Epidemic SIR dynamics on scale-free nets

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⇒ Refer to **Y.Hayashi et al., "Oscillatory epidemic prevalence in growing scale-free networks,"** Physical Review E 69, 016112, 2004,

& Virtual Journal of Biological Physics Research, STATISTICAL AND NONLINEAR PHYSICS, Feb.1, 2004, http://www.vjbio.org

Conventional Epidemic Model

State transition in Kermack-McKendrick models

Susceptible
$\downarrow\uparrow$
Infected

Susceptible
$\downarrow$ contact
Infected
$\downarrow$ immune
Recovered/Removed

assuming lattice or random graphs, and fixed size: equal birth and death rates in a const. population

$\Rightarrow$ Today, we are traveling in world-wide, and communicating on the growing Internet !

Common Struct. in Many Real Nets

Many real nets $\neq$ regular(e.g lattice), uniformly random

$\Updownarrow$

Recently('98-'02), a surprisingly common structure has been found in many real nets

Social: actor-collabo., world trading, acquaintance, citation, language

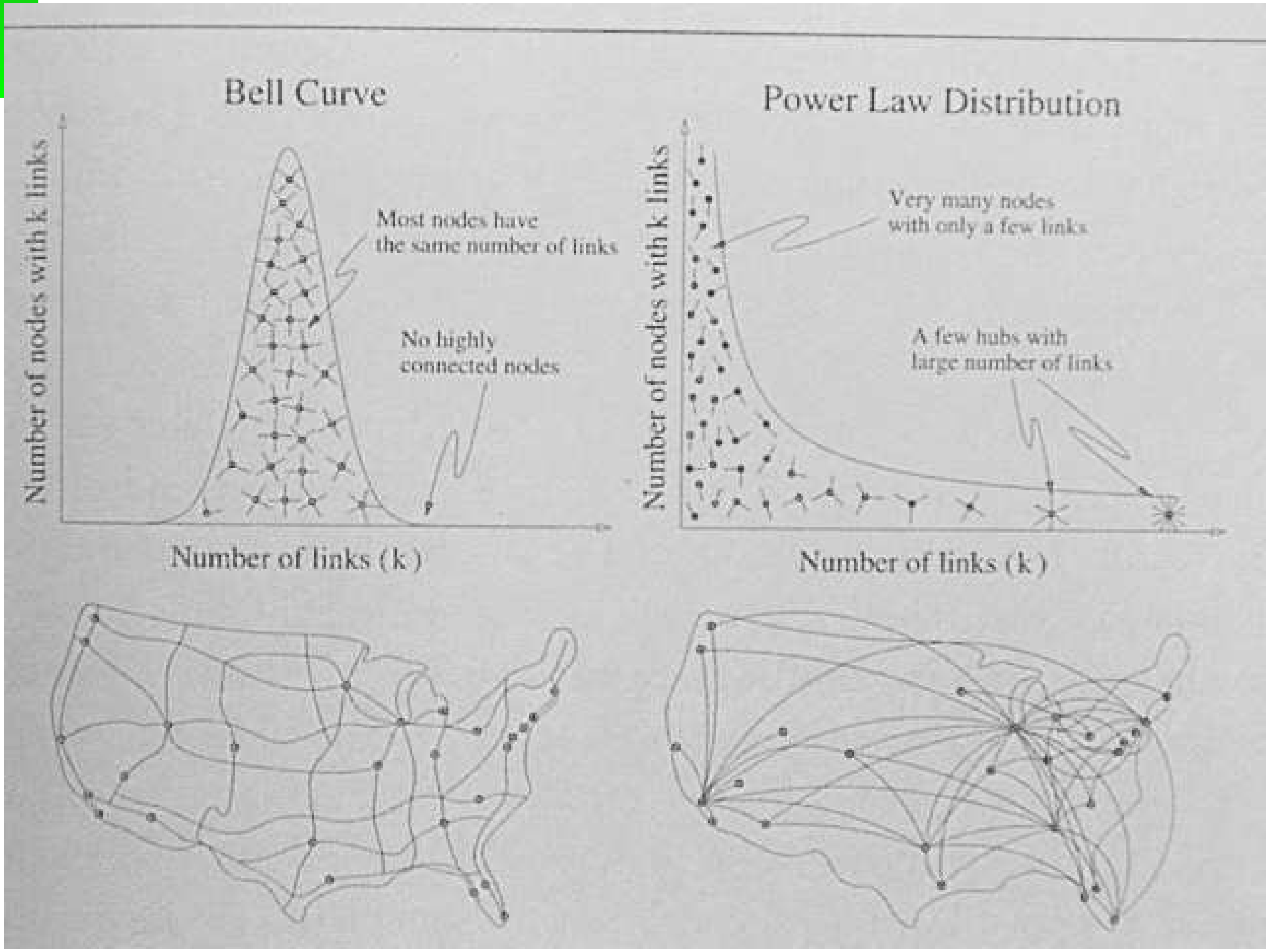
Technological: power grid, Internet, WWW, email

