Well-structure pushdown systems
Case of Dense Timed-Pushdown Automata

Cai Xiaojuan (SJTU)

Mizuhito Ogawa (JAIST)

FLOPS 2014@金沢

2014.6.6

This Talk

- Pushdown Extension of Well-Structured Transition Systems (WSPDS), introduced at CONCUR13
  - ✓Conditions for quasi-coverability, coverability.
  - ✓Well-formed projection for reachability.

- Examples as instances of WSPDS
  - ✓Multiset PDS (without *well-formed projection*) [**Chadha-Viswanathan** CONCUR07]
  - ✓Dense Time PDA (with *well-formed projection*) [**Abdulla**,et.al. LICS12]

Well structured transition system (WSTS)

- **Def.** WSTS M = (S, Δ) consists of
  ✓ WQO (S, ≦) (a possibly *infinite* states)
  ✓ $\Delta \subseteq S \times S$ *monotonic* transitions

  where (S, ≦) is a WQO if any infinite sequence s_1, s_2, s_3, … in *D has a pair* (*i, j*) such that $i < j$ and $s_i \leqq s_j$.

- **Remark**. Well-ordering is WQO; $>$ = $\geqq - \leqq$ is WFO.

- **Theorem**. Coverability of a WSTS is decidable.
  [**Finkel**87, **Abdulla**,et.al.00, **Finkel-Schnoebelen**01]

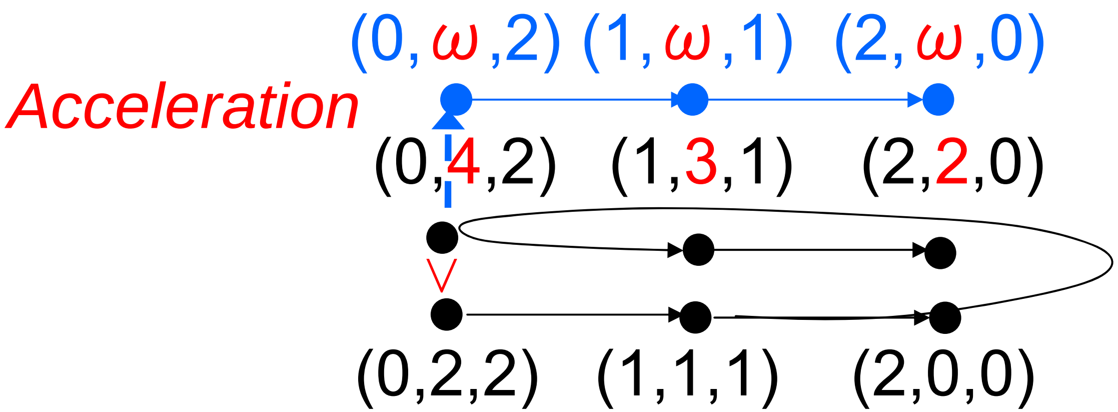
Petri-net (*VAS* = Vector Addition System)

- **VAS** : finite integer vector addition rules on $\mathbb{N}^k$
  e.g., { **m** → **m** + (1,-1,-1), **n** → **n** + (-2,4,2) }

- **Decidability**
  ✓Reachability, e.g., (0,2,2) →* (2,3,0) ?
  [**E.Mayr** 81, **Lambert** 92, **Leroux** 11]
  ✓Coverability, e.g., (0,2,2) →* ∃**m**' ≧ (2,3,0) ?
  [**Karp-Miller** acceleration 69, **Finkel** 93, **GRB** 07]

Example: Coverability (0,2,2) to (2,3,0) where { **m** → **m** + (1,-1,-1), **n** → **n** + (-2,4,2) }

- Forward: Acceleration



(2,4,0) : *reachable*

V

(2,3,0) : *covered*

(3,3,0) : not covered

- Backward: Minimal elements

(1,4,2) (2,4,2)

(0,5,2) → (1,4,1) → (2,3,0)

(3,0,0) (4,0,0)

(0,4,2)

(0,3,2) → (1,2,1) → (2,1,0)

(0,2,2) → (1,1,1) → (2,0,0)

covered

WQO guarantees termination.

