Geographical Scale-free Triangulation

-Related to the communication structure based on population-

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1. Background

Existing a surprisingly common structure: SF net.
the degree dist. exhibits $P(k) \sim k^{-\gamma}$, $2 < \gamma < 3$.

Social: acquaintance, world trading, actor-collabo., citation, language

Technological: Internet, WWW, email, power grid

Biological: neural net, genome, metabolic pathway, foodweb

One of the fundamental generation mechanism has been proposed: Growth & Preferential Attachment

Barabási and Albert, Physica A, 272, 1999

2. Scale-Free Nets with Hubs

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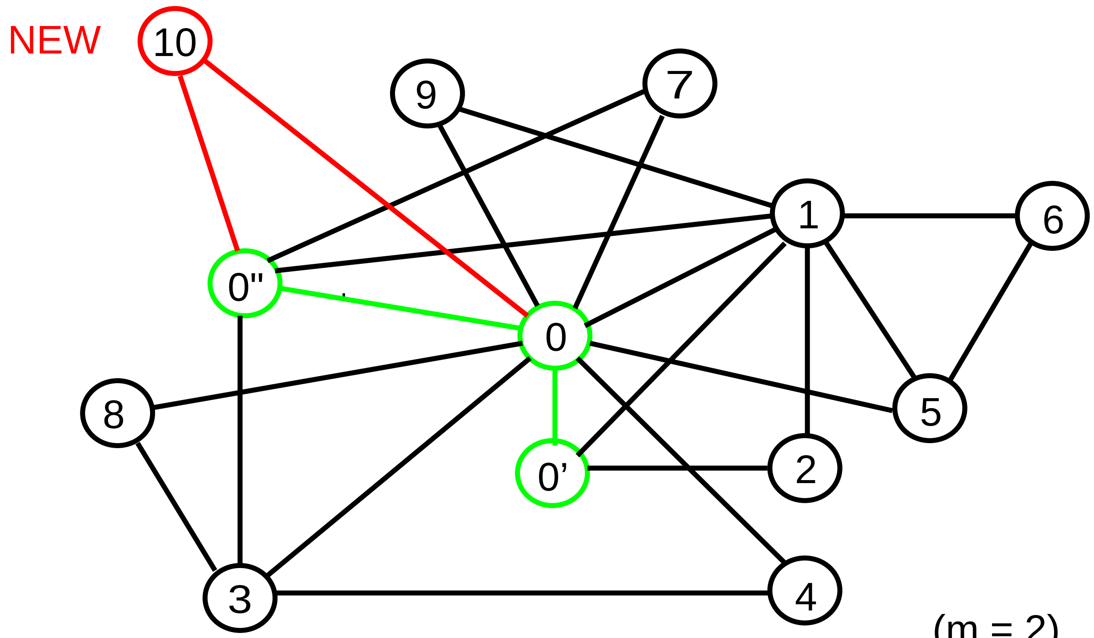
$$\Downarrow$$

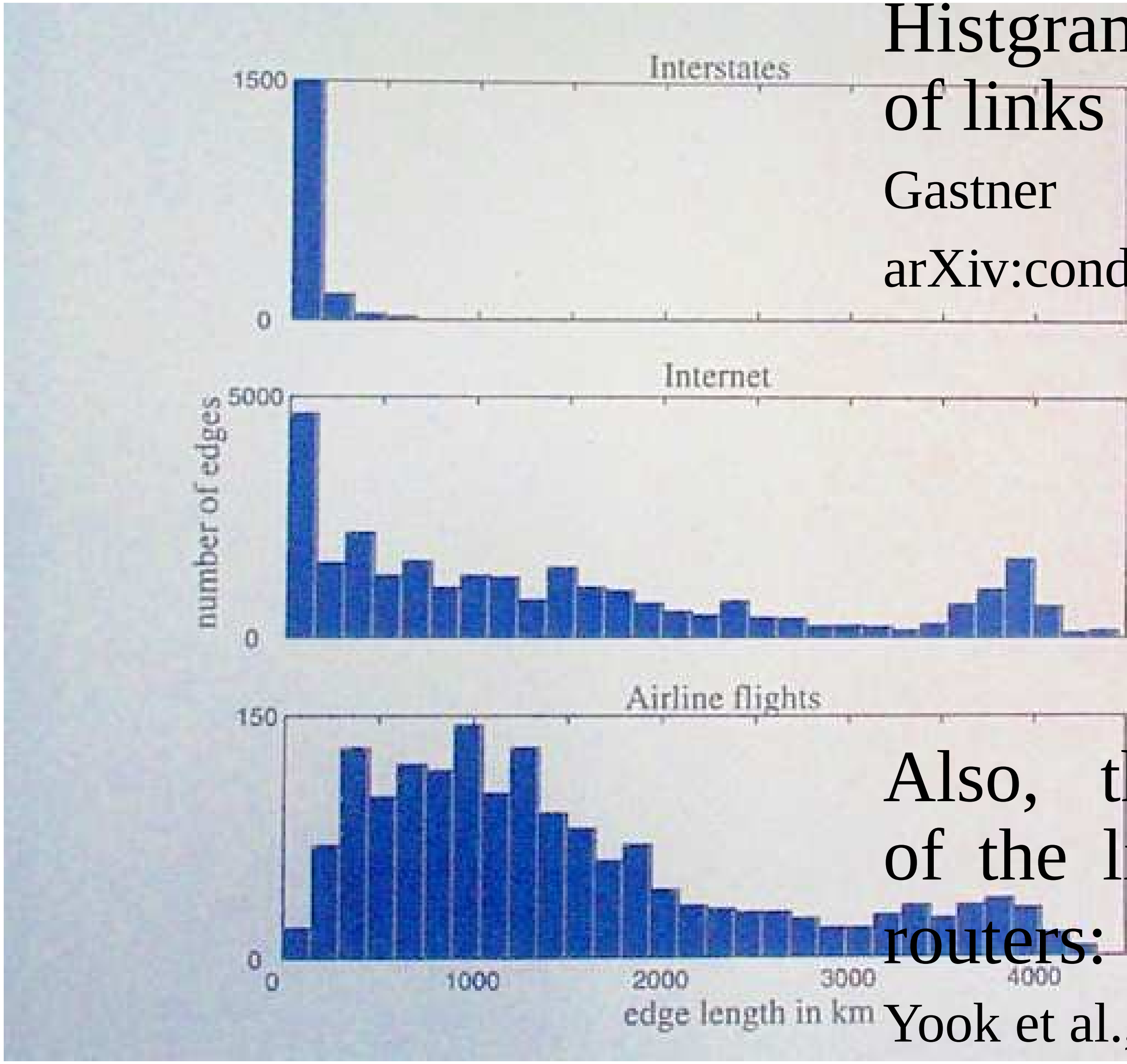
- Therefore, we consider geographical SF nets, especially as planner graphs without crossing links to avoide interference of wireless beams.