Biological: neural net, genome, metabolic pathway, foodweb

The topological struct. are deeply related to the damage of prevalence.

Scale-Free Network

The heterogeneous structure with hubs are called SF net, whose degree dist. exhibits $P(k) \sim k^{-\gamma}$, $2 < \gamma < 3$.



In A.L. Barabási, LINKED, Perseus, 2002

Different Spreading Properties

To be fully connected, more than average

Random: 3 links are necessary

the delivery time $\propto$ the size N

Realistic SF: 5 links are necessary

almost const. time, due to the spread from hubs
(larger net. consts of nodes with larger degrees)

K.Ohkubo, Y.Hayashi, and S.Ninagawa, Trans. of IEICE Vol.J85-D-I, No.2, 2002

$\Rightarrow$ Many researchers are attracted to SF networks and the evolution mechanisms.

Connectivity Correlations

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Assortative: social

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M.E.J Newman, PRE 67, 026126, 2003, A. Vázquez, PRE 67, 056104, 2003.

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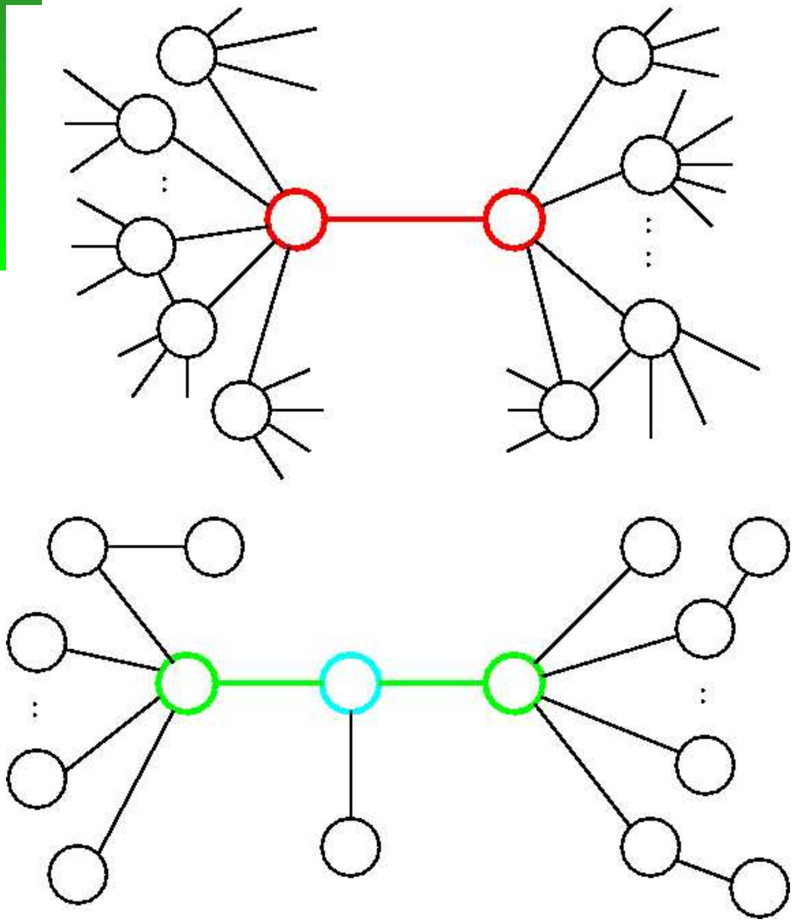
The struct. are crucial for epidemic spreading

Note that our contact relations (email, world trading, etc.) are supported by both social and tech. networks, today !