PDS (Pushdown systems) (S, Γ, Δ)

- **PDS** example

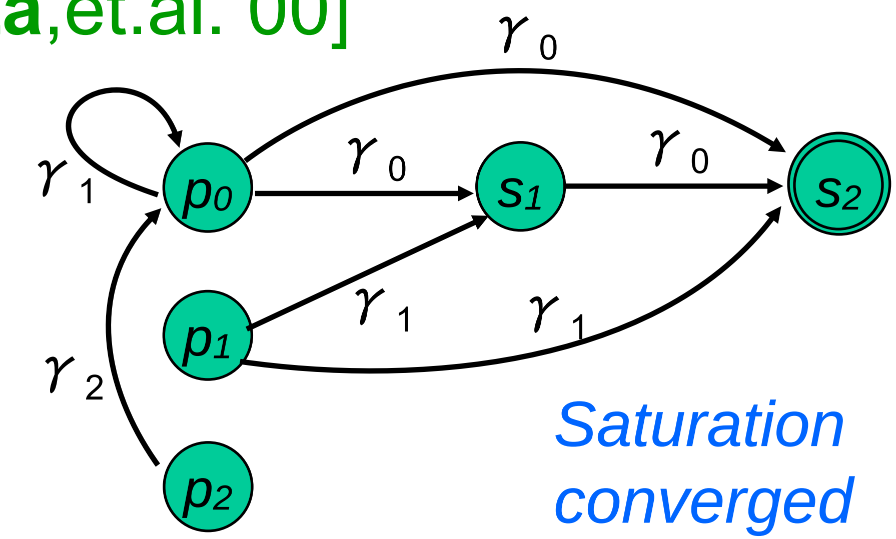
$$(1).\ \langle p_0, \gamma_0 \rangle \rightarrow \langle p_1, \gamma_1 \gamma_0 \rangle$$
$$(2).\ \langle p_1, \gamma_1 \rangle \rightarrow \langle p_2, \gamma_2 \gamma_0 \rangle$$
$$(3).\ \langle p_2, \gamma_2 \rangle \rightarrow \langle p_0, \gamma_1 \rangle$$
$$(4).\ \langle p_0, \gamma_1 \rangle \hookrightarrow \langle p_0, \epsilon \rangle$$

- Reachability is decidable $\langle p_1, \gamma_1 \rangle \rightarrow^* \langle p_0, \gamma_0 \gamma_0 \rangle$?

✓**CYK**-algorithm 65, P-automaton [**Büchi** 64, **Finkel**,et.al. 87, **Esparza**,et.al. 00]

Construct $\mathcal{A}$ with
$L(\mathcal{A}) = Pre^*(\langle p_0, \gamma_0 \gamma_0 \rangle)$

Reachable!



Saturation converged

Well-Structured Pushdown Systems (WSPDS)

PDS
(Pushdown system)

WSTS [**ACJY** 00, **Finkel**,et.al 01]
(Well-structured transition system)

→ WSPDS
(Well-Structured PDS)

- WSPDS (S, Γ, Δ) : $(S, \leq)$, $(\Gamma, \leqq)$ are WQO
  - ✓**Th**. When P-automaton converges, coverability is decidable. (CONCUR13)
- **Forward** : *Post* + acceleration*
  - ✓RVASS, BVAS, VASS with one zero-test
- **Backward**: *Pre* + minimal elements*
  - ✓Multiset PDS, Dense Time PDA (*Pre** diverges)

Coverability and Quasi-coverability

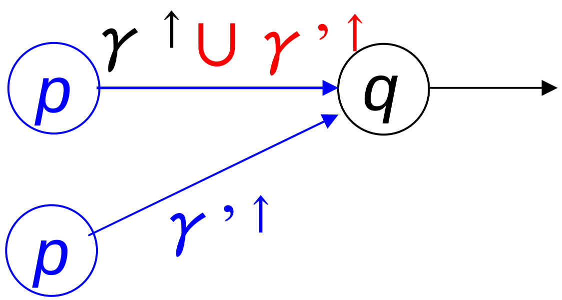
- **Def**. Given source / target configurations $\langle p, w\rangle$, $\langle q, v\rangle$
  - ✓Coverability: $\exists q', v' . \langle p, w\rangle \rightarrow^* \langle q', v'\rangle$, $q \le q'$, $v \leqq v'$
  - ✓Quasi-coverability: $\exists p', w', q', v' . \langle p', w'\rangle \rightarrow^* \langle q', v'\rangle$, $p \le p'$, $w \leqq w'$, $q \le q'$, $v \leqq v'$