2-1. BA model

- A network grows from an initial N_0 nodes with $m < N_0$ links among them.
- At every time step t, a new node is introduced, and is randomly connected to m previous nodes i with an attachment probability $\Pi_i^{BA}(t) \sim k_i(t)$.



2-2. Restriction of Links



Histgrams of the lengths of links

Gastner and Newman, arXiv:cond-mat/0407680, 2004

Also, the length dist. of the links connecting routers: $P(d) \sim 1/d$

Yook et al., PNAS 99, 21, 2002

2-3. Geographical SF Nets

model	generation rule
Modulated BA Manna'02, Xulvi-Brunet'02	competition between pref. attach. and dist. l $\Pi_i(t) \sim k_i(t) l^{\alpha}$, $\alpha < 0$
SF on lattices ben-Avraham'03, Warren'02	confi guration from randomly assigned degree k_j restricted links in the radius $r(k_j) = A k_j^{1/d}$
Apollonian nets. Doye'05, Zhou'04	iterative triangulation on a planner space geo. attach. pref. but $\exists$ long-range links

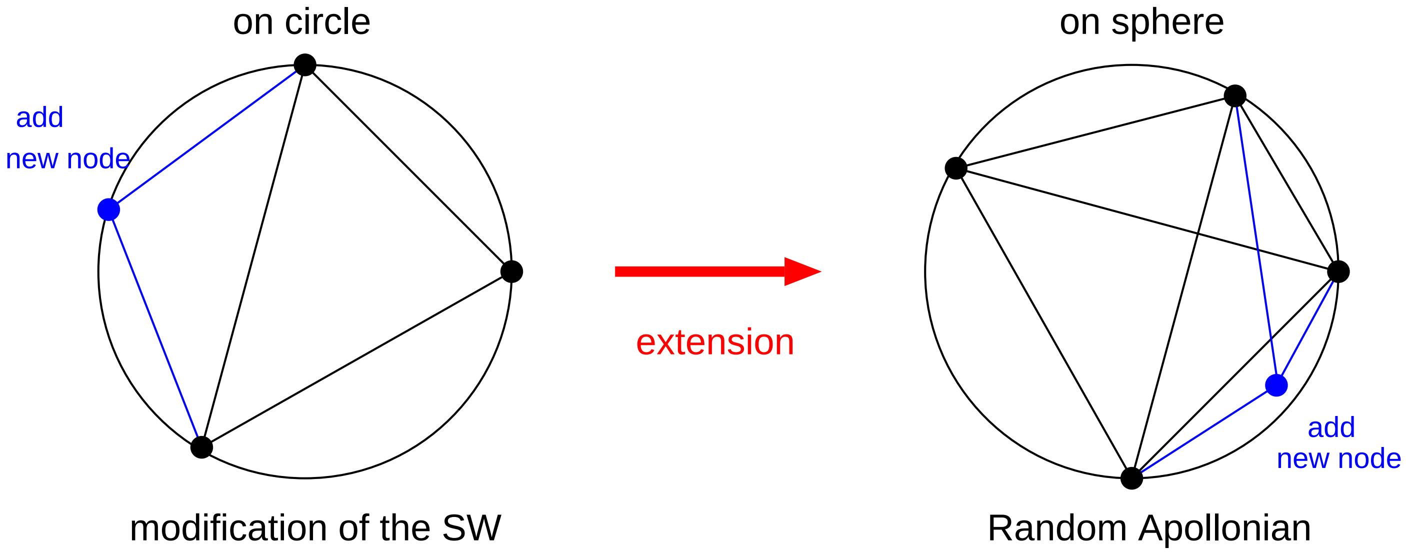
1st & 2nd: crossing links causes interference.