Assortative and Disassortative Correlations



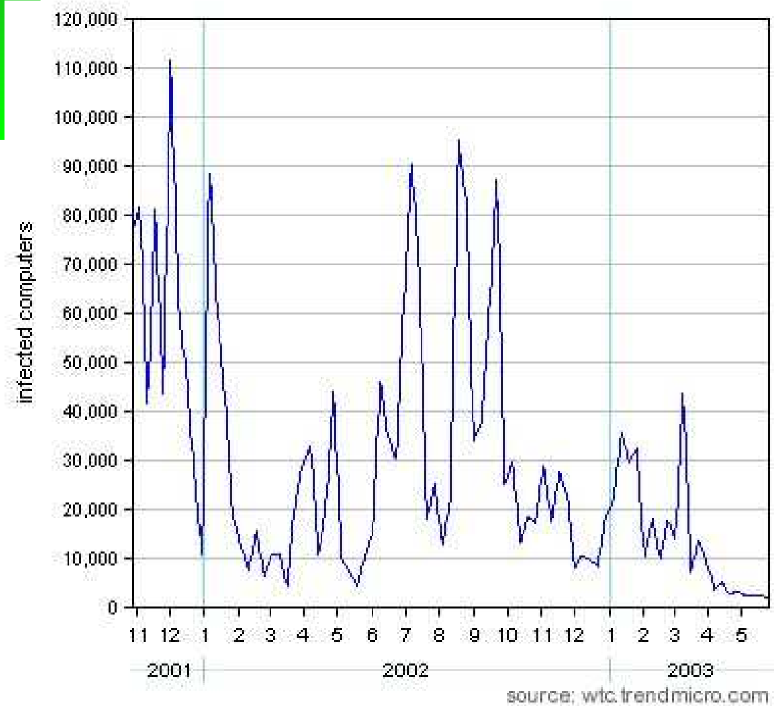
Ass: tend to have connections between similar peers

Dis: hub and peripheral nodes with low degrees

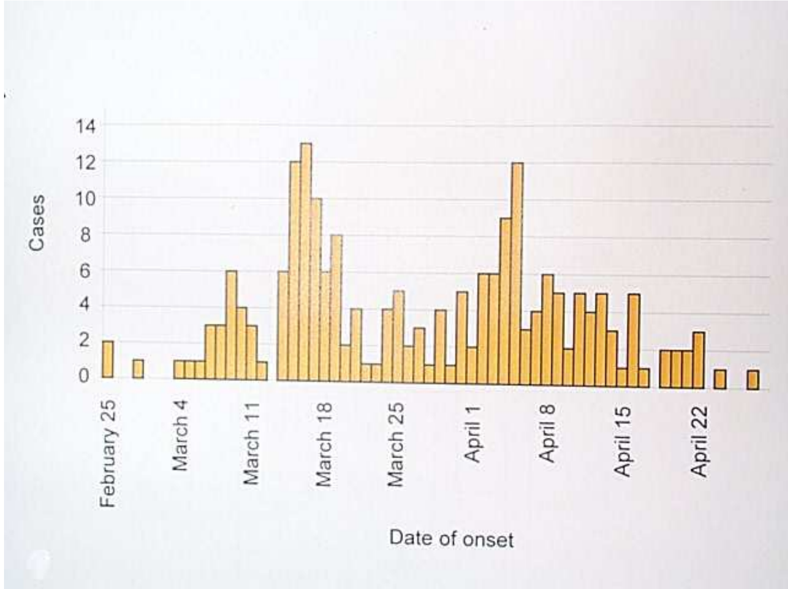
Let us consider the conditional probability $P(k|l)$ of connection of nodes with deg. k, l for each type.

