Scale-free network models for epidemic dynamics

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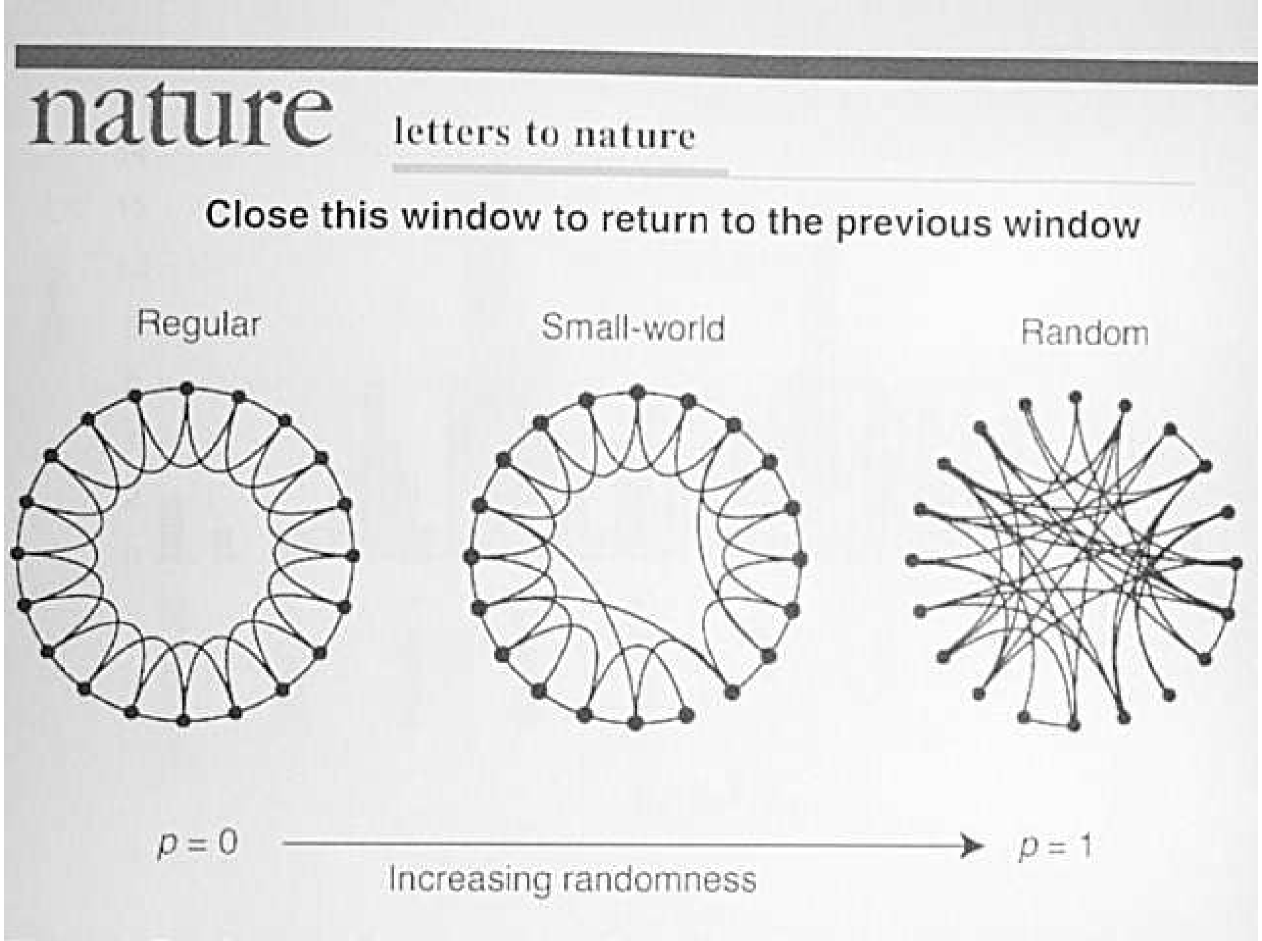
1-1. Small World

Many real networks are positioned between regular and random graphs: highly clustered & short distance

