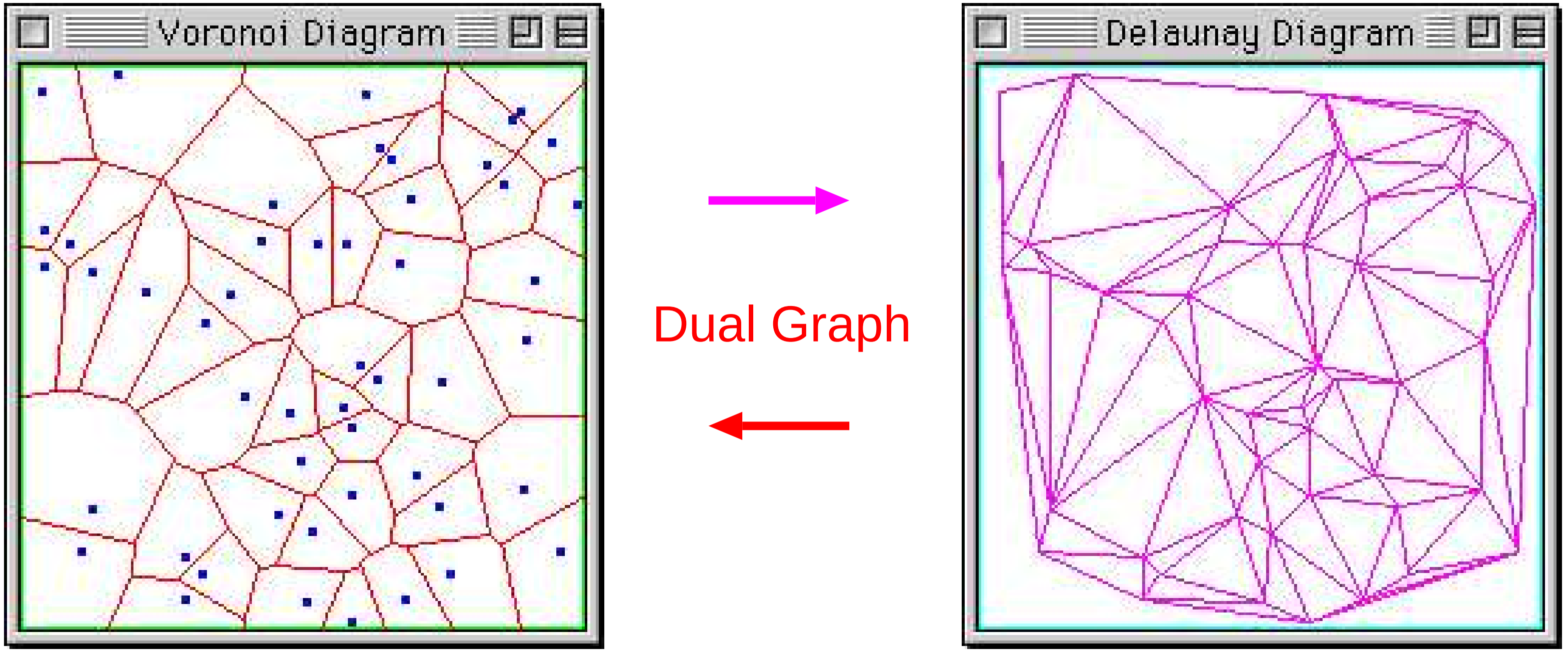


⇒ Some long-range links naturally appear in narrow collapsed triangles near the boundary edges

Zhou, Yan, Wang, Phys. Rev. E 71, 2005

4-2. Voronoi and Delaunay

Optimal triangulation in some geometric criteria: maximim angle, minimax circumcircle, short path length in the same order as the direct Euclidean dist.



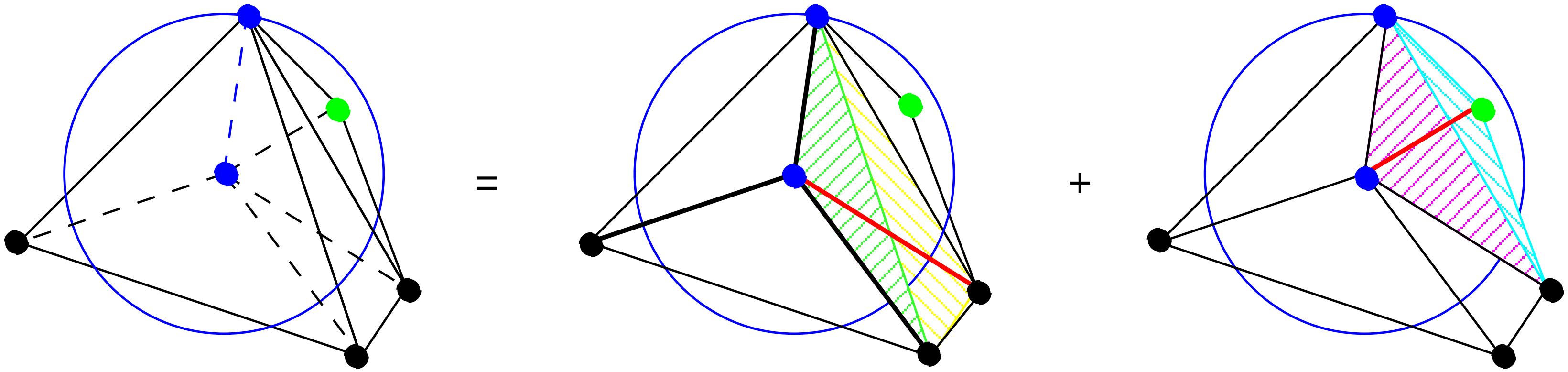
Dual Graph

⇒ Consider the combination of RA (by triangulation on a plane) and DT to avoid the long-range links

4-3. Delaunay-like SF Net

We propose RA+NN:

- Set an initial planar triangulation.
- Select a triangle at random and add a new node at the barycenter. Then, connect the new node to its three nodes of the triangle. By iteratively applying diagonal flips, connect it to the nearest node(s) within a radius as a localization.

