Proof-theoretic semantics and constructivity

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Aim of the talk

- An examination of Dummett's idea that some inference rules confer the meanings of logical constants that they govern.
 - Justification of logical laws or proof-theoretic semantics.
- An alternative picture of meaningconferring different from his own.

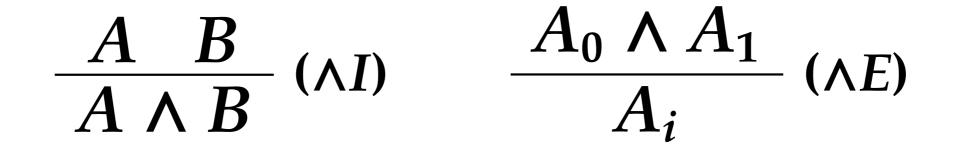
Dummett's picture

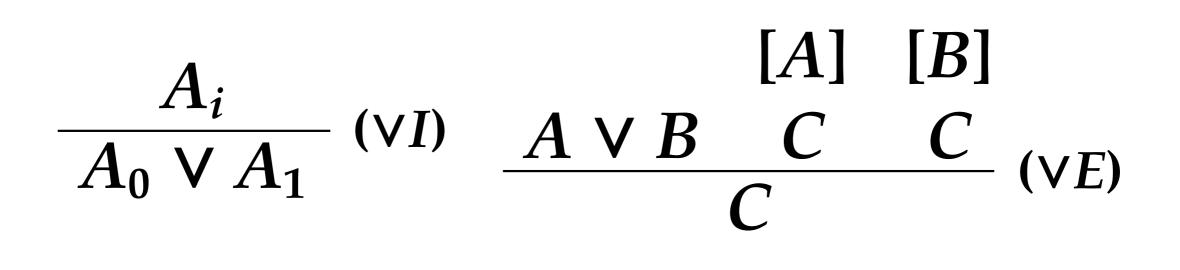
Dummett's claim

in The Logical Basis of Metaphysics (1991)

- Intuitionistic logic (IL) is based on some self-justifying inference rules. But classical logic (CL) is not.
 - A class of basic inference rules of IL confer the meanings of logical constants, hence they are self-justifying.
 - Other basic rules are justified in terms of such rules.

I-rules and E-rules





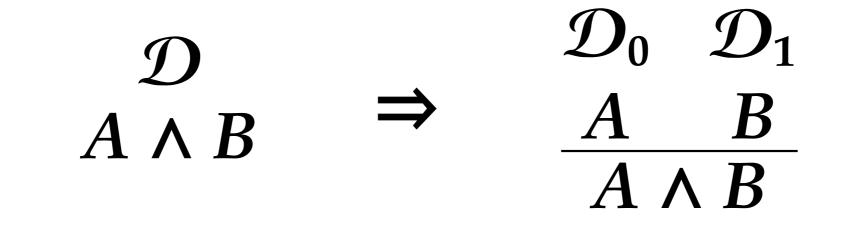
Meaning-conferring rules

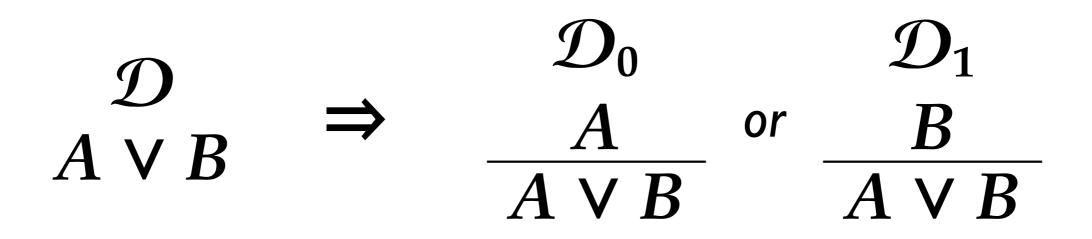
- First, either I-rules or E-rules are taken to be a "base", that is, *meaning-conferring*.
- Meaning-rules are regarded as mere stipulations. Hence they are valid by stipulation, that is, self-justifying.
- Let us take I-rules as a base here.