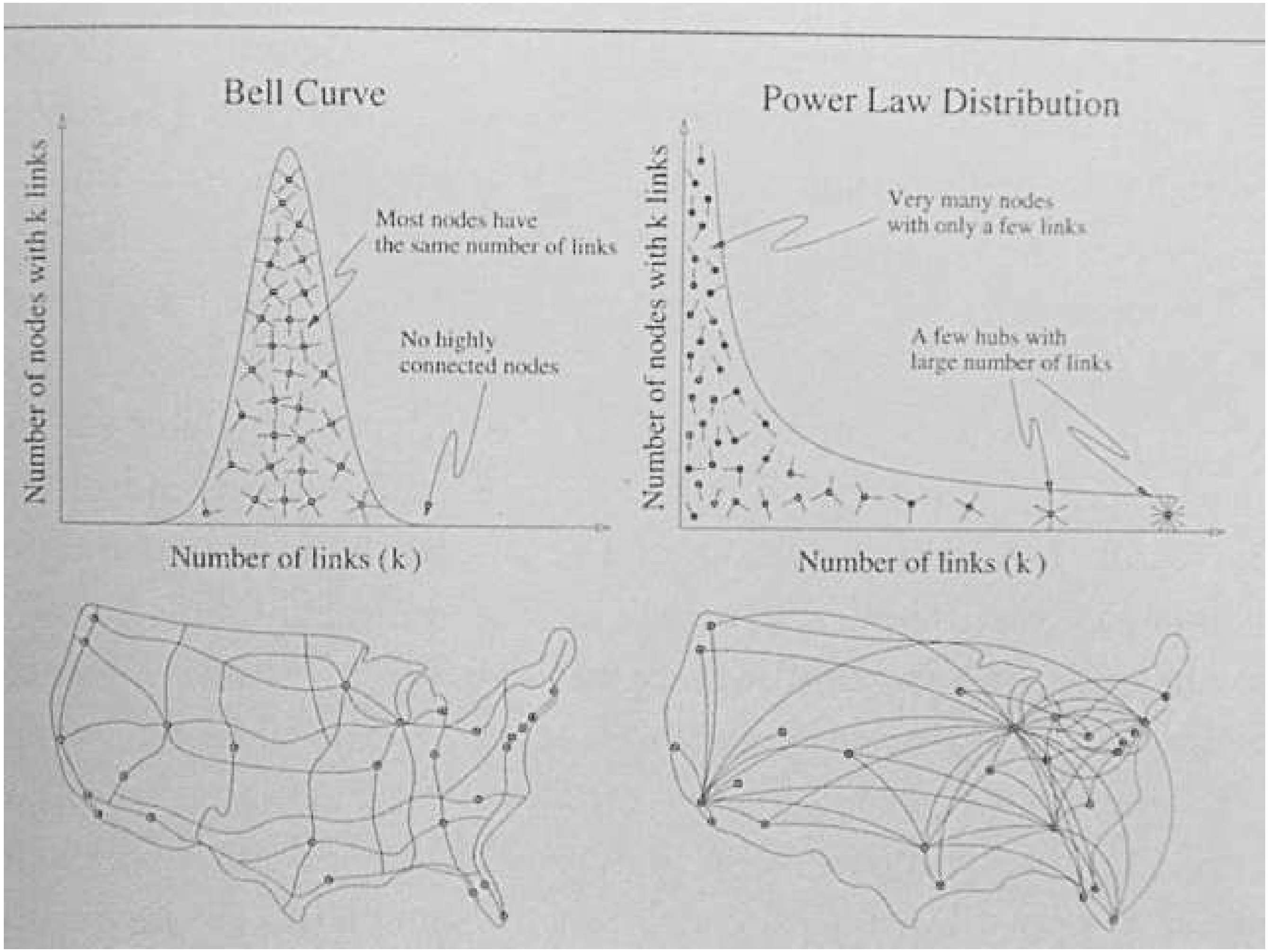


D.J. Watts and S.H. Strogatz, Nature, 393, 1998

1-2. Scale-Free Network

Existing a surprisingly common structure: SF net.

the degree dist. exhibits $P(k) \sim k^{-\gamma}$, $2 < \gamma < 3$.



In A.L. Barabási, LINKED, Perseus Pub., 2002

2-1. Universality

Recently('98-'02), the surprisingly common structure has been found in many real nets

Social: acquaintance, world trading, actor-collabo., citation, language

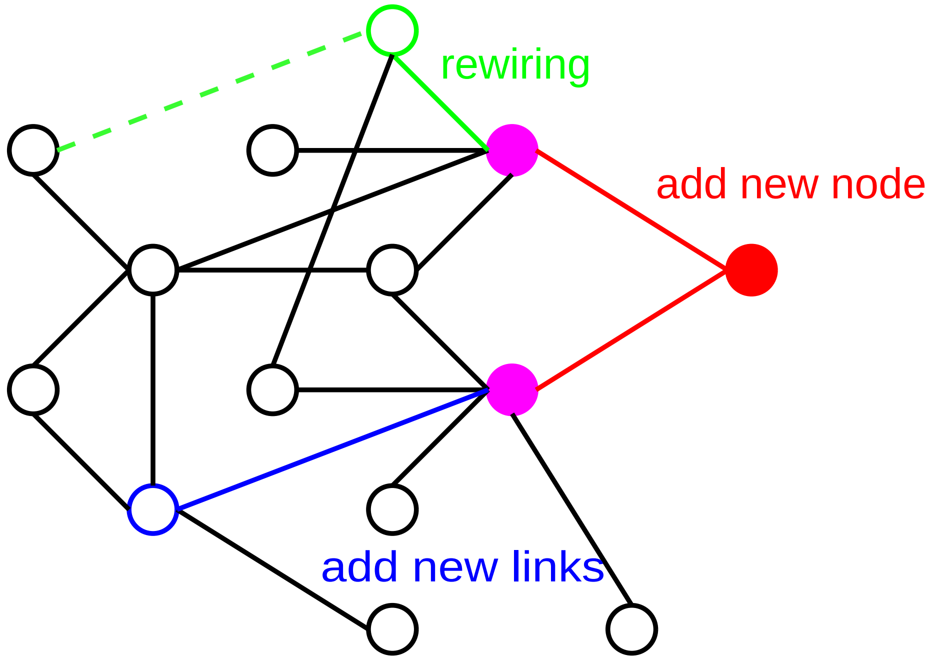
Technological: Internet, WWW, email, power grid