  where $\gamma_1 \gamma_2 \ldots \gamma_n \leqq \gamma'_1 \gamma'_2 \ldots \gamma'_n \Leftrightarrow \forall k. \gamma_k \leqq \gamma'_k$

- **Th**. For WSPDS (S, Γ, Δ), assuming computability of immediate predecessor sets ($pre(w^{\uparrow})$),
  - ✓If *Pre** automaton converges (e.g., $|S| < \infty$), coverability is decidable. (CONCUR13)
  - ✓If a WSPDS is *growing,* quasi-coverability is decidable. (This work)

Idea for coverability

- WSTS techniques on edges of *Pre** automaton.
  - ✓Example: Coverability of Multiset PDS



- If *Pre** automaton does not converge, strengthen quasi-coverability to reachability by finding a *compatible well-formed projection*. (*Later*)
  - ✓Example: Dense Time PDA (DTPDA)

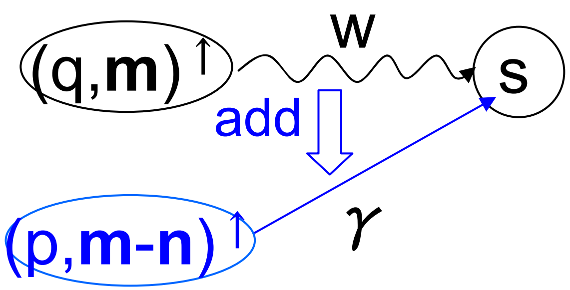
Multiset PDS [Chadha-Viswanathan07]

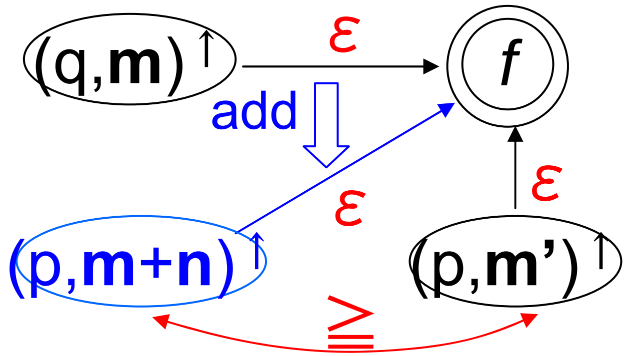
- Multi-set PDS (S, Γ , Δ) has
  - ✓S = finite control states × $\mathbb{N}^k$ (WQO)
  - ✓ Γ = finite stack alphabet
  - ✓ Δ : special forms (*Pre** converges)

Coverability is decidable

$$\frac{(p,\gamma,q,w,\boldsymbol{n}) \in \delta_1}{\langle(p,\boldsymbol{m}),\gamma w'\rangle \hookrightarrow \langle(q,\boldsymbol{n}+\boldsymbol{m}),ww'\rangle}$$

$$\frac{(p,q,\boldsymbol{n}) \in \delta_2, \boldsymbol{m}-\boldsymbol{n} \in \mathbb{N}^k}{\langle(p,\boldsymbol{m}),\epsilon\rangle \hookrightarrow \langle(q,\boldsymbol{m}-\boldsymbol{n}),\epsilon\rangle}$$