Oscillatory Epidemic Prevalence

Typically observed, but unknown the mechanism



computer virus NIMDA



SARS in Singapore, Sciencexpress May 23, 2003

⇒ SIR (susceptible-infected-recovered/removed state transition) model <u>on SF nets</u> for epidemic spreading

Epidemic dynamics for the macro. eq. at the MF level

$$\frac{dS_k}{dt} = -bk \underbrace{S_k \Theta_k}_{contact} + a_k, \quad \frac{dI_k}{dt} = -\delta I_k + bk \underbrace{S_k \Theta_k}_{contact},$$

where b and δ denote the infection and immune rate, $a_k = Ak^{-\gamma}$, $A > 0$, provides a constant increasing of S_k,

Mean-Field infection: $\Theta_k(t) \stackrel{\text{def}}{=} \sum_l \frac{l-1}{l} P(k|l) \frac{I_l(t)}{N_l(t)}$.

From $N_k = S_k + I_k + R_k$ and $\frac{dR_k}{dt} = \delta I_k$, $\frac{dN_k}{dt} = a_k$, the growing $N_k(t) \sim a_k t$ gives asymptotic $P(k) \sim k^{-\gamma}$,

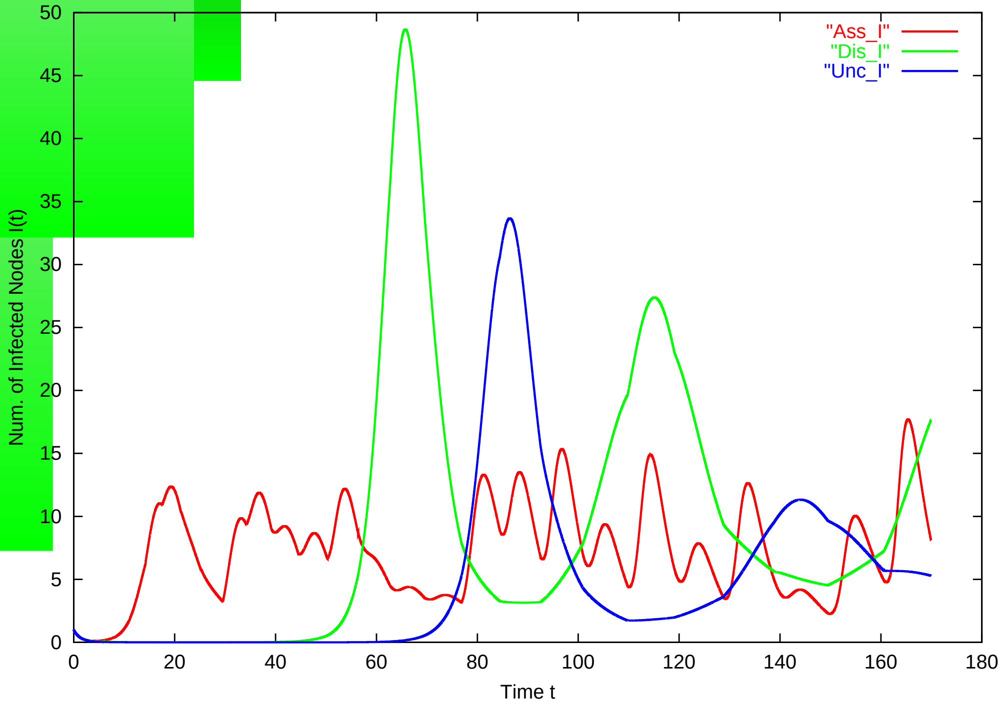
$$N(t) = \sum_k N_k \sim \left(\sum_k a_k \right) t.$$

$\Rightarrow$ linearly growing SF net, and simultaneously progress of epidemic spreading

Simulation Result

Different behavior depend on the correlation types

Trade-off: persistency and breaking size



Time evolution of $I(t) = \sum_k I_k(t)$

Ass: persistently survived with fluctuation

Dis: later outbreaks

Unc: corresponded to the conventional SF models without correlations

$\Rightarrow$ the behavior on Dis or Unc is also consistent with a stochastic SIR model, but it on Ass has’nt been found