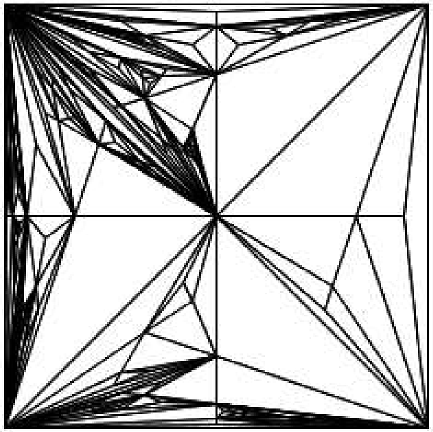


1st diagonal flip

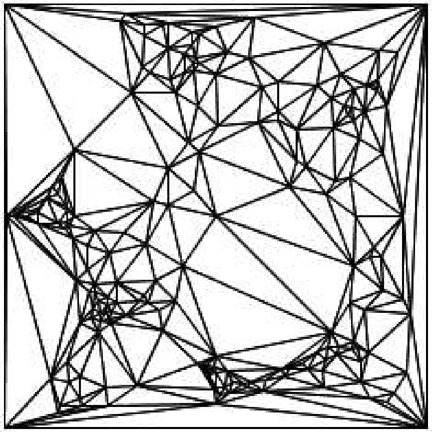
2nd diagonal flip

5-1. Topological Structure

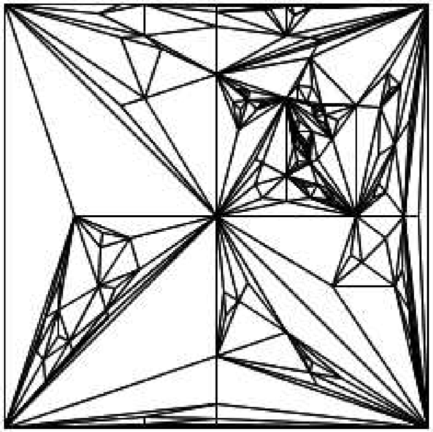
RA+NN has the intermediate structure



RA



DT

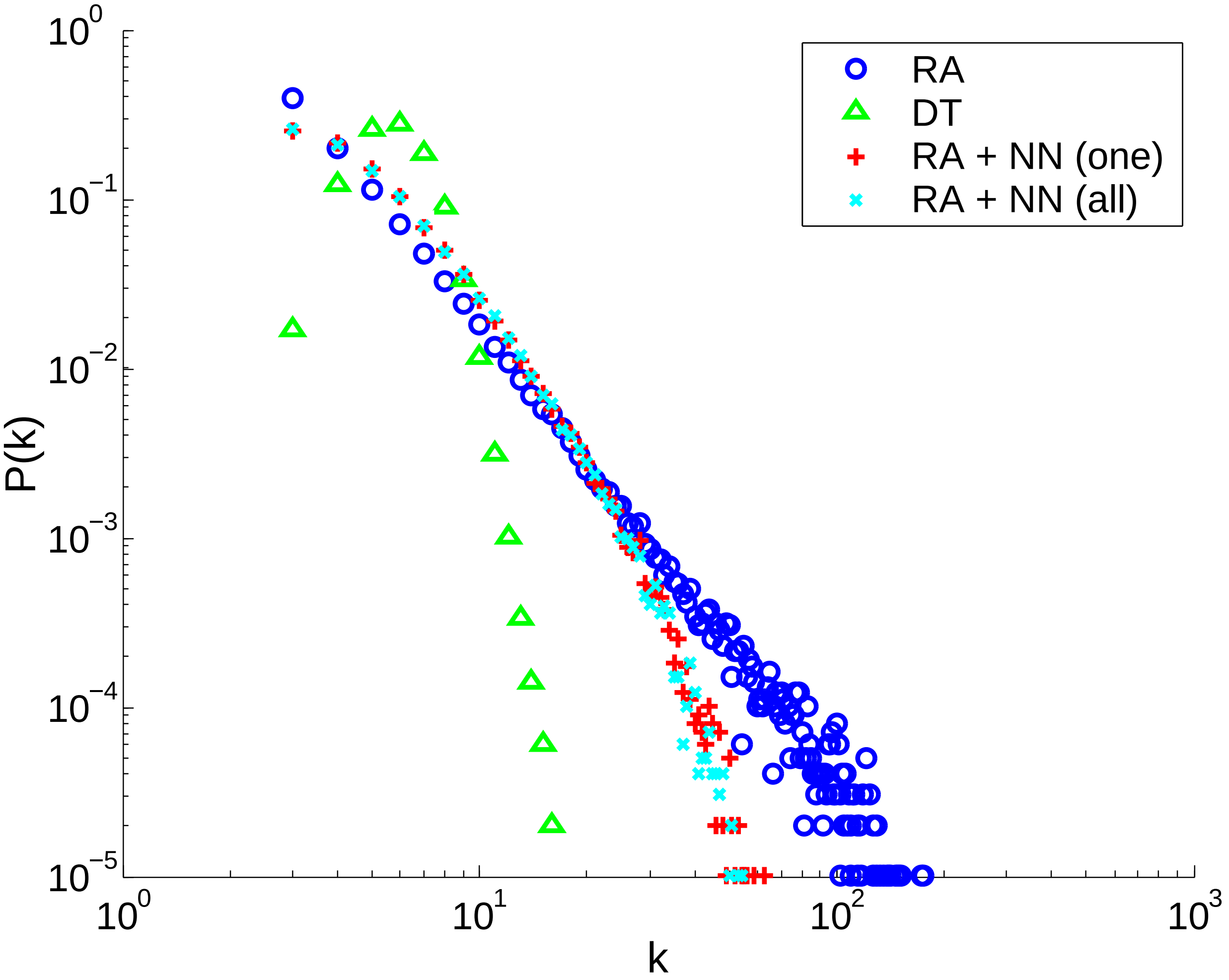


RA+NN(one)

We should remark that there exists

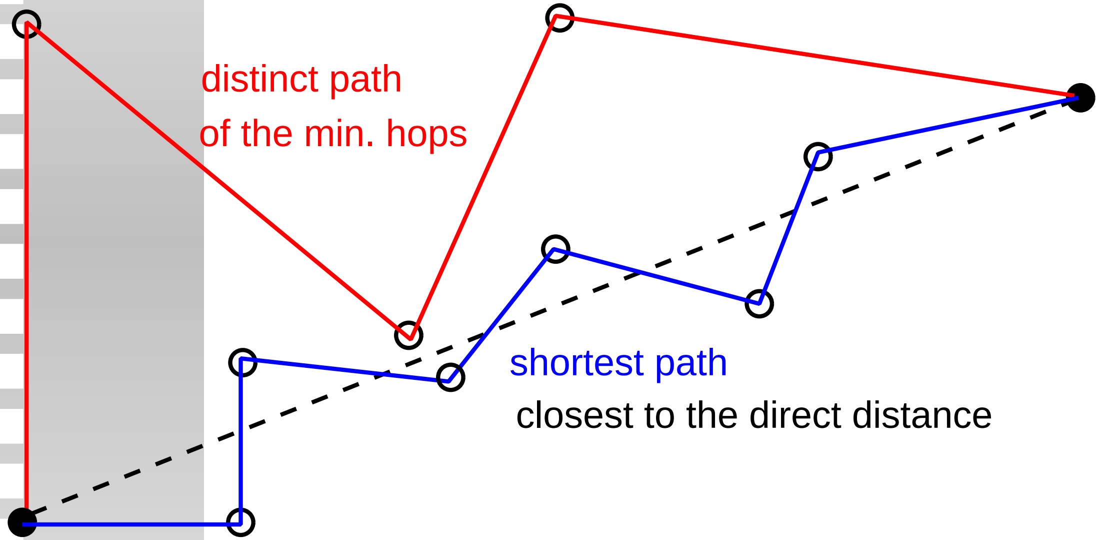
- star-like stubs at four corner and the center nodes
- mixing of dense and sparse areas

5-2. Degree Distribution



RA: power law, DT: lognormal, RA+NN: power law with exponential cutoff (the deg. of hubs are reduced)