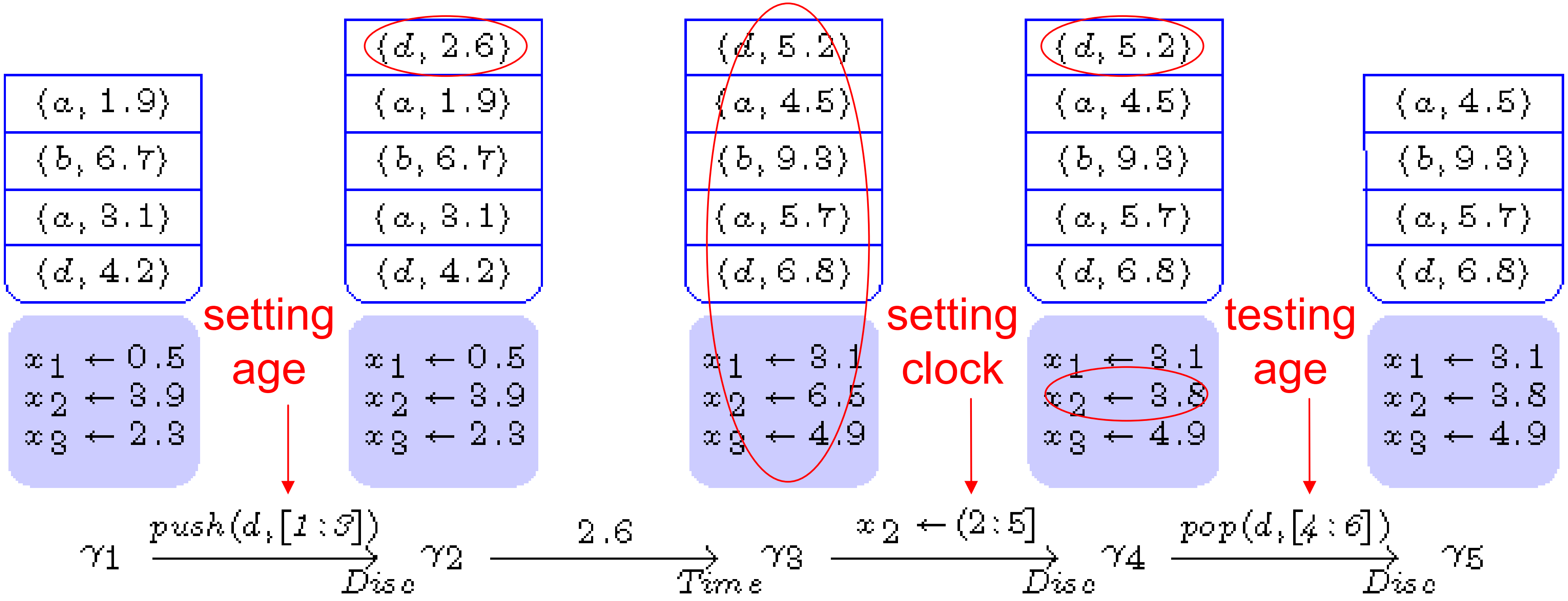




Only with the empty stack

Dense Time PDA (DTPDA) [**Abdulla**,et.al.12]

- Timed PDA with *global clocks* and *local ages*
  - ✓*Discrete* transitions: Control transitions (with testing/setting time), and no time proceeds.
  - ✓*Time* transitions: Time progress.

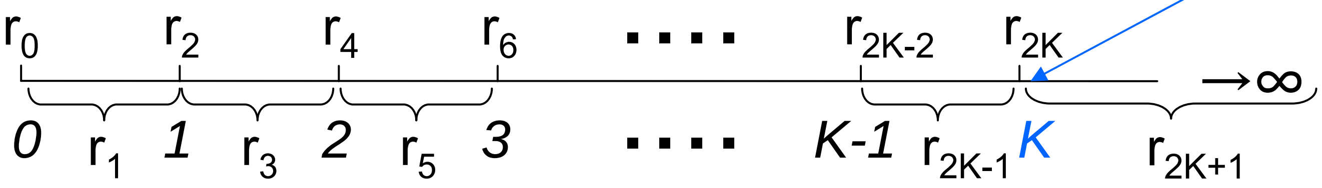


Difficulty: Local ages in the stack also proceeds

Dicretization (Region word)

- Word representation of region construction **[Ouakline-Worrell**04]

Max K appearing in time constraints



Local ages in the top and second frames are marked

(d,5,2)
(a,4.5)
(b,9.8)
(a,5.7)
(d,6.8)