Biological: neural net, genome, metabolic pathway, foodweb

Universal evolution mechanism has been elucidated: Growth & Preferential Attachment

A.L. Barabási et al., Physica A, 272, 1999

2-2. Generalized BA Model

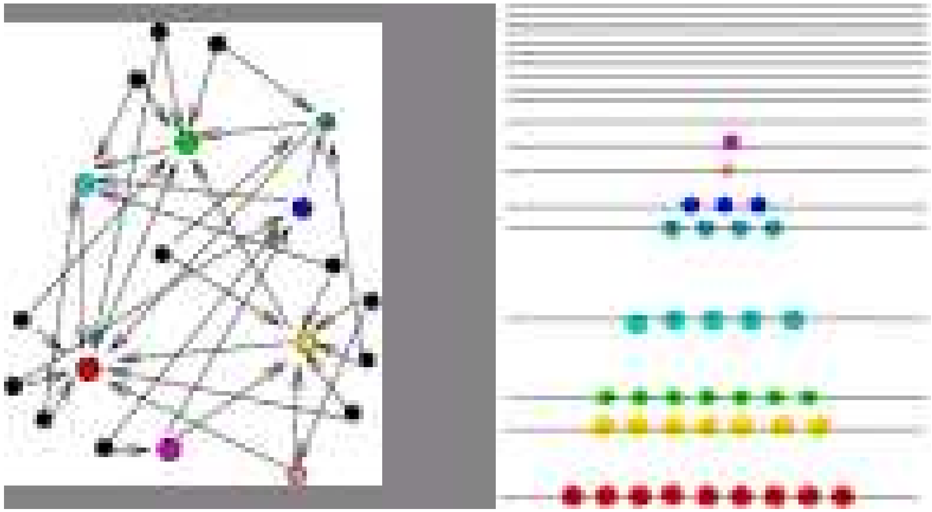


For the degree k_i of a node i inserted at time t,

$$\frac{\partial k_i}{\partial t} = pm\frac{1}{N} + pm\frac{k_i+1}{\sum_l (k_l+1)} - qm\frac{1}{N} + qm\frac{k_i+1}{\sum_l (k_l+1)} + (1-p-q)m\frac{k_i+1}{\sum_l (k_l+1)}.$$

R. Albert, and A.L. Barabási, PRL 85, 2000.

2-3. Fitness Model



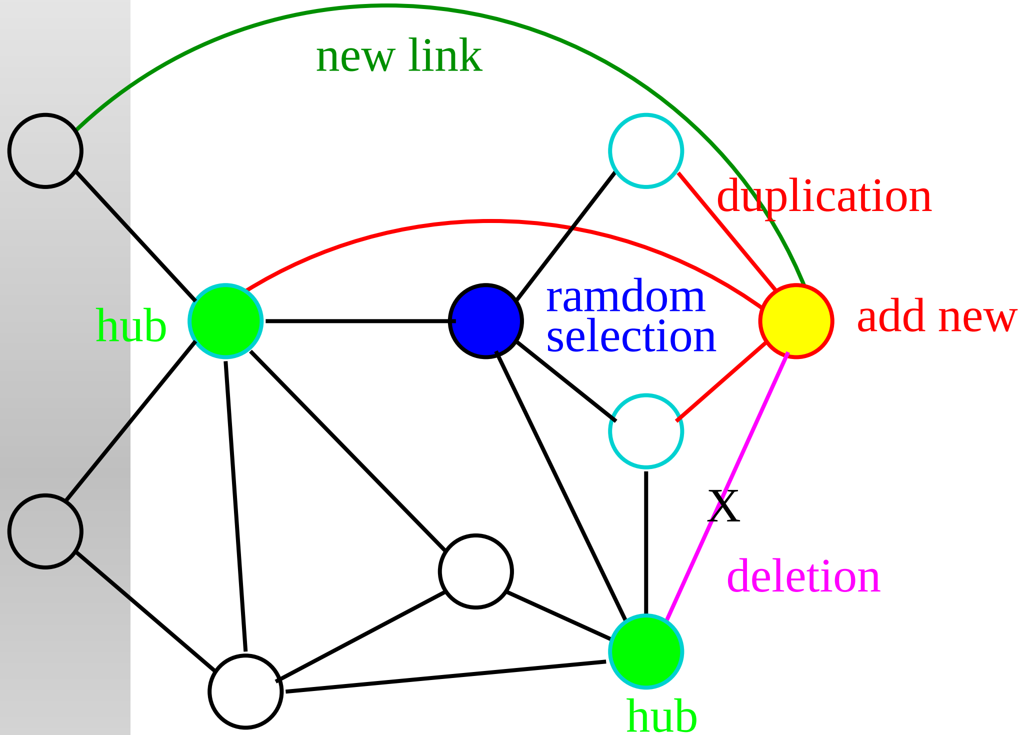
G. Bianconi et al., PRL 86, 2001.

monopolization of links as similar phenomenon to Bose-Einstein condensation

<u>Other models</u>

- age-effect, S.N. Dorogovtsev et al., PRL 85, 2000
- hierachical organization, E. Ravas, A.L. Barabási et al., Science 297, 2002

2-4. Duplication Model



In spite of random node selection, the neighbor hub node has many chance to get duplicate connections (the prob. is prop. to k_i).