6-1. Communication Efficiency



$\langle D \rangle$: average distance on the shortest paths

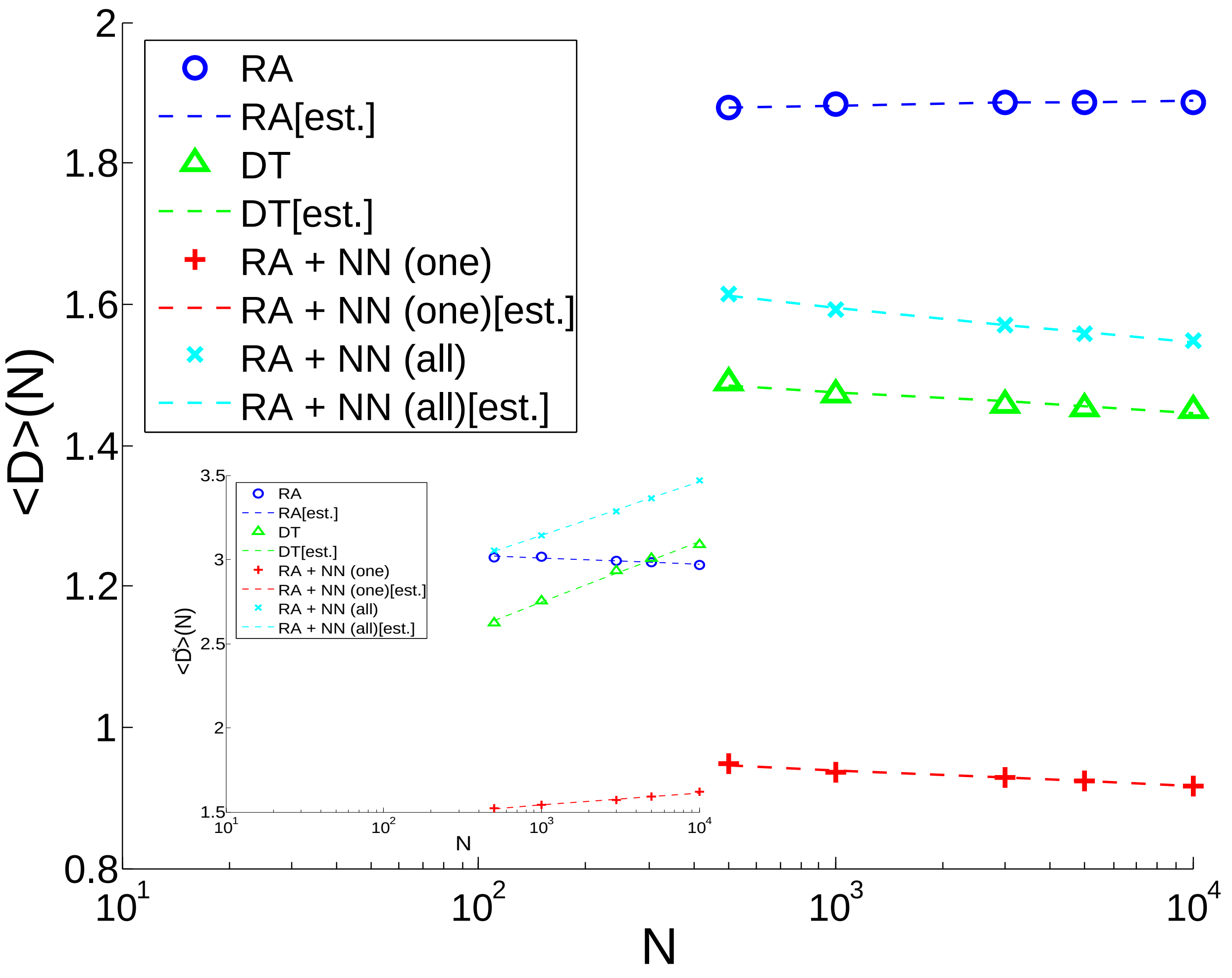
$\langle D' \rangle$: average distance on the paths of the min. hops

$\langle L \rangle$: average number of hops on them

$\langle L' \rangle$: average number of hops on the shortest paths

6-2. Distance on the Opt. Path

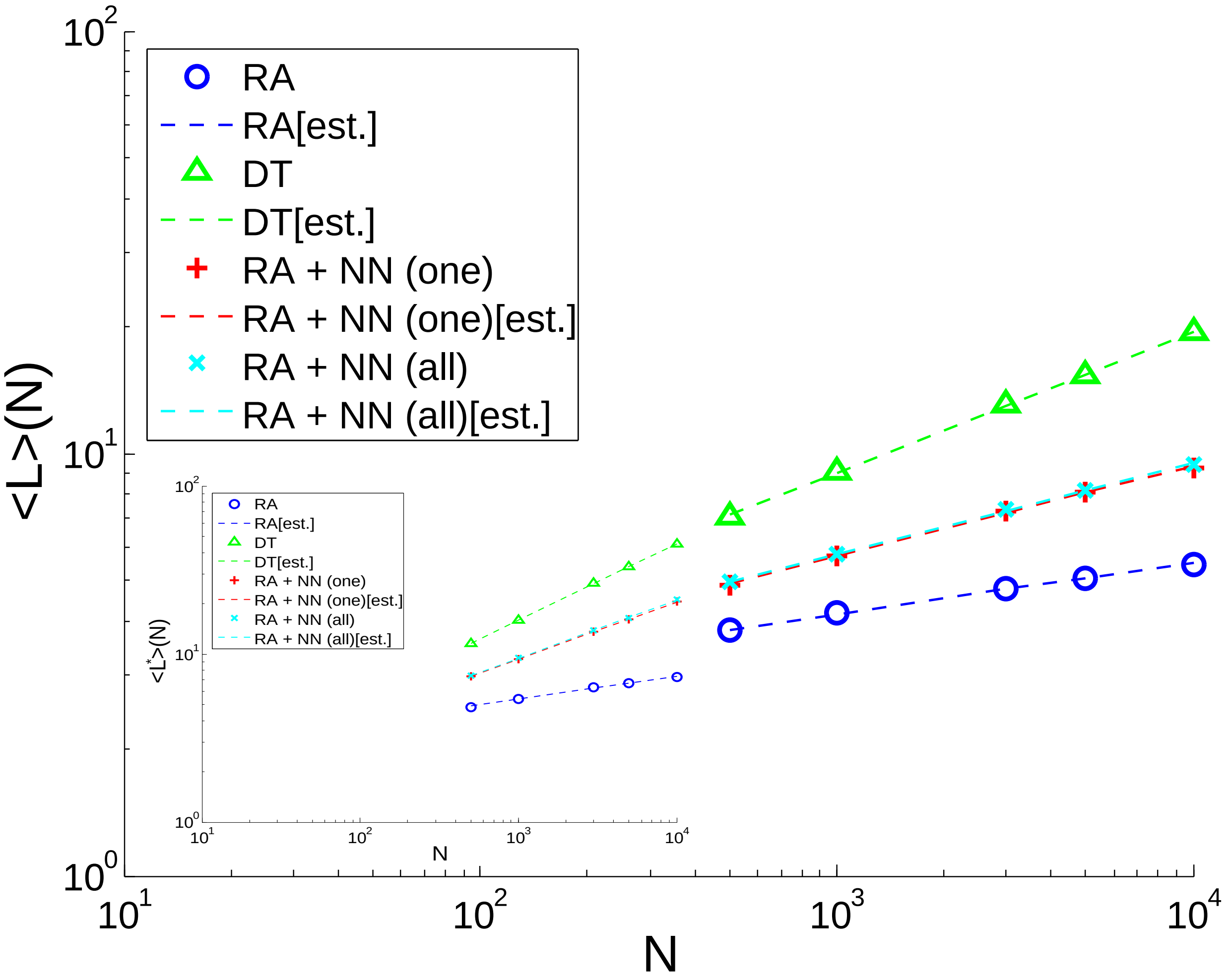
Scaling relation: $\langle D \rangle \sim (\ln N)^{\beta_d}$, $\langle D' \rangle \sim (\ln N)^{\beta_{d'}}$