Fundamental Assumption

- Non-meaning rules are justified by the Fundamental Assumption (FA).
- FA: If we have a valid argument for a complex statement, we can construct a valid argument for it which finishes with an application of one of the introduction rules governing its principal operator.

Fundamental Assumption



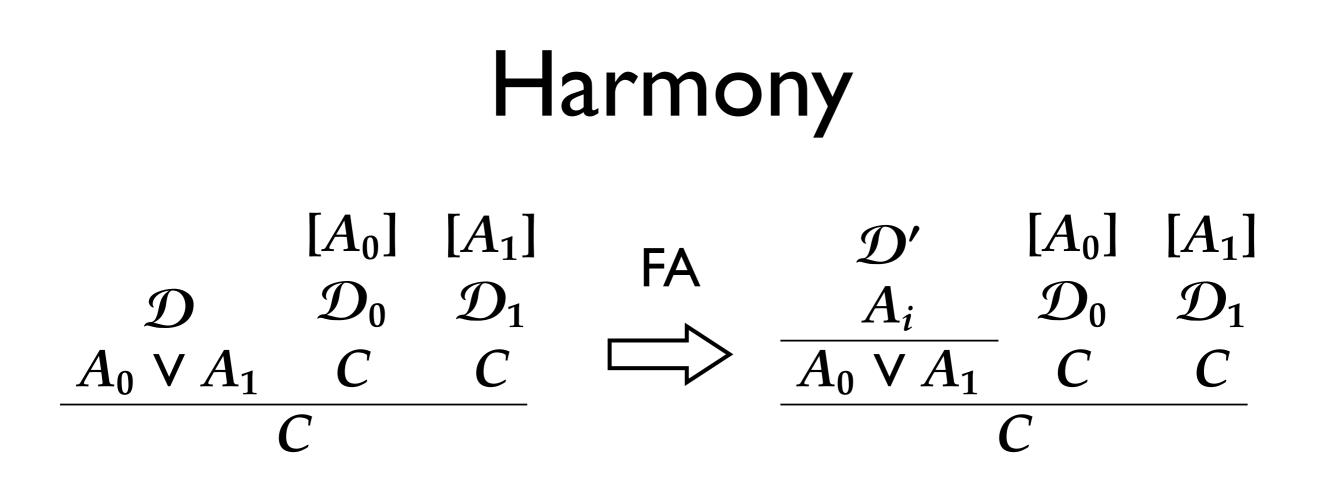


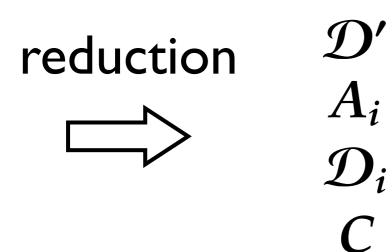
Fundamental Assumption

- $\vdash A \land B \iff \vdash A \text{ and } \vdash B$
- $\vdash A \lor B \iff \vdash A \text{ or } \vdash B$
- FA: an assumption that I-rule is invertible.
 - The range of direct grounds of the complex statement is fixed.
 - I-rules and FA jointly embody definitional biconditionals.

Harmony as justification

- Non-meaning rules (E-rules) are justified if harmony obtains.
- Harmony: Any consequence that E-rules entitle us to draw from the complex statement is already a consequence of its direct grounds.





the conclusion of E-rule can be obtained directly.

FA as a precondition

- Finally, check whether FA holds in the entire system.
 - ✦ Justification of E-rules depends on FA.
 - If FA fails in the system as a whole, justification lose its significance.

$\mathsf{CL} \,\mathsf{vs} \,\mathsf{IL}$ $\vdash A \,\lor B \implies \vdash A \,\mathrm{or} \vdash B$

- FA for disjunction holds in the full system of IL (disjunction property), but fails in CL.
- IL: self-justifying I-rules + justified E-rules.
- CL: since FA fails, justification doesn't work.

FA as a global feature

- Failure of FA in CL is caused in part by non-conservative formulation of negation.
- FA as a precondition for justification is a global feature of entire systems.

Stability

Harmony

 Harmony: Any consequence that E-rules entitle us to draw from the complex statement is already a consequence of its direct grounds.