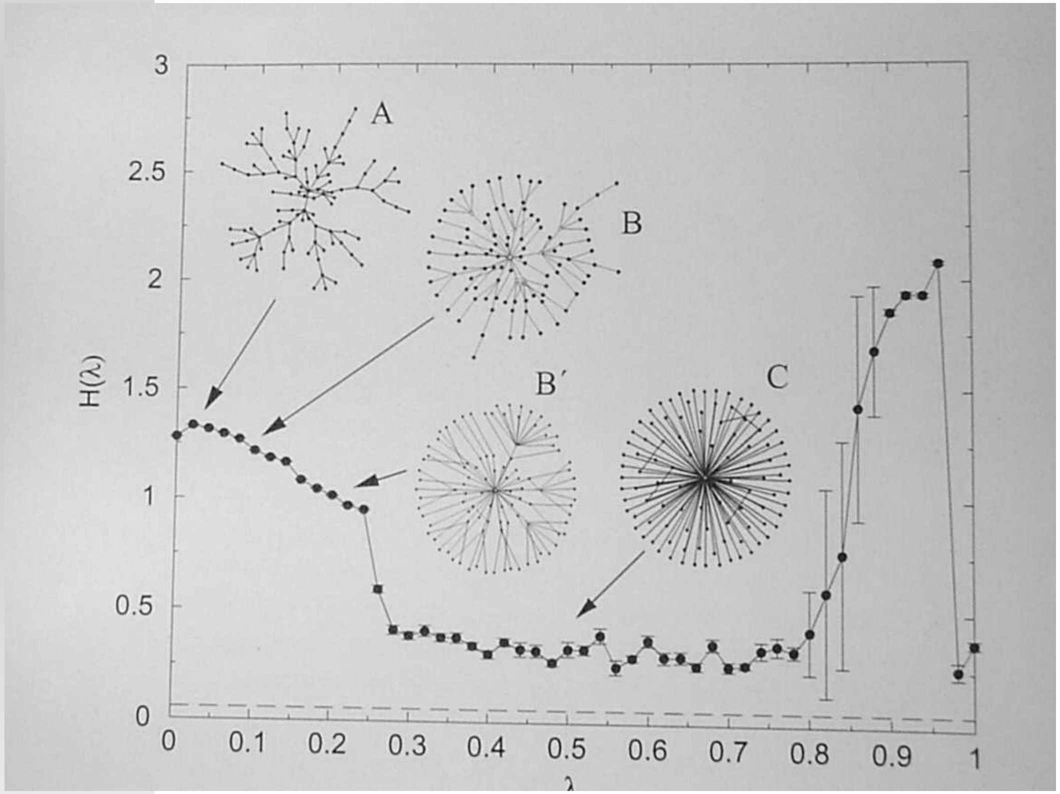
$\Rightarrow$ Biologically plausible networks realize Preferential Attachment in a local rule !

R.V. Solé et. al., Advances in Complex Systems, 5, 2002

3-1. Optimal Topology

economy, # of links ρ $\leftarrow 0 < \lambda < 1 \rightarrow$ efficiency, distance d

Random (tree) - Pref. (SF) - Forced (star, clique)



SF appears in random generations for min.

$E(\lambda) = \lambda d + (1 - \lambda)\rho$,

$d \stackrel{\text{def}}{=} \frac{\sum_{i<j} D_{ij}}{{}_nC_2} / D_{max}$,

$\rho \stackrel{\text{def}}{=} \frac{\sum_{i<j} a_{ij}}{{}_nC_2}$,

with a weight λ

entropy $H(\lambda)$ vs. weight λ

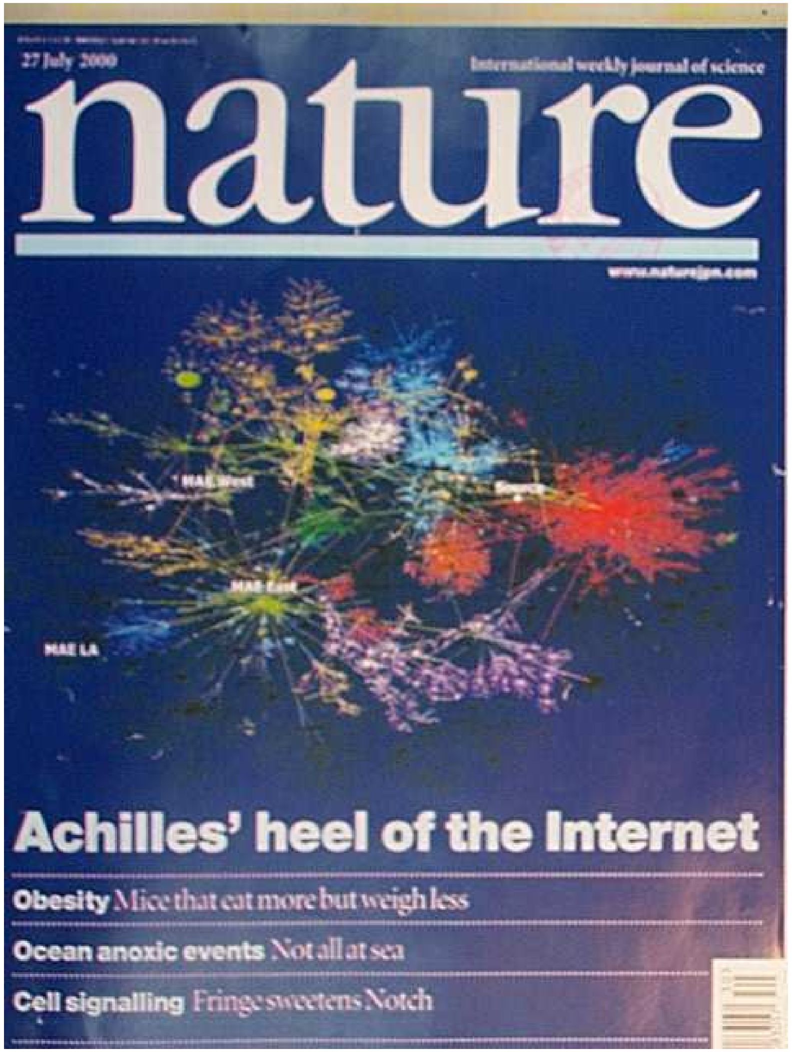
R.F. i Cancho and R.V. Solé, SantaFe Inst. working paper, 2001