Summary

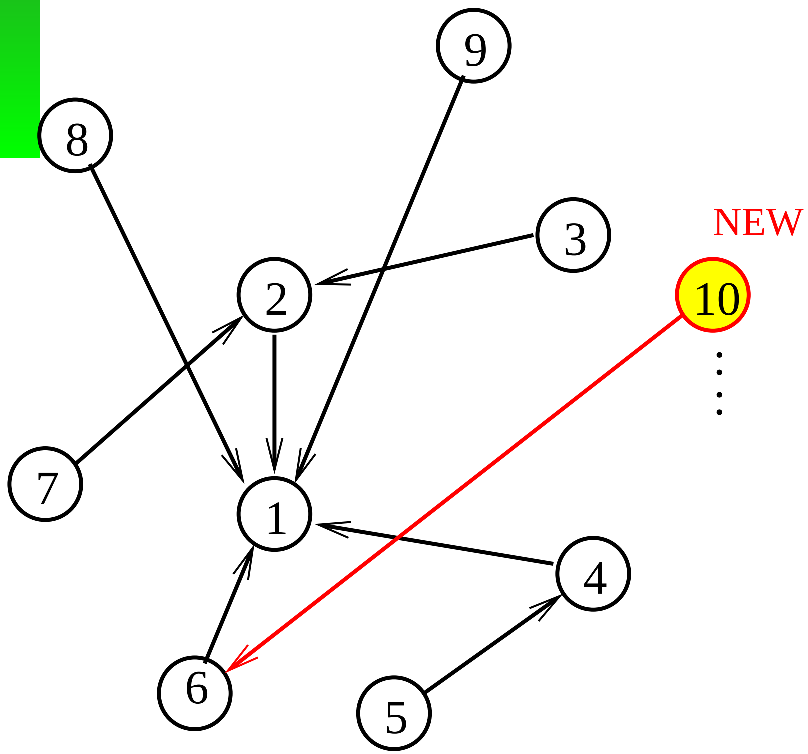
- Instead of the conventional homogeneous contacts in a const. population, we've considered a more realistic SIR model on growing SF nets with new participants.
- Moreover, we've extended it with Ass (social, between peers) and Dis (tech. or bio., hub-periph.) connectivity correlations.
- In our simulation of the SIR dynamics, the correlations cause quite different behavior for epidemic spreading.

$\Rightarrow$ a good struct. will be used for preventing the spread with huge damage in distributed network evolutions.