3rd: long-range links causes dissipation of the power.

3. Random Apollonian Net

Two models based on geo. attach. pref.

At each time step, a new node is added in an uniform-randomly chosen interval (or surfaces), and conneted to its m nearest neighbors w.r.t distance.



$\Rightarrow$ We can derive the $P(k)$ for each case.

3-1. Evolution Eq. (Circle)

Subdivision of interval:
By randomly chosen interval with equal prob. m/N, we have the evolution equation

$$n(k,N+1) = \left(1 - \frac{m}{N}\right) n(k,N) + \frac{m}{N} n(k-1,N) + \delta_{k,m}$$

From an approximation $n(k,N) \approx NP(k)$ for sufficient large N, we obtain the exp. degree dist.

$$P(k) = \frac{1}{m+1}\left(\frac{m}{m+1}\right)^{k-m}$$

for $k \gg m$ ($P(k) = 0$ for $k < m$).

Ozik et al., PRE 69, 026108, 2004

3-2. Evolution Eq. (Sphere)

Subdivision of surface (Triangulation):

$$n(k+1, N+1) = \frac{k}{N_\triangle} n(k, N) + \left(1 - \frac{k+1}{N_\triangle}\right) n(k+1, N),$$

where $N_\triangle$ denotes the number of triangles.
In the $P(k) \approx n(k, N)/N$, it can be rewritten as

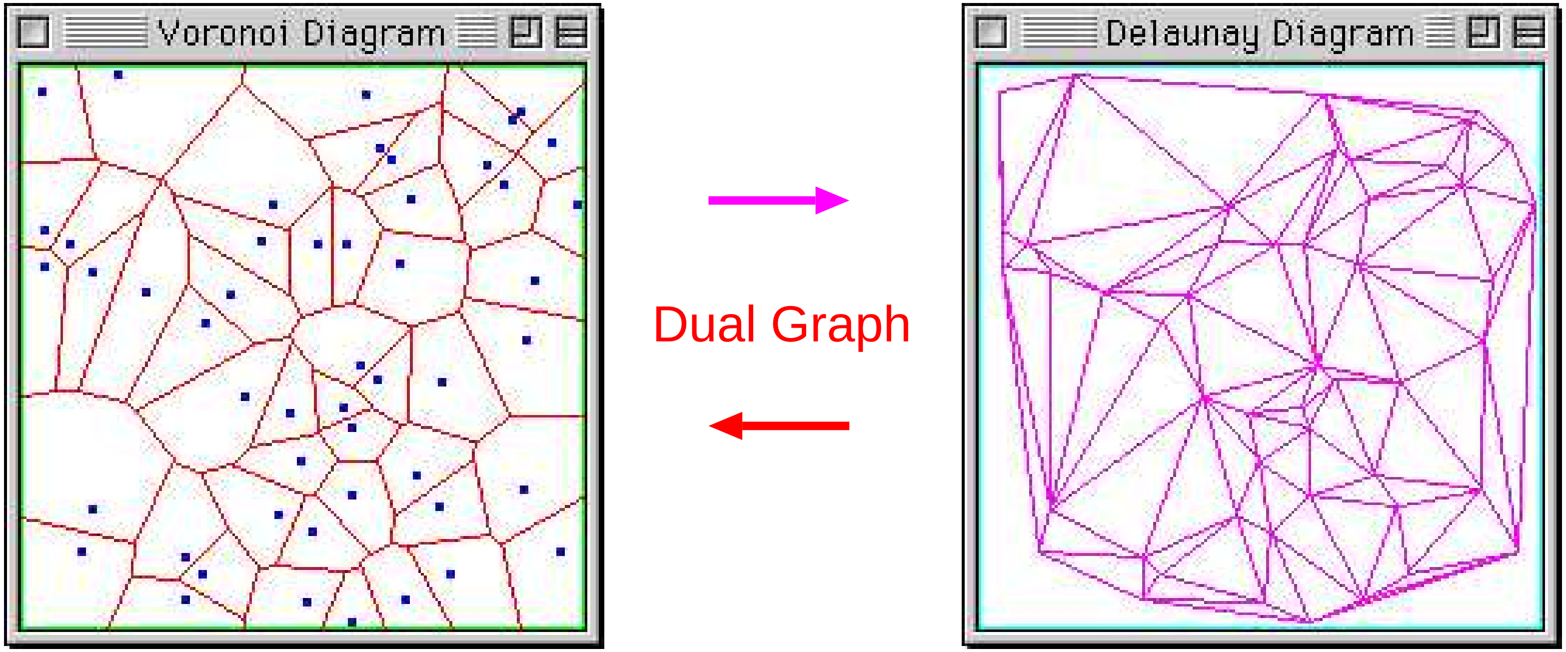
$$\begin{aligned}(N+1)P(k+1) &= NkP(k)/N_\triangle + NP(k+1) \\ &\quad -N(k+1)P(k+1)/N_\triangle.\end{aligned}$$

By the continuous approx., we obtain $P(k) \sim k^{-\gamma_{RA}}$
with $\gamma_{RA} = (N_\triangle + N)/N \approx 3$ for large N.