⇒ Our model (marked by +) is the best, shortest

6-3. Min. Hops for Transfer

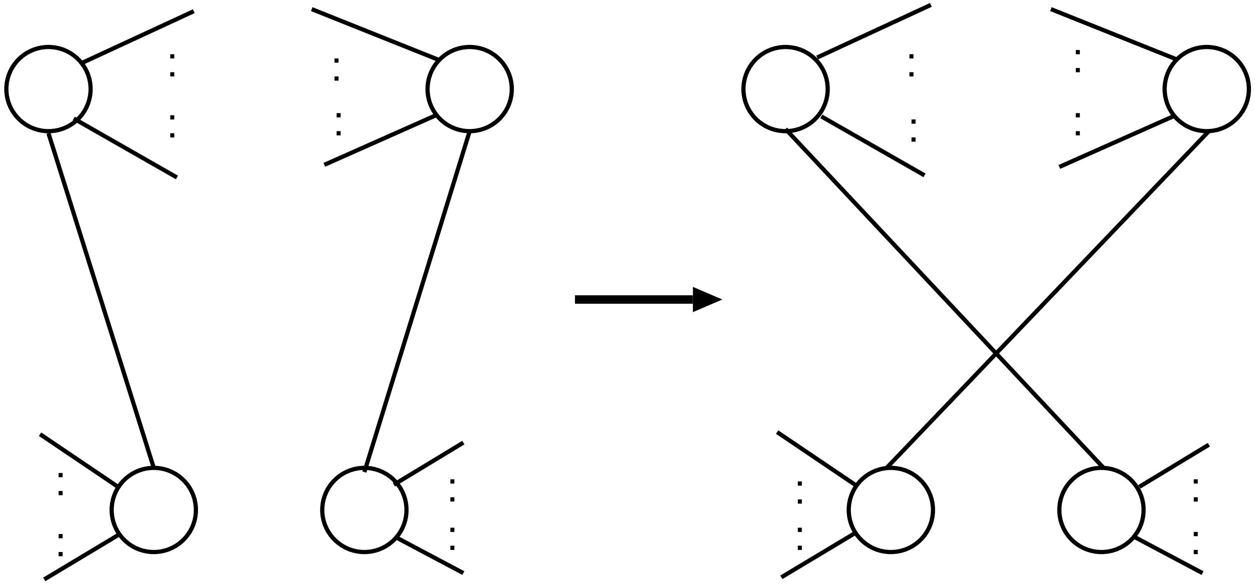
Scaling relation: $\langle L \rangle \sim N^{\alpha_l}$, $\langle L' \rangle \sim N^{\alpha_{l'}}$



$\Rightarrow$ Our model (marked by $+$) is the intermediate

7-1. Randomly Rewired Nets

We compare the tolerance to random failures of nodes and targeted attacks on hubs in the geographical and non-geographical randomly rewired networks, when a fraction f of nodes is removed.

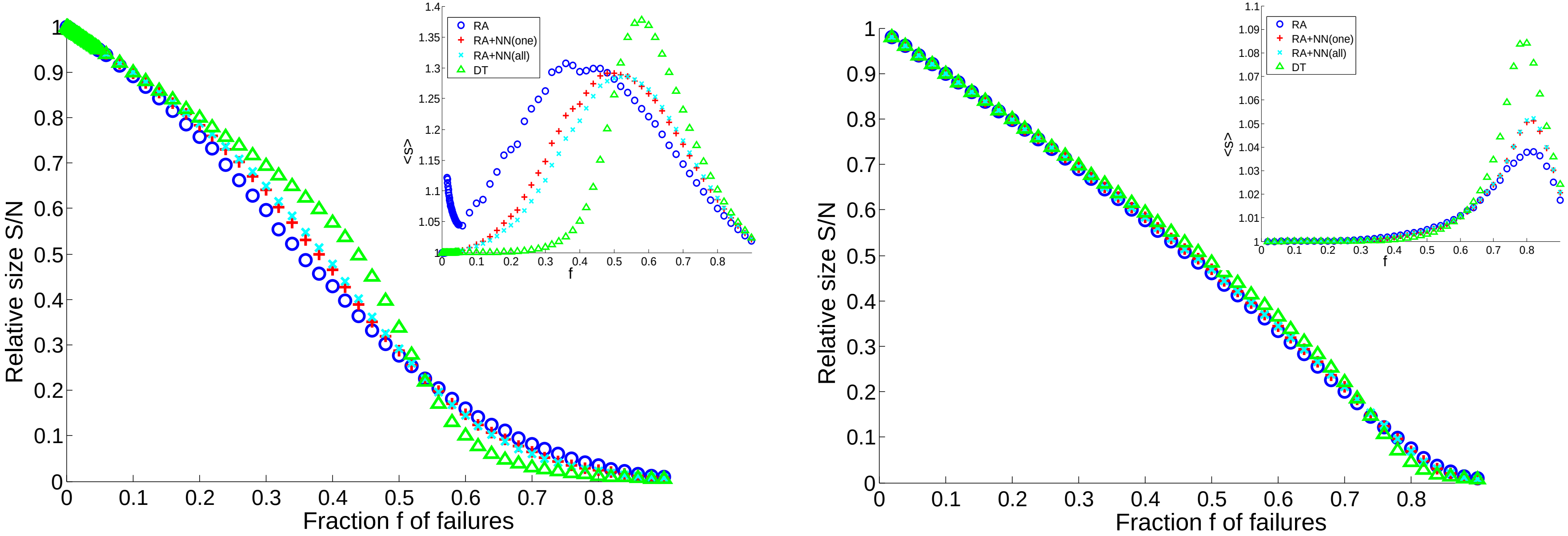


Rewiring a pair of links with the same degree at each node

Maslov et al., Physica A 333, 2004

7-2. Tolerance to Failures

Relative size S/N of the giant component
Inset: the average size $\langle s \rangle$ of isolated clusters