3-2.

Robust and Vulnerable Connectivity

Robust: for random failure, remaining the connectivity

Vulnerable: for targeted attack against hubs, disconnecting into isolated parts



4-1. Conventional SIS, SIR

State transition

Susceptible
↓↑
Infected

Susceptible
↓ contact
Infected
↓ immune
Recovered/Removed

assuming lattice or random graphs, and fixed size:
equal birth and death rates in a const. population

However, our traveling and communication are not in homogeneous, but in (social or technological) SF nets

4-2. Absence of the Threshold

For SIS on SF, the density of nodes with degree k

$$\dot{\rho}_k(t) = -\rho_k(t) + \lambda k(1 - \rho_k(t))\Theta(t), \quad s_k(t) + \rho_k(t) = 1$$

Substitute the solution $\rho_k = \frac{\lambda k \Theta}{1 + \lambda k \Theta}$ for $\dot{\rho}_k = 0$ into the expectation (mean-field) of infection

$\Theta \stackrel{\text{def}}{=} \sum_k \frac{kP(k)\rho_k}{\langle k \rangle}$: denoted by $f(\Theta)$, the condition of

$\exists \rho_k \neq 0$ is given by $\frac{df(\Theta)}{d\Theta}|_{\Theta=0} \geq 1$.

The epidemic threshold of infection rate λ_c is

$$\lambda_c \leq \frac{\langle k \rangle}{\langle k^2 \rangle} \sim \frac{1}{\ln N} \to 0 \;\; (N \to \infty).$$

R. Pastor-Satorras and A. Vespignani, PRE 65, 2001

5-1. Connectivity Correlations

- Is the connection structure essentially same in social, tech., and bio. networks ?

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  Assortative: social

  Disassortative: technological or biological

  M.E.J Newman, PRE 67, 2003, A. Vázquez, PRE 67, 2003.

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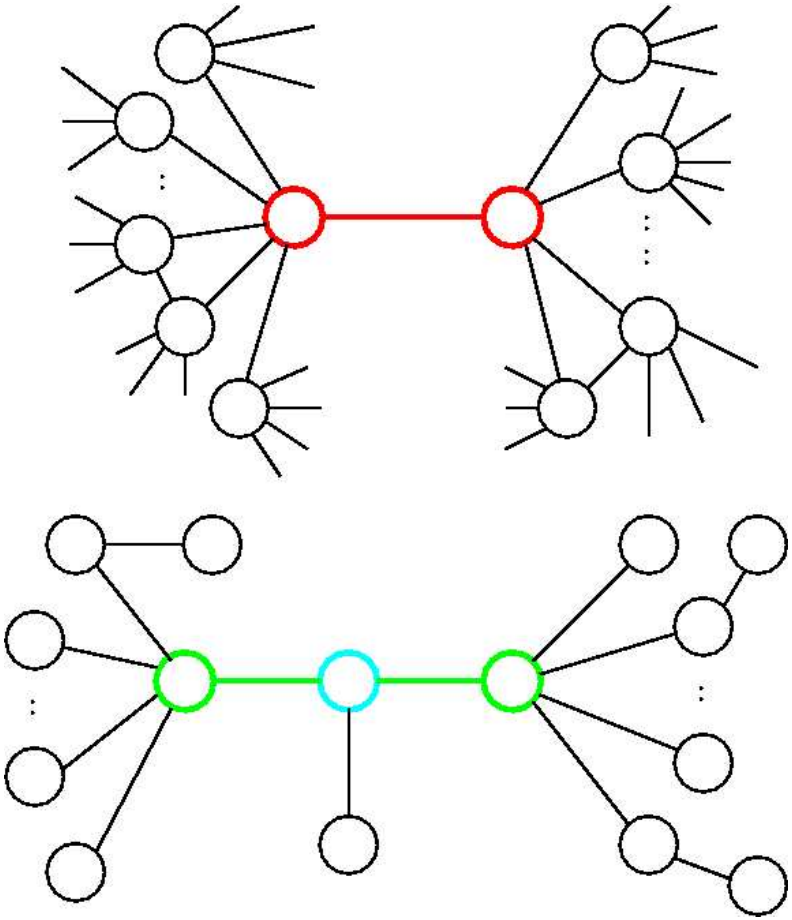
  The struct. are crucial for epidemic spreading

Note that our contact relations (email, world trading, etc.) are supported by both social and technological networks, today !