$x_1 \leftarrow 3.1$
$x_2 \leftarrow 6.5$
$x_3 \leftarrow 4.9$

$\{(x_1,r_7)\}, \{(d,r_{11})\}, \{(a,r_9),(x_2,r_{13})\}, \{(a,r_{11})\}, \{(b,r_{19}), (d,r_{13})\}, \{(x_3,r_9)\}$
$\{(x_1,r_7)\}, \{(a,r_9),(x_2,r_{13})\}, \{(a,r_{11})\}, \{(b,r_{19}),(d,r_{13})\}, \{(x_3,r_9)\}$
$\{(x_1,r_7)\},\{(x_2,r_{13})\},\{(a,r_{11})\}, \{(b,r_{19}), (d,r_{13})\} ,\{(x_3,r_9)\}$
$\{(x_1,r_7)\},\{(x_2,r_{13})\},\{(a,r_{11})\},\{(d,r_{13})\},\{(x_3,r_9)\}$
$\{(x_1,r_7)\},\{(x_2,r_{13})\},\{(d,r_{13})\},\{(x_3,r_9)\}$

0.1 *0.5* *0.8* *0.9*

Encoding
Time progress: Rotation of the top frame.
Call: Put a local age into the region word and push.
Return: Porpagate time progress to the next frame and pop

Well-formed projections

- **Def**. A monotonic projection ⇓ such that, if not undefined, compatible with transitions.

- **Remark**. If the source / target configurations $\langle p,w \rangle$, $\langle q,v \rangle$ hold $\langle p,w \rangle{\Downarrow} = \langle p,w \rangle$ and $\langle q,v \rangle{\Downarrow} = \langle q,v \rangle$, quasi-coverability becomes reachability.

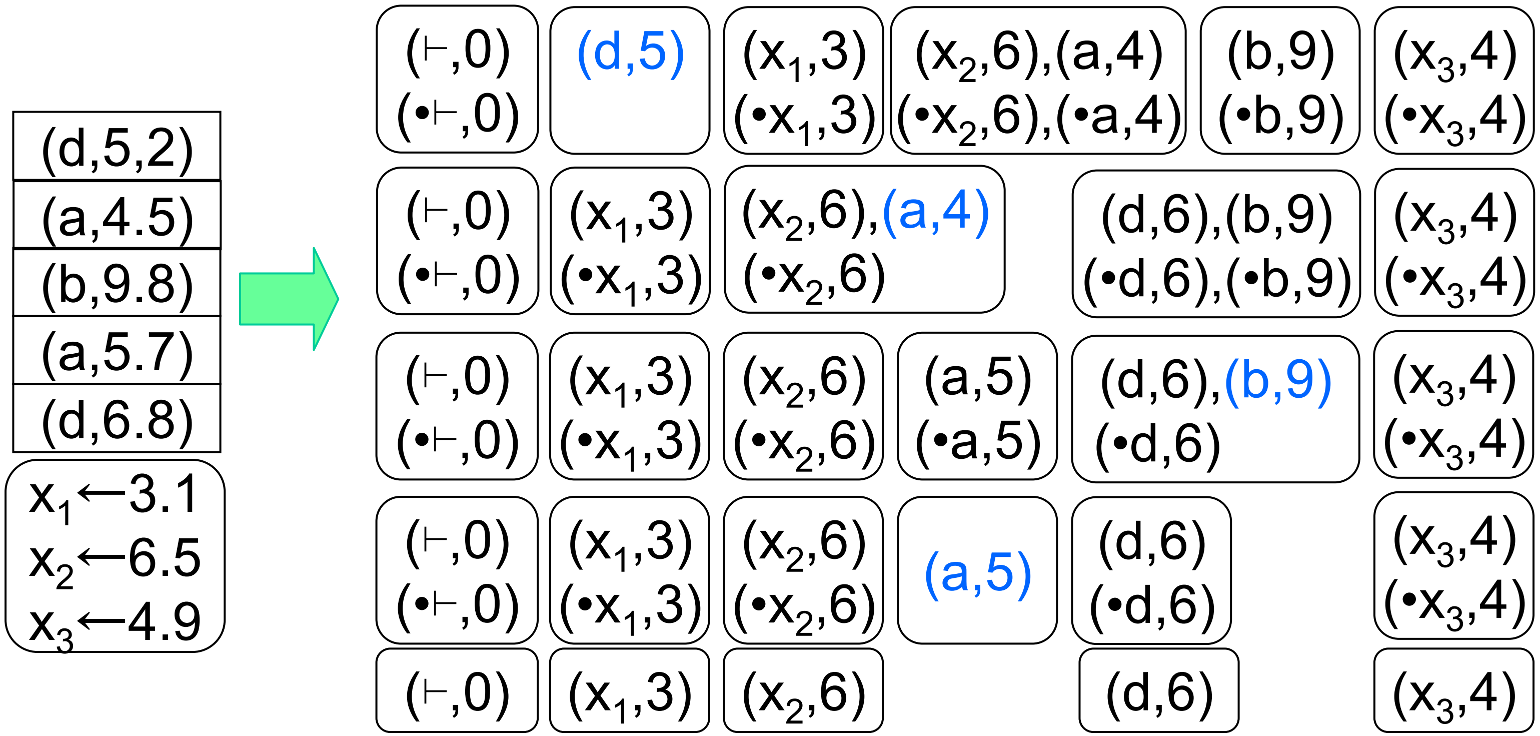
- For the discretization of DTPDA, it is