$Cn(A) \subset Cn(DG[A])$

Stability

$Cn(DG[A]) \subset Cn(A)$

 Stability: E-rules enable us to draw all consequences that the direct grounds of the statement entitle us to draw.

$\begin{array}{c|c} Stability \\ [A] & [B] \\ \hline A \lor B & C & C \\ \hline C & (\lor E) \end{array}$

- We can see from the form of the rule that stability obtains.
- How about other constants?

Stability $\begin{bmatrix} A, B \end{bmatrix}$ $\frac{A \land B \qquad C}{C} (\land E_G)$

- General elimination rules (cf. Schroeder-Heister 1984; Negri 2002, von Plato 2003)
 - Equivalent to standard E-rules.
 - Stability can be seen from the form of the rules.

Harmony and stability

Cn(A) = Cn(DG[A])

- Harmony: the inverses of I-rules (FA) justify Erules (through operations on deductions). The former is not weaker than the latter.
- Stability: E-rules are not weaker than the inverses of I-rules.
- H&S:A symmetry between I-&E-rules.

$\begin{array}{c} \text{Read-off rules} \\ \hline A_0 \land A_1 \\ \hline A_i \end{array} (\land E) \implies \begin{array}{c} \vdash A \land B \implies \\ \vdash A \text{ and } \vdash B \end{array}$

- Consider whether E-rules justify the inverses of I-rules.
- Such pairs of I-&E-rules are equivalent to the definitional biconditionals.
- Let us call such pairs read-off rules.

Read-off rules

$$\begin{array}{cccc} [A] & [B] & & \vdash A \lor B \Rightarrow \\ \hline A \lor B & C & C \\ \hline C & & (\lor E) & & & \vdash A \text{ or } \vdash B \end{array} \end{array}$$

- Disjunction: E-rule cannot be taken to justify the inverse of I-rules.
- The rules for disjunction are harmonious and stable, but not read-off rules.

Read-off rules

• "Read-off": a stronger symmetry than H&S.

Dummett doesn't mention it.

- An alternative picture of meaningconferring emerges if we consider systems consisting solely of read-off rules.
- The picture is essentially what is presented in Sambin, Battilotti and Faggian (2000).

In merely H&S systems

I-rules + E-rules ≠ definitional biconditionals

I-rules + FA = definitional biconditionals

- One-half of definitional biconditionals are realized only as a *global* feature (FA).
- E-rules are only justified. The symmetry between the rules doesn't play any role in meaning-conferring.

In "read-off" systems

I-rules + E-rules = definitional biconditionals

- Definitional biconditionals are *locally* realized.
- The symmetry between the rules has a significance in meaning-conferring.
- FA as a global feature must hold to warrant that the "definitions" realized by rules are correct.

Why harmony and stability?

$H\&S: Cn(A \lor B) = Cn(DG[A \lor B])$

$\Gamma, A \vdash C \quad \Gamma, B \vdash C$ $\Gamma, A \lor B \vdash C$

 If we take E-rules (L-rules) as the "base", then the rules can be seen as read-off rules.

definitional $\Gamma, A \lor B \vdash C \iff$ biconditional: $\Gamma, A \vdash C$ and $\Gamma, B \vdash C$

$$\begin{array}{c|c} & A_0 \lor A_1 \vdash A_0 \lor A_1 \\ \hline \Gamma \vdash A_i & A_i \vdash A_0 \lor A_1 \\ \hline \Gamma \vdash A_0 \lor A_1 \end{array} \end{array} \begin{array}{c} \text{The inverse} \\ \text{of L-rule} \end{array}$$

The inverse of L-rule justifies R-rule.

Conversely,

$$(\lor R) \frac{A_i \vdash A_i}{A_i \vdash A_0 \lor A_1} \Gamma, A_0 \lor A_1 \vdash C$$
$$\Gamma, A_i \vdash C$$

R-rules justifies the inverse of L-rule.

Therefore L-&R-rules are equivalent to the definitional biconditional.