Zhou et al., arXiv:cond-mat/0409414, 2004

3-3. Voronoi and Delaunay

Consider the combination of RA (by triangulation on a plane) and DT to avoid long-range links

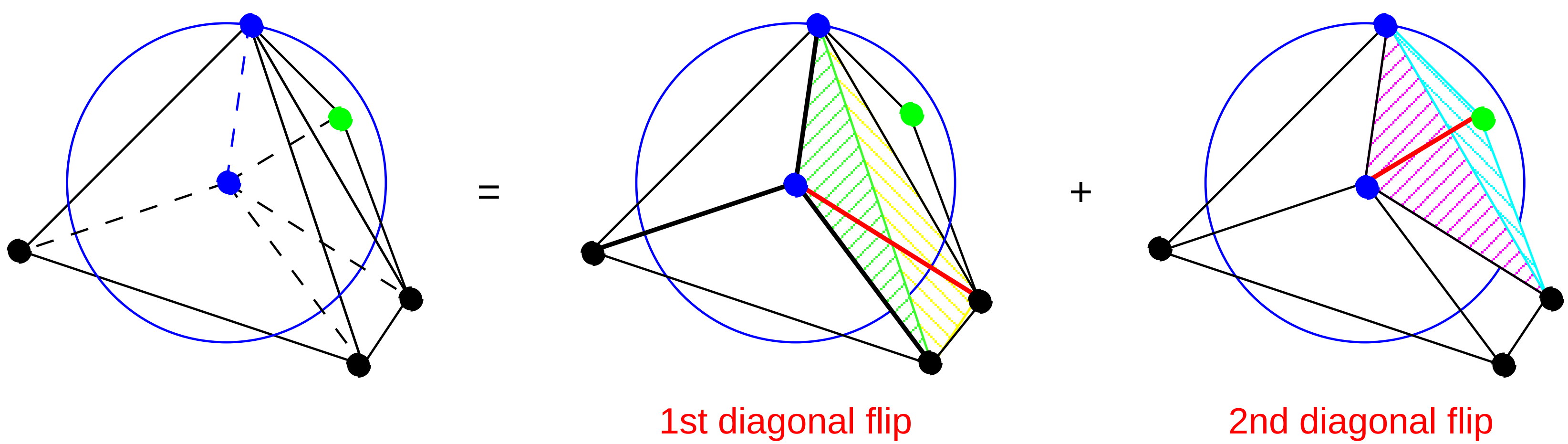


⇒ optimal triangulation in some criteria: maximim angle, minimax circumscribed circle, short path length close to the direct Euclidean dist., etc.

4. Delaunay-like SF Net

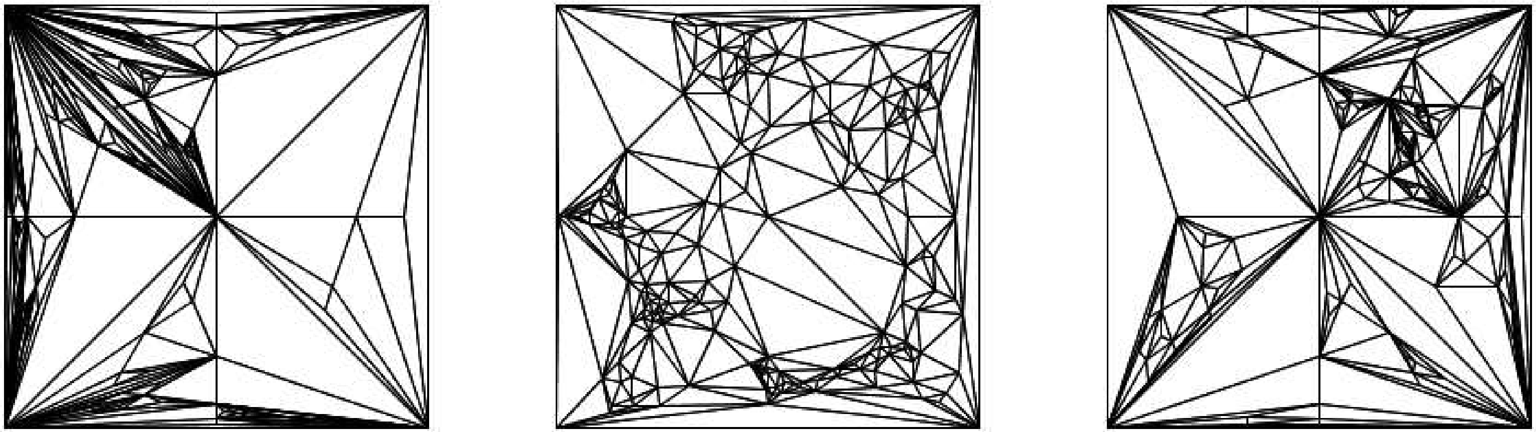
We propose RA+NN:

- Set an initial planar triangulation.
- Select a triangle at random and add a new node at the barycenter. Then, connect the new node to the three nodes of its triangle. By iteratively applying diagonal flips, connect it to the nearest node(s) within a radius.