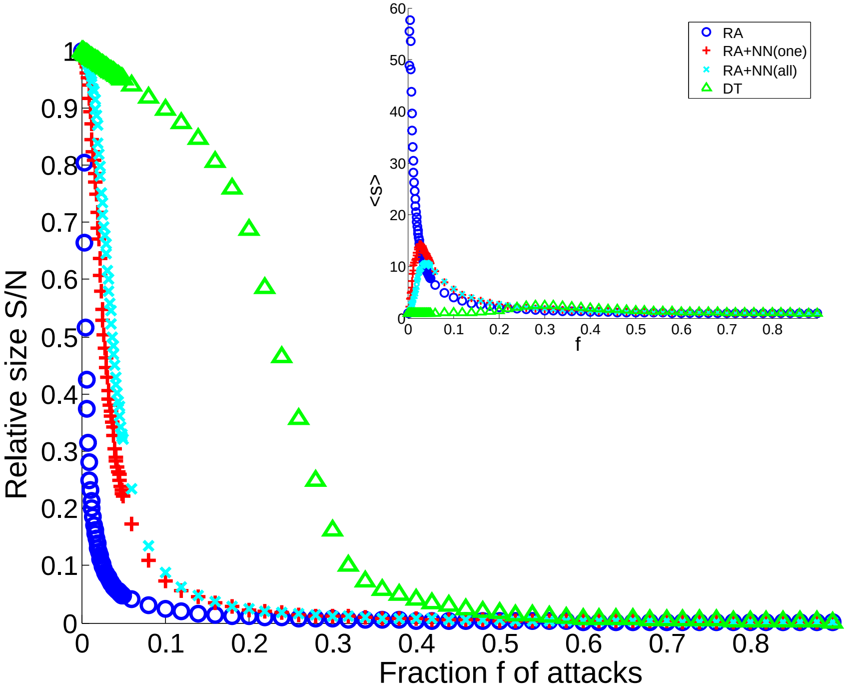


(a) geographical nets (b) rewired nets

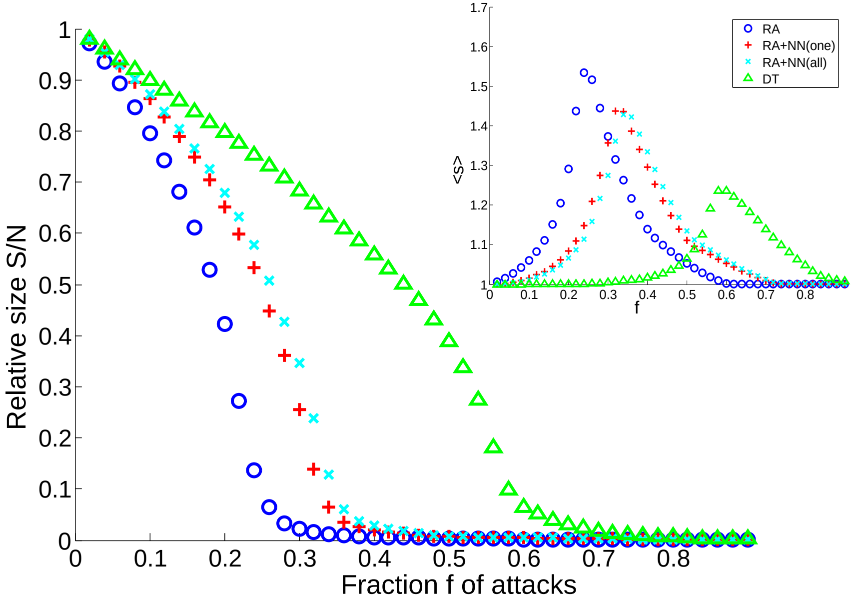
$\Rightarrow$ Similar robustness in these models

(RA: ○, DT: △, RA+NN(one): +)

7-3. Tolerance to Attacks



(a) geographical nets



(b) rewired nets

⇒ Improvement from the extremely vulnerable RA

7-4. Related Topics

Similar improvement relaxed from geographical constaints by rewirings has been shown in SFL.
From a theoretical viewpoint, small-order cycles significantly affect the vulnerability.
At no-cycle $L_c = 0$, the percolation threshold $q_c^* = \langle k \rangle / (\langle k^2 \rangle - \langle k \rangle)$ is well-known.
It is generalized at any cycle length e.g. $L_c = 3$,

$$q_c = \frac{\langle k \rangle}{\langle k(k-1) \rangle - \left(1 - q_c \frac{\langle k(k-2) \rangle}{\langle k \rangle} \right) \langle C(k) k (k-1)^2 \rangle},$$

$1 - f_c = q_c > q_c^*$ predicts decreasing of robustness.

Huang et al., Europhy. Lett. 72, & arXiv:physics/0503147, 2005

8. Summary

Based on the advanced SF properties,

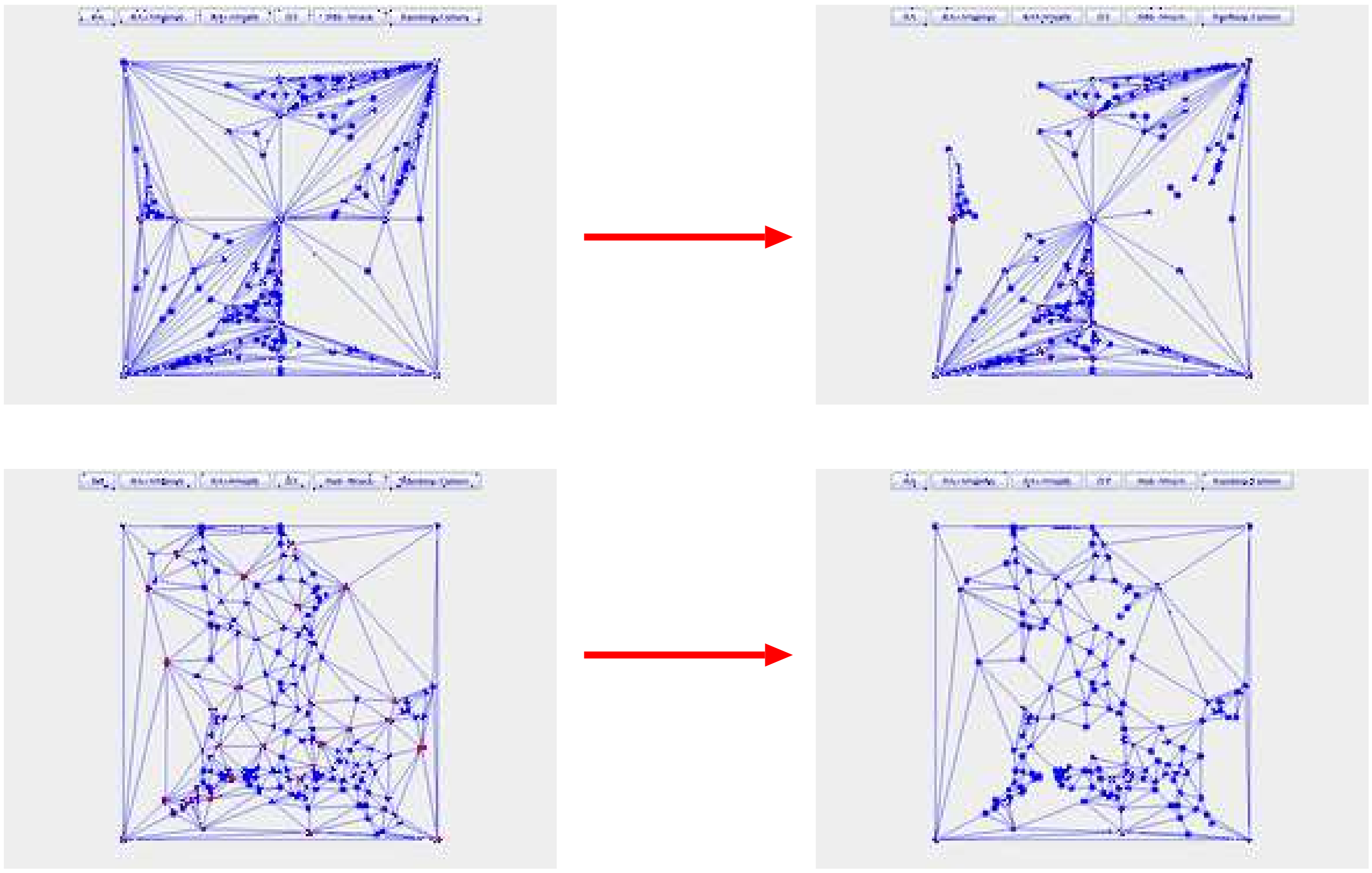
- we've proposed a modified model from RA in complex net. science and DT in computer science to reduce long-range links on a planar space for sensor or ad hoc networks.
- In our model, $\langle D \rangle \sim (\ln N)^{\beta_d}$ is the shortest, and $\langle L \rangle \sim N^{\alpha_l}$ is the intermediate on the optimal paths in two criteria (A pure SF is not the best).
- The tolerance to failures and attacks is weakened by the geo. effect, although it is improved in our model from the extreme vulnerability in RA. DT is the most robust, it requires global config. $\Rightarrow$ trade-off: localization vs robustness