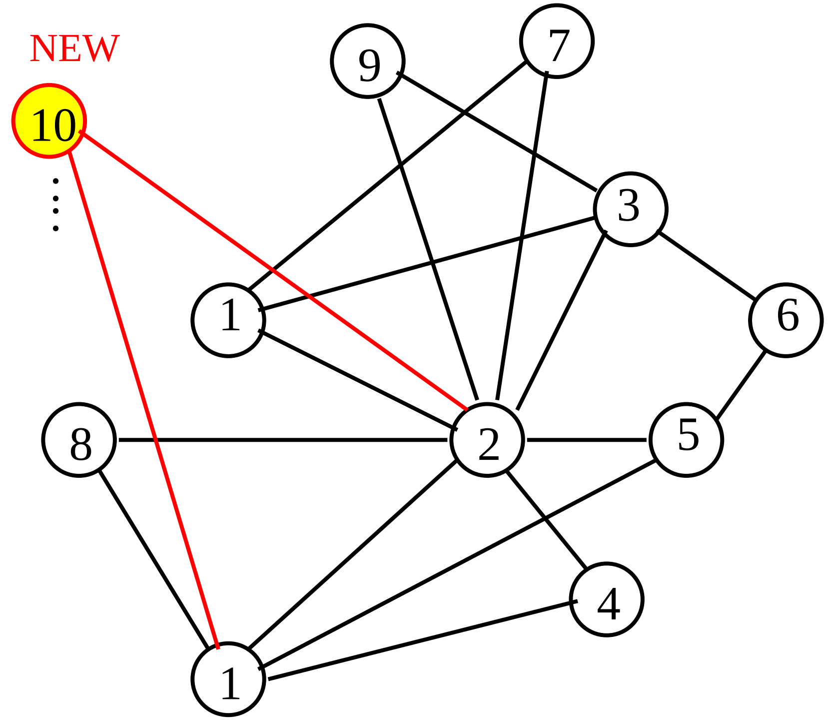
Appendix 1: Evolutional Net. Models

$P(k) \sim k^{-\gamma}$, $\gamma \sim 3$, $m = 1$

$\gamma = 3$, $m = 2$



Krapivsky-Render's GN model

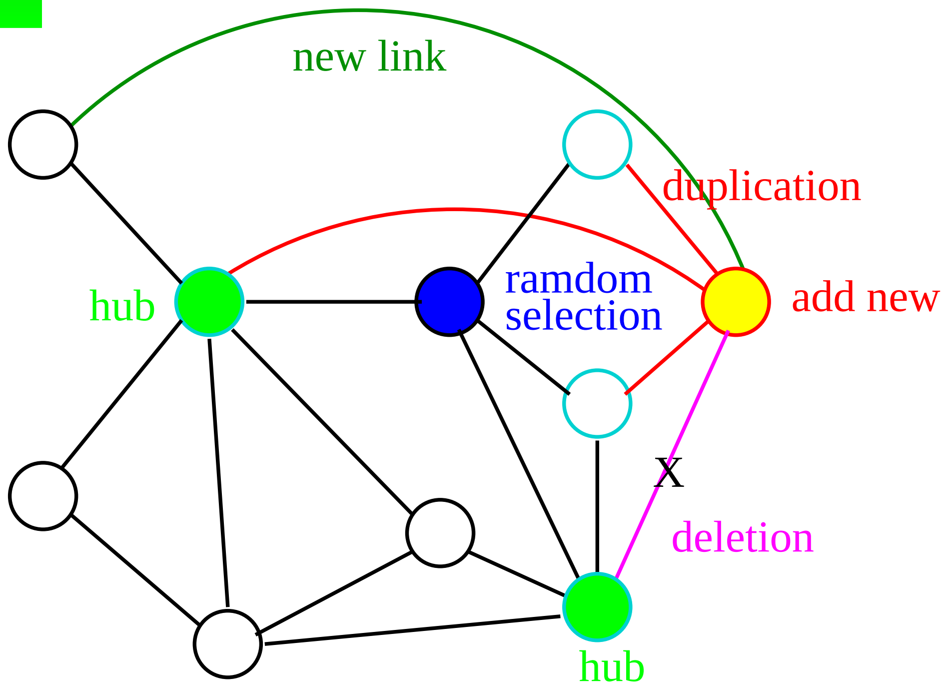


BA model

$\Rightarrow$ age-effect (S.N. Dorogovtsev et al. PRL 85, 2000), rewire(A. Albert, PRL 85, 2000), fi tness(G. Bianconi, PRL 86, 2001)

Appendix 2: Duplication Model

In spite of random node selection, the neighbor hub node has many chance to get duplicate connections (proportional to the degree).