5-2. Ass and Dis Correlations

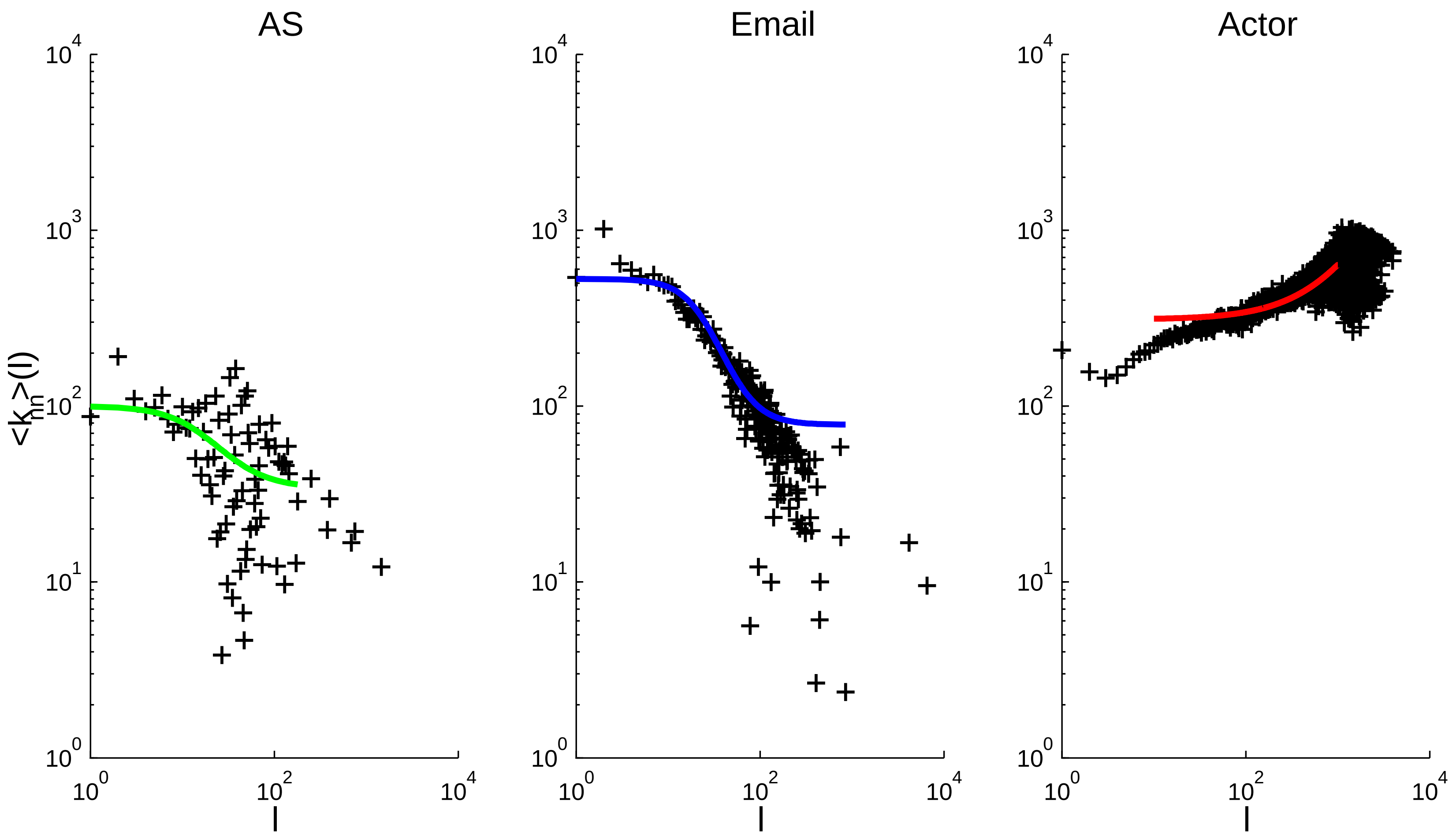


Ass: tend to have connections between similar peers

Dis: hub and peripheral nodes with low degrees

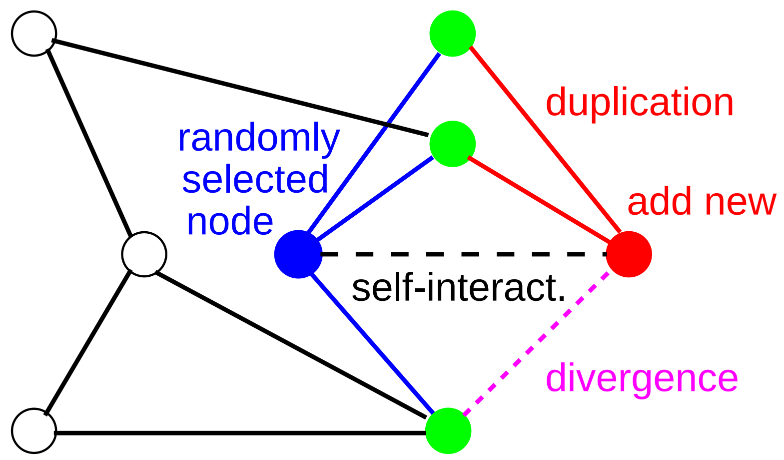
Let us consider the conditional probability $P(l|k)$ of connection of nodes with deg. k, l for each type.