Thanks

Thank you for your kind attention !

For more details: Geographical effects on the path length and the robustness in complex networks, Physical Review E 73, May (2006)

A1. Damages by Failures



Initial N=200

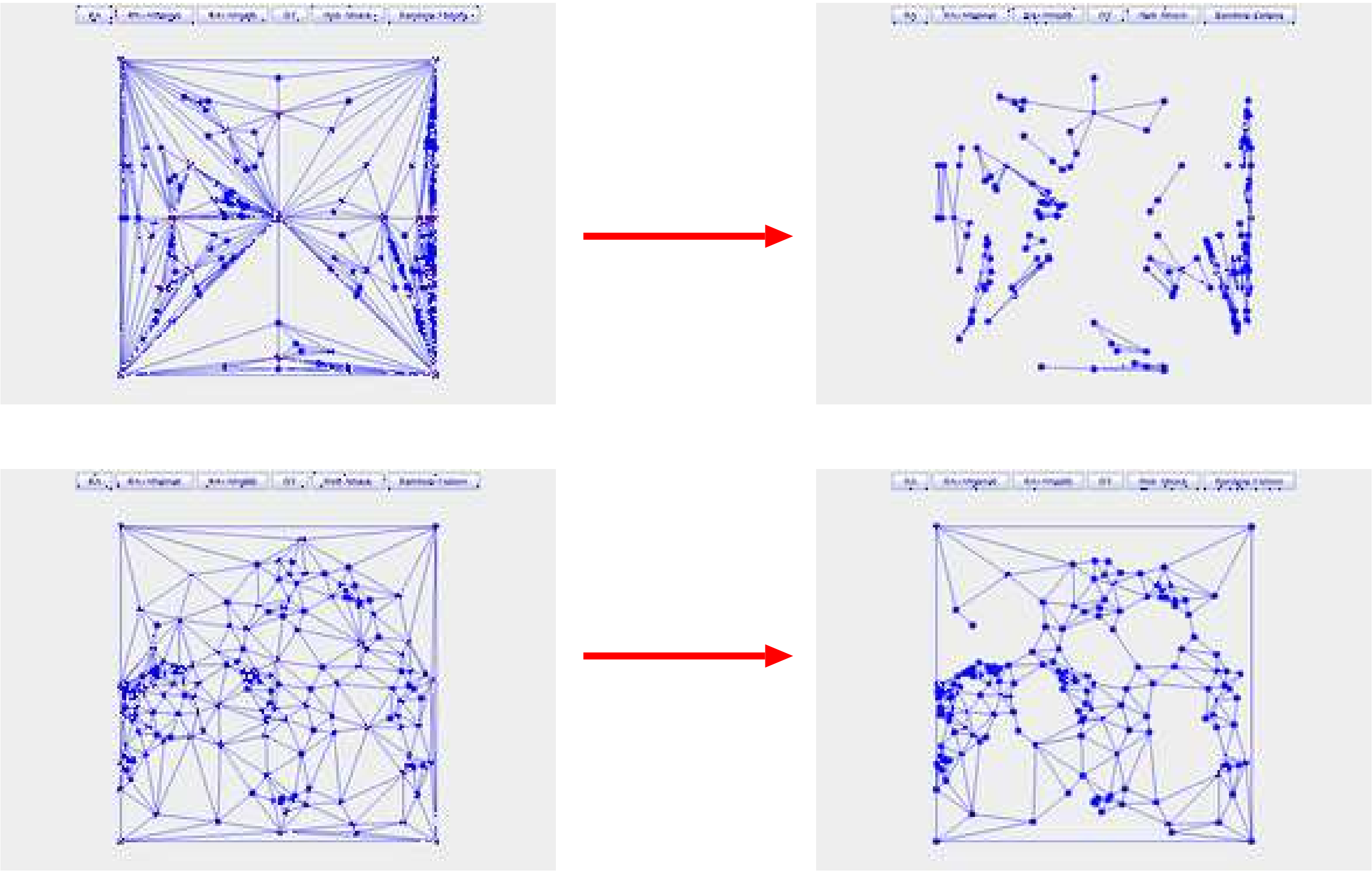
randomly removed
32 nodes

RA

DT

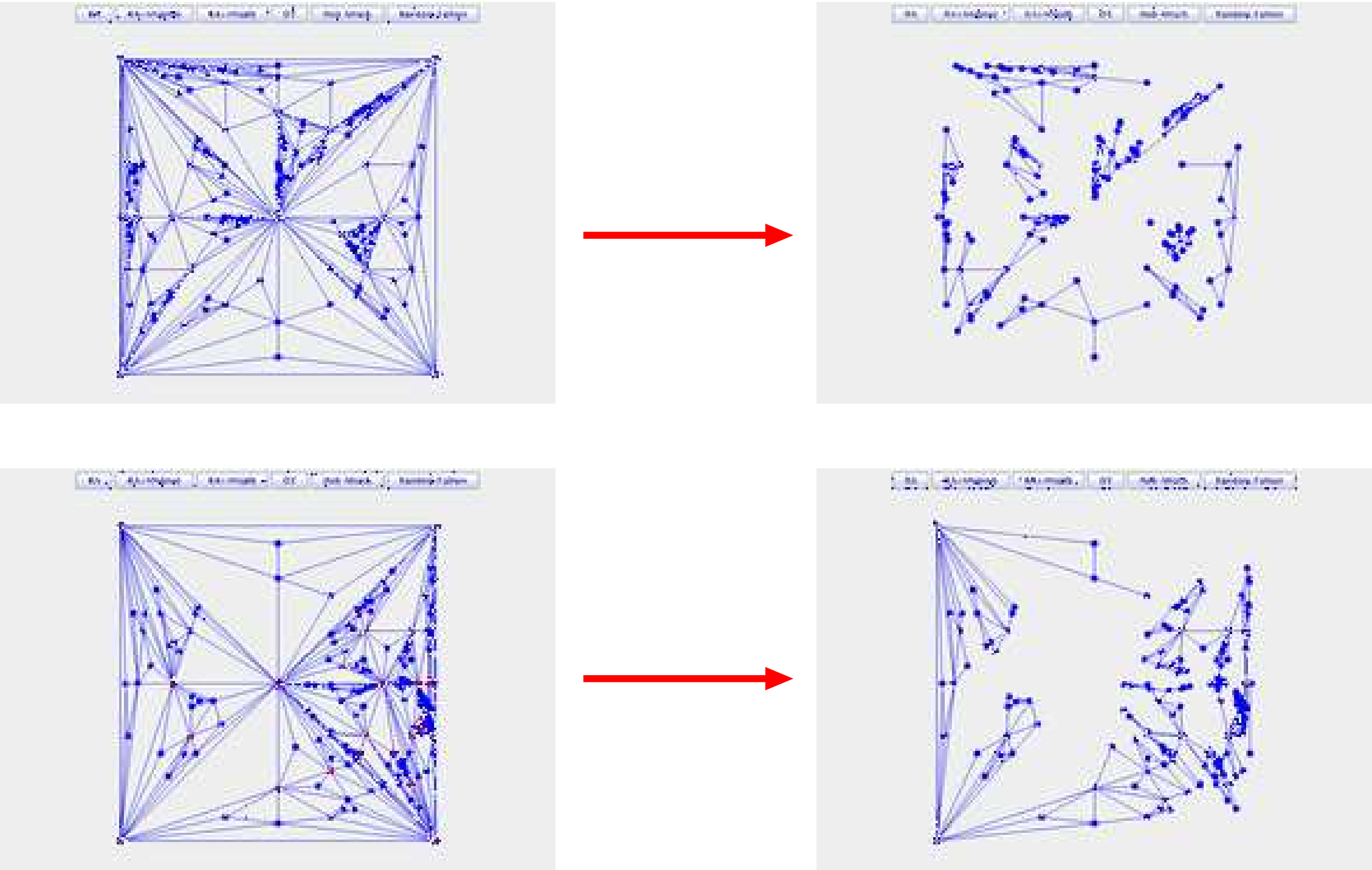
Similarly remaining the initial connected component