5. Simulation

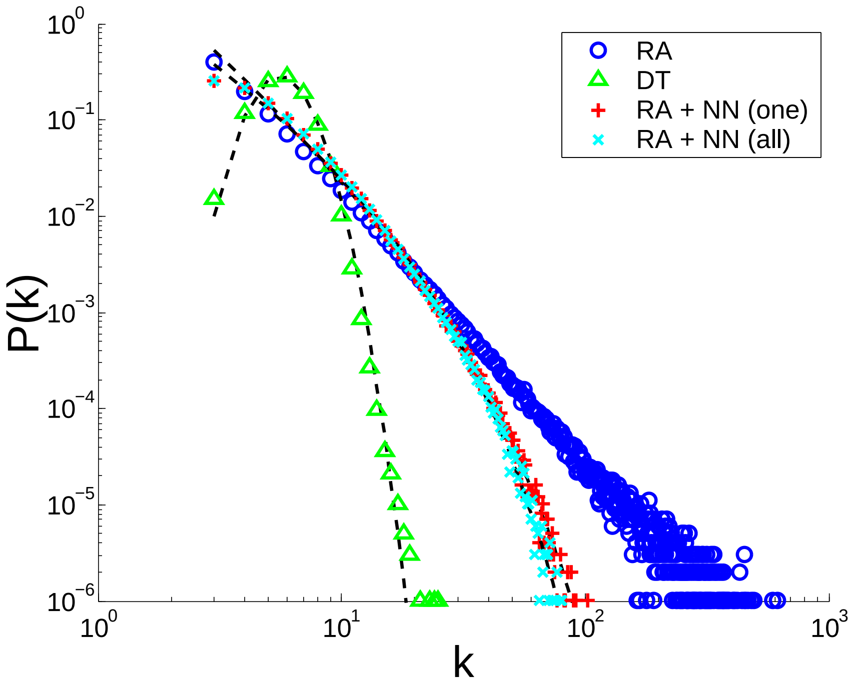
We compare the topological properties for the models: RA, DT, and our proposed RA+NN in the averaging of 100 realization at size $N = 10,000$.

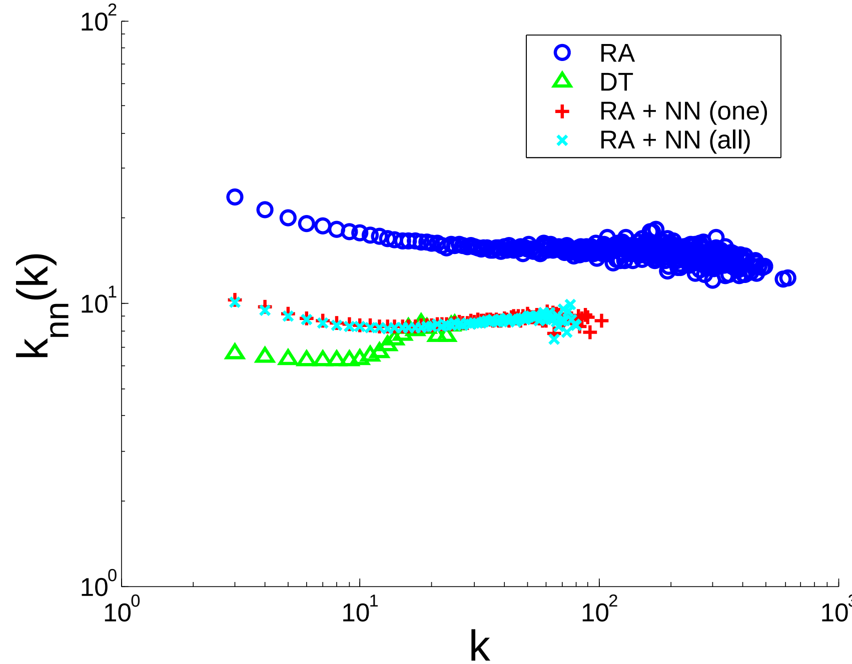


RA DT RA+NN(one)

$\Rightarrow$ RA+NN has the intermediate structure.