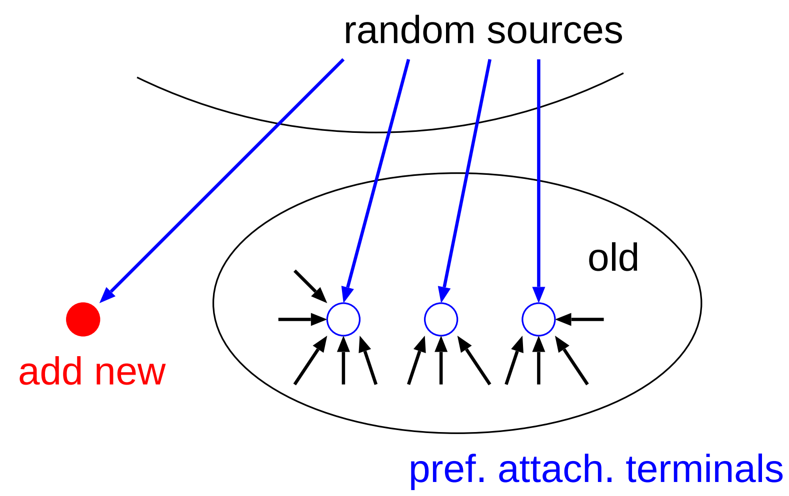
5-3. Emprical Data of Correl.



⇒ estimating the cond. prob. is intractable because of the poor statistical measurement

5-4. Correlated Models





Duplication-divergence model

A. Vázquez, PRE 67, 2003

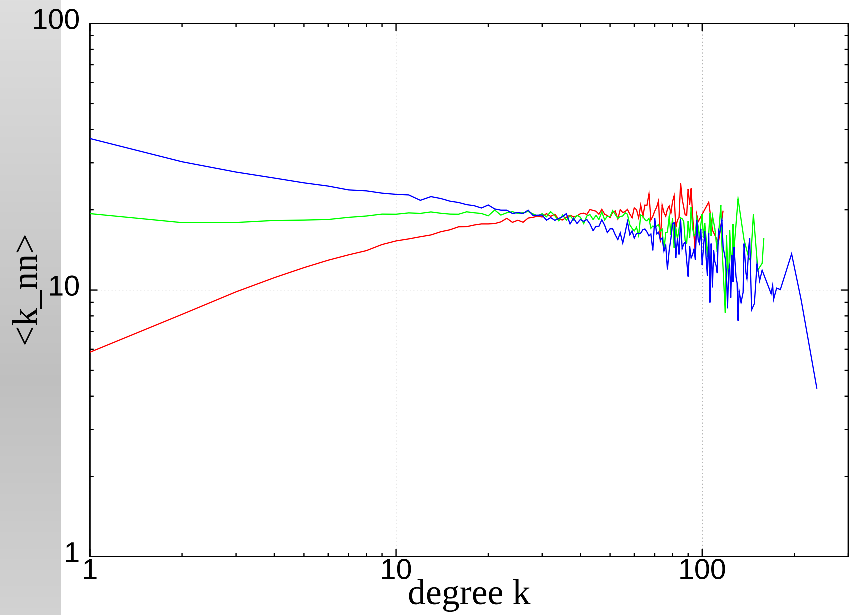
Directed growing model

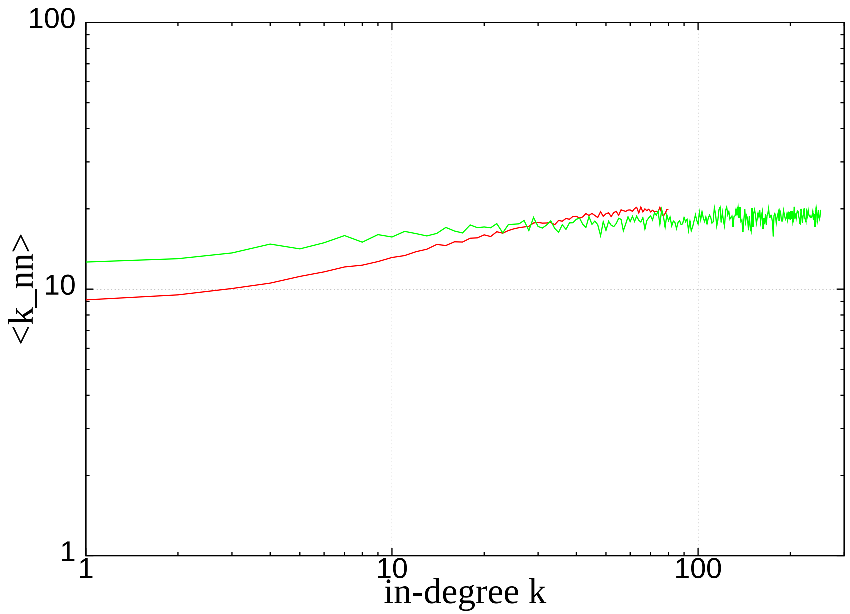
S.N. Dorogovtsev and J.F.F. Mendes, Evolution of Networks, Oxford Univ. Press, 2003

The $P(l|k)$ is estimated from the average of random realizations for each model.