A2. Damages by Attacks



Extreme vulnerable RA with many isolated clusters

A3. Damages by Attacks (cont.)



Initial N=200

targeted attacks on
16 hubs

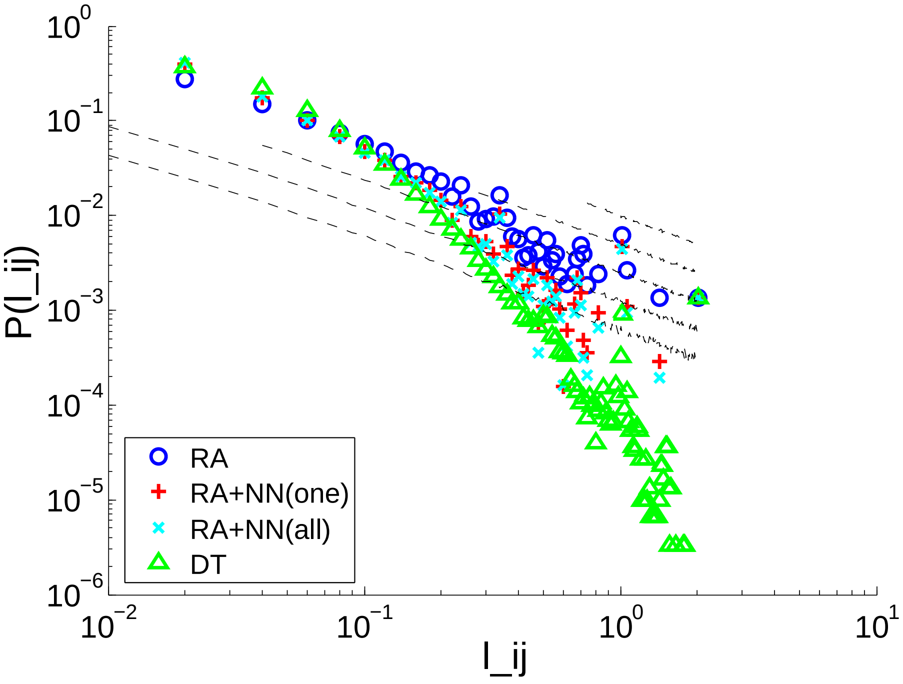
RA+NN(one)

RA+NN(all)

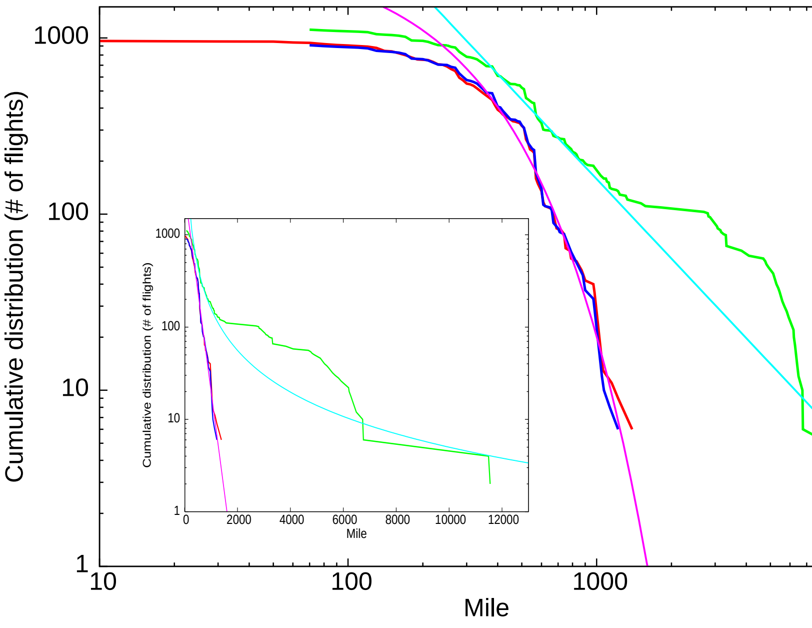
Slightly improved robustness from that in RA

A4. Weak Disorder

Distribution of path lengths -exp. decaying- the dashed lines from top to bottom correspond to $\delta = 1, 2, 4, 8, 16$ in the assumed weight $\exp(\delta\varepsilon)$ of link with a random number $\varepsilon \in (0, 1)$