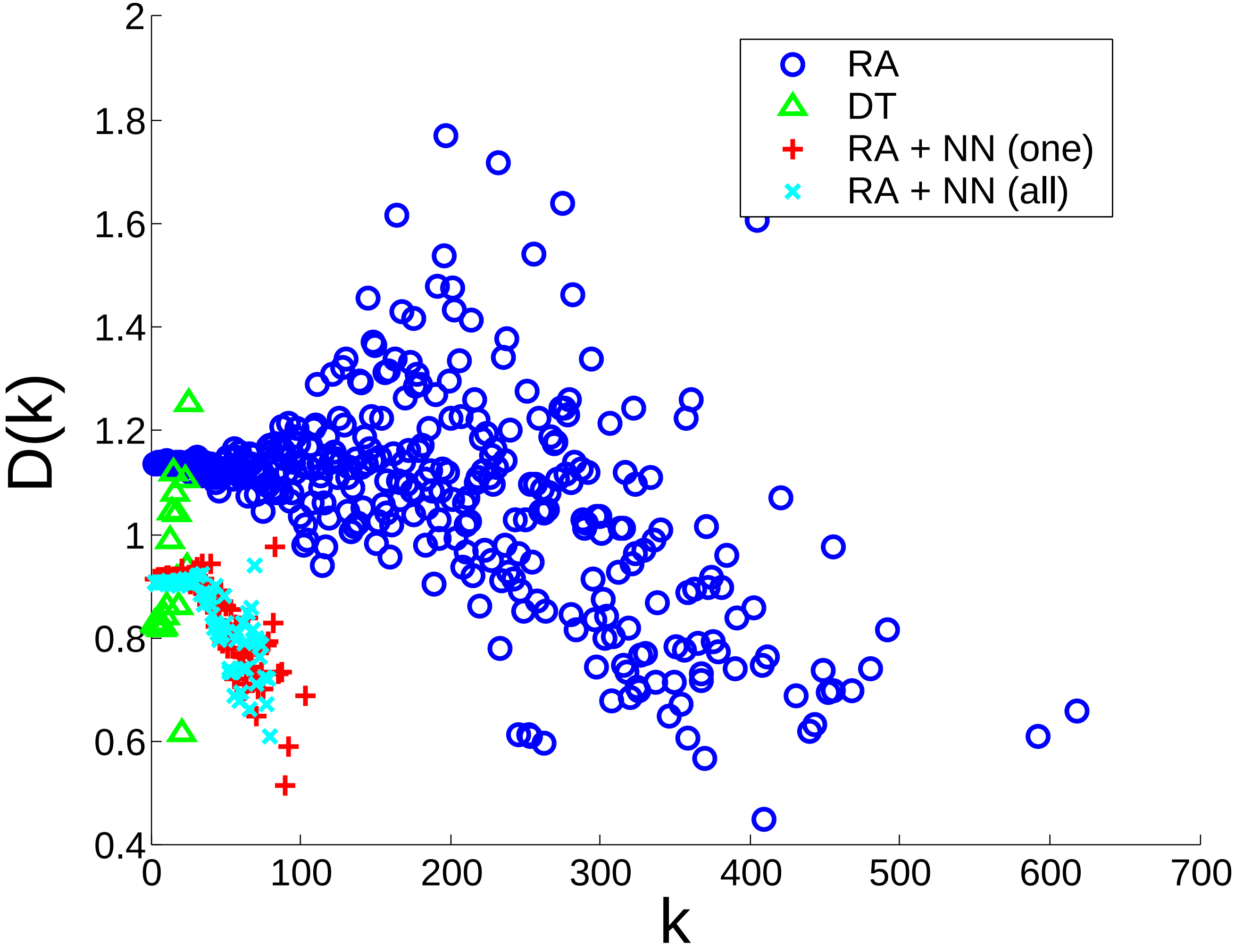
5-1. Degree Dist. & Correlation





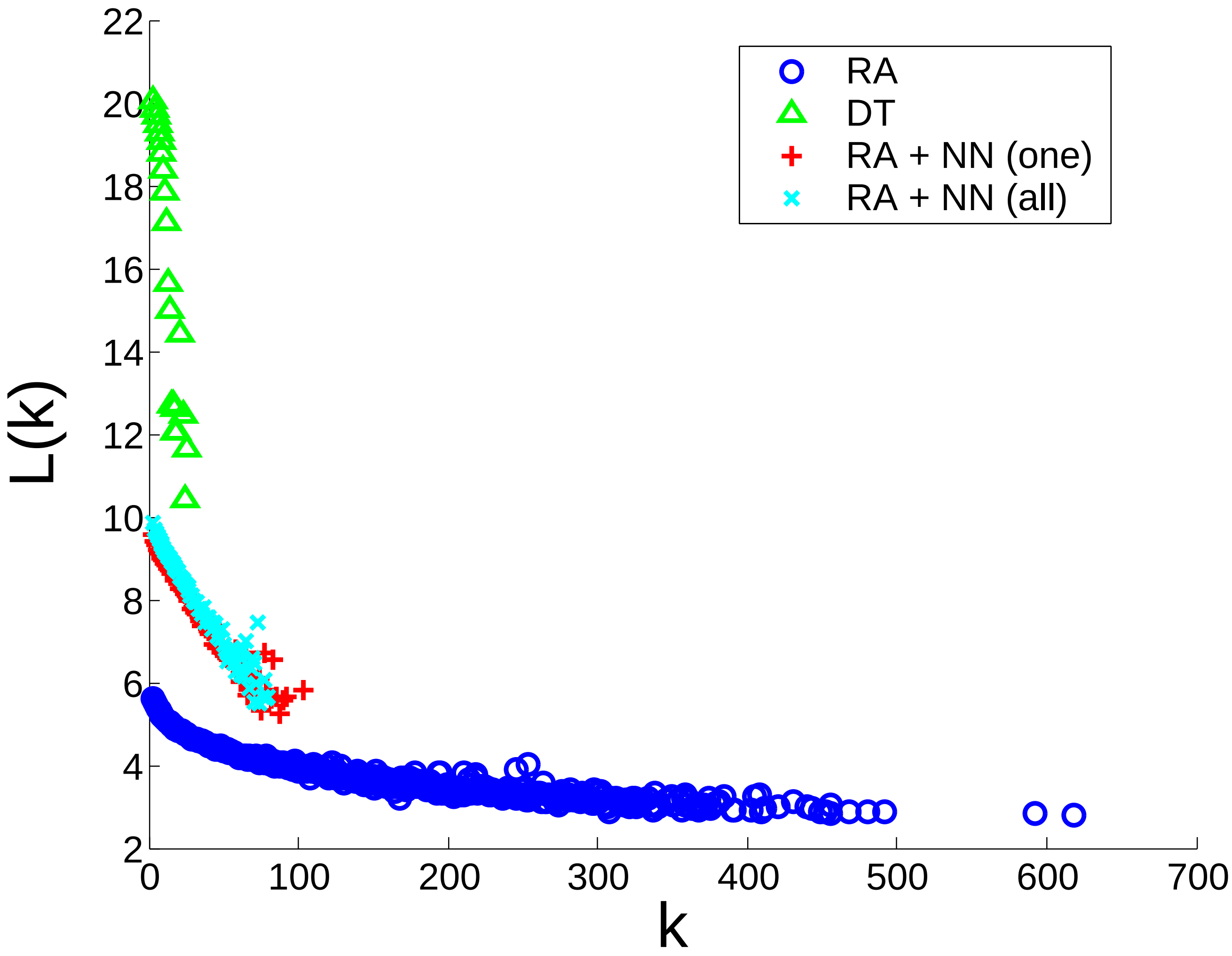
$\Rightarrow$ RA+NN has a power law dist. with exp. cutoff, and weak negative correlation.