5-5. Control of the Correlations

For the Dup (left) and Dir (right) models





the prob. of self-interact. $q_v = 0.1, 0.3, 0.9$

$\gamma \stackrel{\text{def}}{=} 2 + (w+1)/m' = 2.1, 3.0$

$\Rightarrow$ the correlations between Ass-Dis or Ass-Unc

5-6. SIR on SF net with Correl.

Epidemic dynamics at the mean-field level

$$\dot{\rho}_k(t) = -\delta\rho_k(t) + bk\,\underbrace{s_k(t)\Theta_k(t)}_{contact},$$

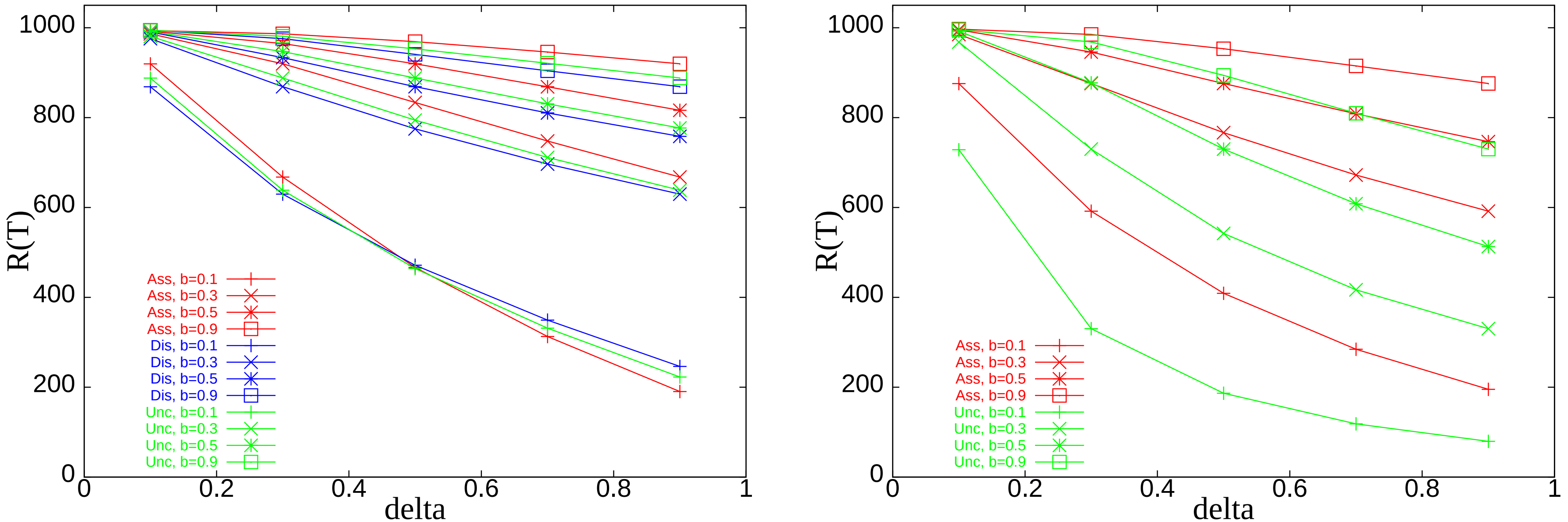
$$\dot{s}_k = -bk\,\underbrace{s_k(t)\Theta_k(t)}_{contact}, \quad \dot{r}_k(t) = \delta\rho_k(t),$$

where b and δ denote the infection and immune rate, $s_k(t) + \rho_k(t) + r_k(t) = 1$,

$$\Theta_k(t) \stackrel{\text{def}}{=} \begin{cases} \sum_{l=1}^{k_c} \frac{l-1}{l} P(l|k)\rho_l(t) & \text{for Dup} \\ \sum_{l=1}^{k_c} P(l|k)\rho_l(t) & \text{for Dir} \end{cases}$$

5-7. Simulation Results

Epidemic incidence $R(T) \stackrel{\text{def}}{=} \sum_k r_k(T) \times N_k(T)$ for the Dup (left) and Dir (right) models



Connections between similar peers (such as hub-hub) tend to enhance the spread of infection

It is more reamarkable in Dir (through directed links).

6. Summary

- We've briefly reviewed recent studies for a commonly existing SF structure in social, technological, and biological networks.
- The properties of SF network have been shown as the optimal topology, robustness-vulnerability, absence of epidemic threshold, etc.
- Besides the power law dist., there exist Ass (social, between peers) and Dis (tech. or bio., hub-peripheral node) connectivity correlations. Our simulation results for the SIR dynamics suggest that the epidemic spreading is enhanced by assortative connections between similar peers.

Further study for biol. & socio. inspired net. design.