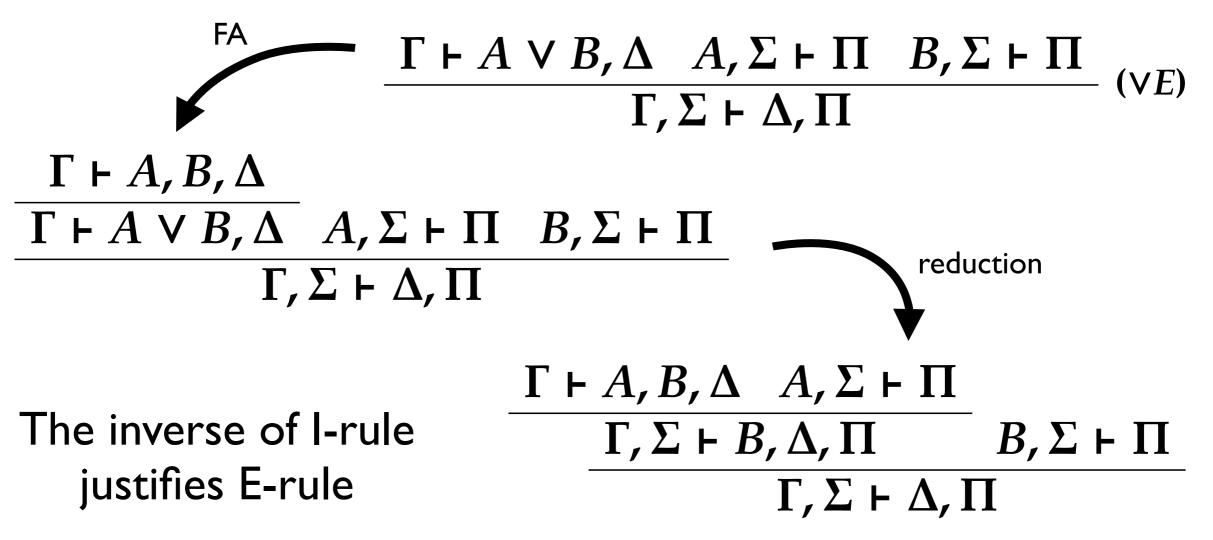
- This feature does not differentiate IL from CL, so has no use for Dummett's purpose.
- It must be always possible to take I-rules as the base.
- The "reading-off" picture is not suitable for him here.

Multiple-conclusion

- How to obtain a pair of read-off rules for disjunction with I-rules as the base?
- A natural way is to allow for multipleconclusion (cf. Read 2000).

$$(\lor I) \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \lor B, \Delta}$$
$$(\lor E) \frac{\Gamma \vdash A \lor B, \Delta \quad A, \Sigma \vdash \Pi \quad B, \Sigma \vdash \Pi}{\Gamma, \Sigma \vdash \Delta, \Pi} 31$$

Definitional $\Gamma \vdash A \lor B, \Delta \iff \Gamma \vdash A, B, \Delta$ biconditional:



Definitional $\Gamma \vdash A \lor B, \Delta \iff \Gamma \vdash A, B, \Delta$ biconditional:

$\begin{array}{c} \Gamma \vdash A \lor B, \Delta \quad A, \Sigma \vdash \Pi \quad B, \Sigma \vdash \Pi \\ \hline \Gamma, \Sigma \vdash \Delta, \Pi \end{array} (\lor E) \\ \hline & \swarrow \quad \Sigma := \emptyset, \Pi := \{A, B\} \\ \hline & \Gamma \vdash A \lor B, \Delta \quad A \vdash A, B \quad B \vdash A, B \\ \hline & \Gamma \vdash A, B, \Delta \end{array}$

E-rule justifies the inverse of I-rule.

Thus, I-&E-rules are equivalent to the definitional biconditional.

Multiple-conclusion

- Multiple-conclusion rules make possible a "reading-off" formulation of disjunction.
- But they open the way for a formulation of classical logic based on self-justifying (meaning-conferring) rules.
- The "reading-off" picture is not suitable for Dummett's purpose here, too.

Conclusion

Conclusion

- Two different symmetry between the rules: H&S and reading-off.
- Two different pictures of meaningconferring by inference rules.
- The "reading-off" picture is not suitable for Dummett's intent.
- But it is surely a viable alternative.

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