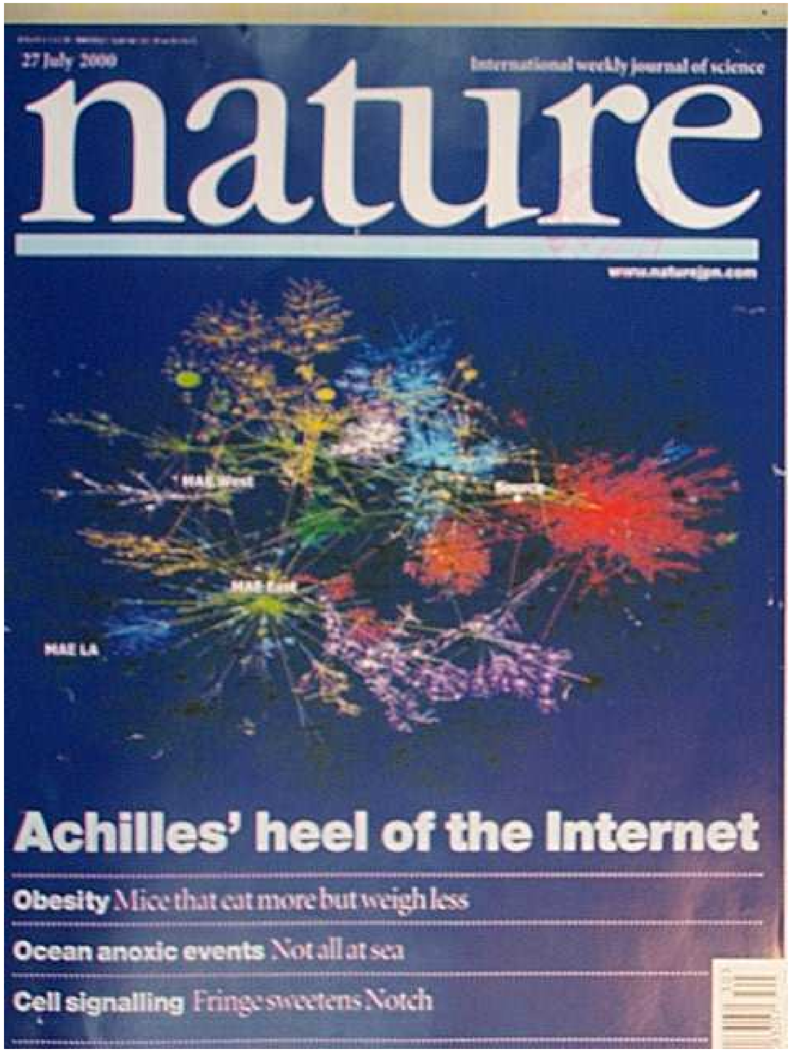


$\Rightarrow$ Biologically plausible networks realize Preferential Attachment in a local rule !

Appendix 3: Robust and Vulnerable Connectivity

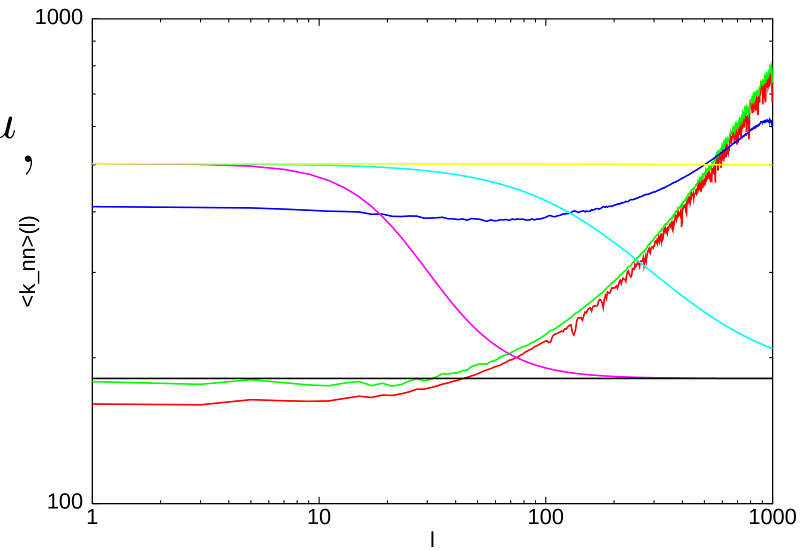
Robust: for random failure, remaining the connectivity

Vulnerable: for targeted attack against hubs, disconnecting into many isolated parts



Appendix 4: Variety of Correlations

$$P(k|l) \sim \alpha k^{1-\gamma} + \frac{1-\alpha}{|k-l|^{\nu}+1},$$
$$\sim \alpha k^{1-\gamma} + (1-\alpha) l^{-\mu},$$



and adding random noise as arbitrary correlations

$$< k_{nn} > (l) \stackrel{\text{def}}{=} \sum_{k=1}^{K} k P(k|l)$$

ave. correl. $< k_{nn} >$ vs. degree l

$\alpha = 0.2$, $\gamma = 2.5$, Ass: $\nu = 2.5, 1.5, 0.5$, Dis: $\mu = 2.5, 1.5, 0.5$

estimated from real data of actor-collabo. (Ass), Internet-AS level-, and email (Dis).