$\{(x_1,r_7)\},\{(c,r_7)\},\{(d,r_{11})\}, \{(a,r_9),(x_2,r_{13})\}, \{(a,r_{11})\}, \{(b,r_{19}), (d,r_{13})\},\{(a,r_5)\},\{(x_3,r_9)\}$
$\{(x_1,r_7)\},\{(c,r_7)\},\{(a,r_9),(x_2,r_{13})\}, \{(a,r_{11})\}, \{(b,r_{19}),(d,r_{13})\},\{(a,r_5)\},\{(x_3,r_9)\}$
$\{(x_1,r_7)\},\{(c,r_7)\},\{(x_2,r_{13})\},\{(a,r_{11})\}, \{(b,r_{19}), (d,r_{13})\},\{(a,r_5)\},\{(x_3,r_9)\}$
$\{(x_1,r_7)\},\{(c,r_7)\},\{(x_2,r_{13})\},\{(a,r_{11})\},\{(d,r_{13})\},\{(a,r_5)\},\{(x_3,r_9)\}$
$\{(x_1,r_7)\},\{(c,r_7)\},\{(x_2,r_{13})\},\{(d,r_{13})\},\{(a,r_5)\},\{(x_3,r_9)\}$

⇓ keeps global clocks, and ages propagated from marked ages.

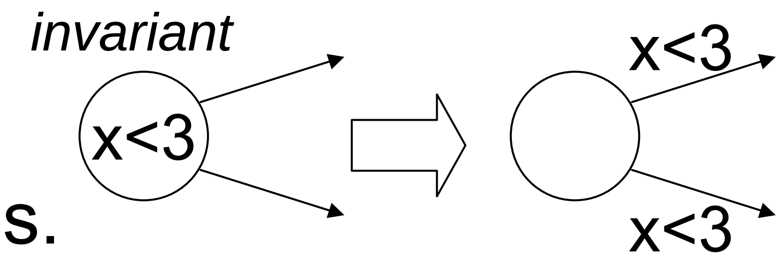
Comparison with original DTPDA encoding

- The similar idea of region words, but overwrites local ages with the same stack alphabet. (• shows the pointers to the next frame) ⇒ (finite) PDS encoding
  - ✓Reachability was shown.



Conclusion

- WSPDS reduces coverability to convergence of P-automaton.
  - ✓**Forward**: "*Post** + acceleration" reproves RVASS, BVAS, VASS with one zero-test.
  - ✓**Backward**: "*Pre** + minimal elements" reproves Multiset PDS, DTPDA (with well-formed projection).



- Extension with invariants
  - ✓For TA, not much differences.
  - ✓For DTPDA, *invariants on local ages* are hidden in the stack, which can be handled by our encoding.