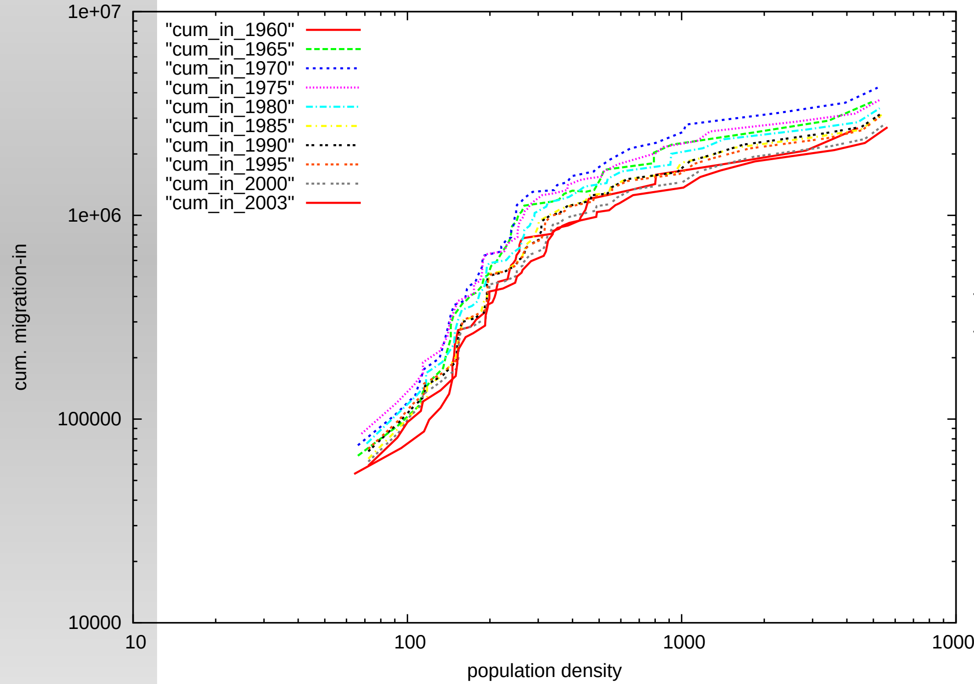
(a) geographical nets

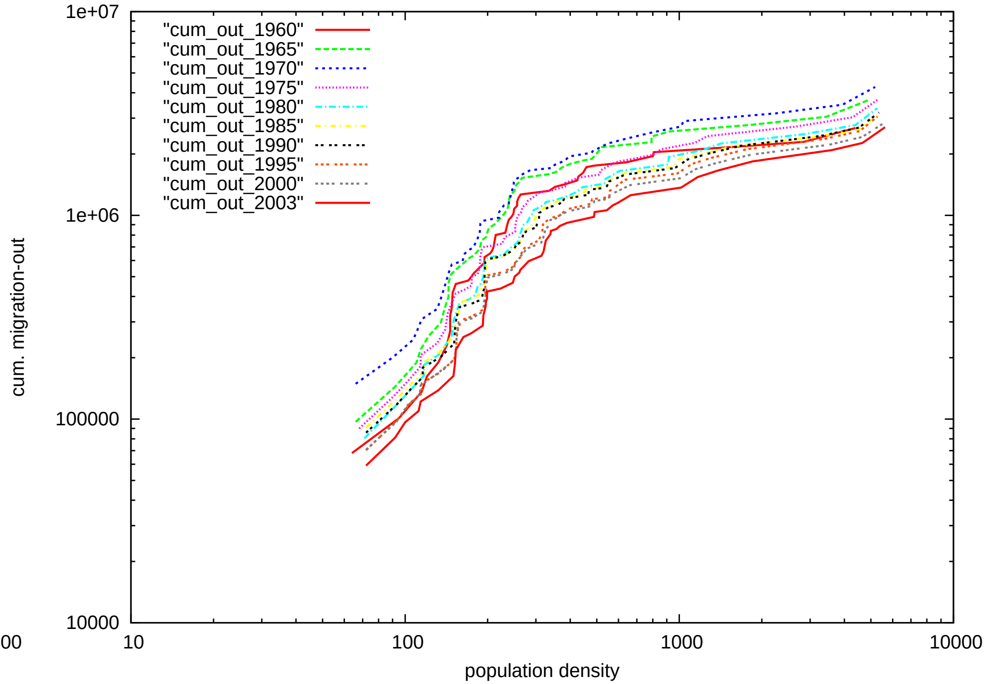


(b) flight connections in JAL and ANA

A5. Dense-get-denser

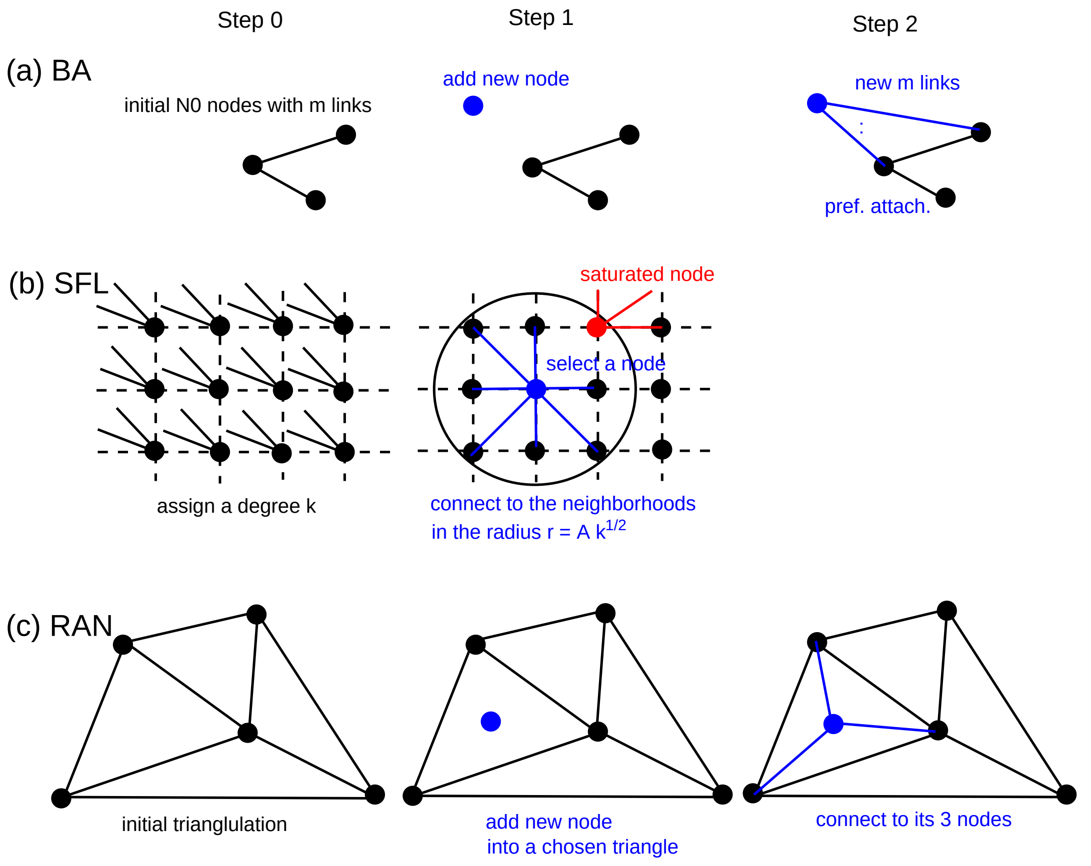
Migration-in/out versus population density of Japanese prefectures in each year





A6. Classes of Geo. SF Nets

- Modulated BA: $\Pi_i \sim k_i \times l^{\alpha}$, rand. position of node
- SF on lattices: connect within $r = A \times k_i^{1/d}$
- Space-filling: subdivision of a region (heterogeneous dist. of nodes)



A7. Planarity and Shortness

class	planarity of net	shortness of links
Modulated BA Manna’02, Xulvi-Brunet’02	× ∃ crossing links (not prohibited)	○ with disadvantaged long-range links
SF on lattices ben-Avraham’03, Warren’02	× cross of regular links and shortcuts	△ ∃ long shortcuts from hubs
Space-filling Apollonian nets. Doye’05, Zhou’04	○ by subdivision of a selected region	△ ∃ long-range links in narrow triangles