5-2. Distance on the Shortest



⇒ The distances in RA+NN are smaller than that in DT and many of broad dist. in RA.

5-3. The Number of Min. Hops



$\Rightarrow$ The numbers in RA+NN are improved to less than the half in DT, but slightly larger that in RA.

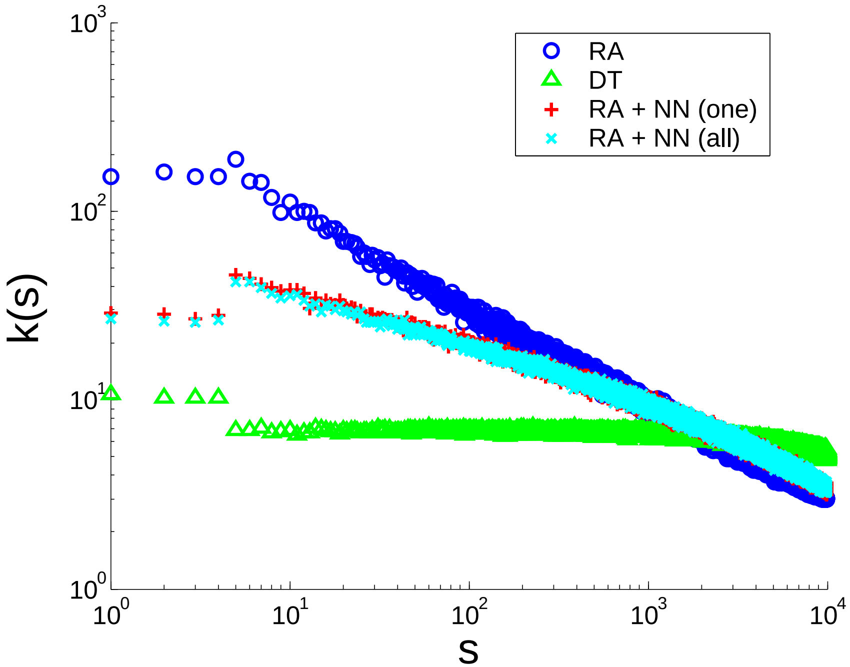
6. Summary

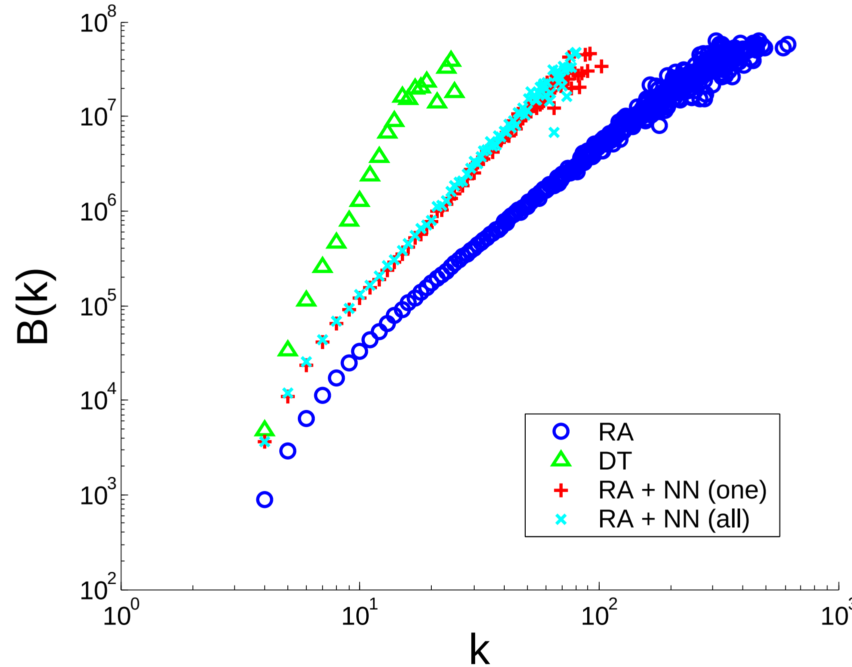
- We've briefly reviewed recent studies of geographical SF net models, and proposed a modified one to reduce long-range links.
- The Delaunay-like SF net without crossing links is generated by the iterative triangulation and diagonal flipping based on local rules.
- Simulation results have shown that our proposed model has short path lengths and small num. of hops, which are suitable topological properties for efficient communications.

We'll further investigate the dynamical traffic proper-ties (e.g. delivery time) and the fault-tolerance (e.g. in cascaded failures)

Appendix 1.

Average degree $k(s)$ of the node inserted at time s, and the betweeness $B(k)$ of the nodes with degree k





$\Rightarrow$ Old nodes of RA and RA$+$NN tend to be hubs, and the traffic load of RA$+$NN is the intermediate

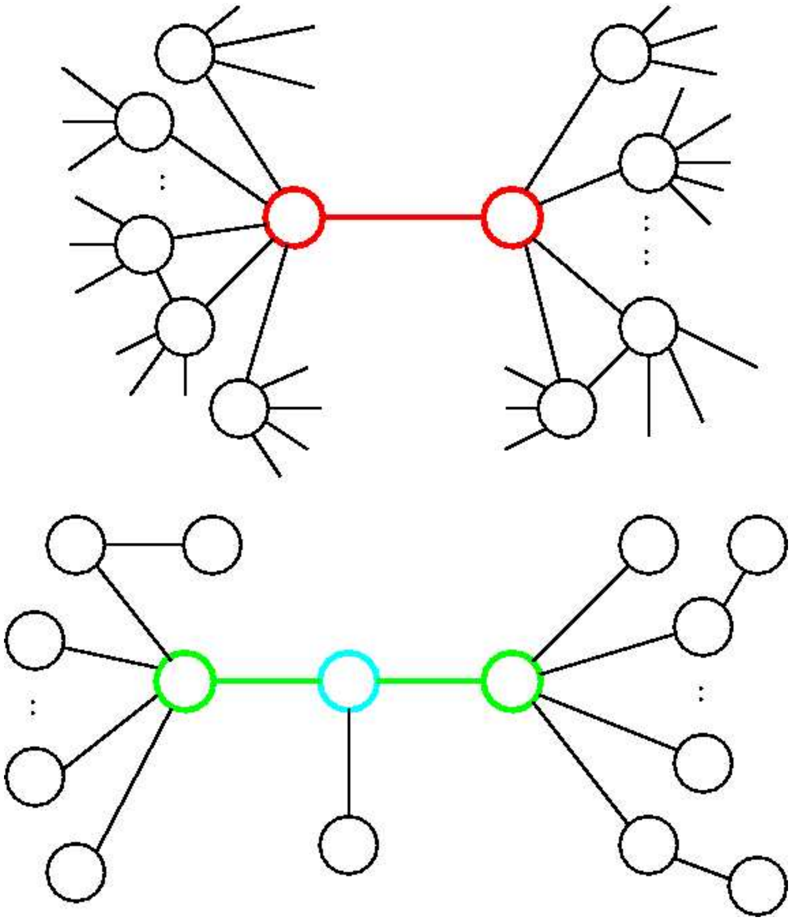
Appendix 2.

Estimated function for the data of degree distribution

model	estimated function	parameters
RA	$P(k) \sim k^{-\gamma_{RA}}$	$\gamma_{RA} \approx 3$
DT	$P(k) \sim \exp\left(-\frac{(\ln k - \mu)^2}{2\sigma^2}\right)$	$\mu = 1.7755$, $\sigma = 0.2383$
RA+NN(one)	$P(k) \sim k^{-\gamma} \exp(-ak + b)$	$\gamma = 2.26$, $a = 0.0647$, $b = 2.045$
RA+NN(all)	$P(k) \sim k^{-\gamma} \exp(-ak + b)$	$\gamma = 1.7248$, $a = 0.0979$, $b = 1.2286$

Appendix 3.

Assortative and Disassortative correlations observed in social and technological/biological networks



Ass: tend to have connections between similar peers

Dis: between hub and peripheral nodes with low degrees

Appendix 4.

planar triangulation: reasonable math. abstraction of ad hoc net. (each triangle forms a service region) Moreover, a memoryless, never defeat, and competitive online routing algorithm has been developed for networks on triangulation.