Workshop on Proof Theory and Computability Theory

Date: 21-24 February 2011

Contents

Program

2011/2/21

	Speaker	Title
11:20-12:00	Yokoyama	On Ramsey's theorem for pairs
Lunch		
13:20-14:00	Discussion time	
14:10-14:50	Sanders	Reverse Mathematics & Nonstandard Analysis
Break		
15:10-15:50	Wong	Omitting types theorem and reverse mathematics
Break		
16:10-16:30	Termimi	A New Network Game with Many Attackers and Defenders
16:30-17:00	Higuchi	Weihrauch Lattice and Intuitionism
17:00-17:30	Kihara	Degrees of difficulty of disjunctions

2011/2/22

	Speaker	Title
9:30-10:10	Kashima	Canonical models for some modal logics
10:20-11:00	Wilken	Derivation lengths classification of Goedel's T revisited
Break		
11:20-12:00	Ueno	Proof Methods for Formula Size Lower Bounds Classics and Frontier
Lunch		
13:20-14:00	Kumabe	TBA
14:10-14:50	Suzuki	No-free-lunch theorem and uniqueness of eigen-distribution of a game tree
Break		
15:10-15:50	Tadaki	A computational complexity-theoretic elaboration of weak truth-table reducibility
Break		
16:10-16:30	Horihata	Ring of integers in second order arithmetic
16:30-17:00	Kurahashi	Nonstandard model of arithmetic and QGL
17:00-17:30	Peng	The distributional query complexity of unbalanced game trees

2011/2/23

	Speaker	Title
9:30-10:10 10:20-11:00	Arai Kuroda	Quick cut-elimination in intuitionistic logic calculi Circuits and Bounded Arithmetic for LOGDCFL
Break		
11:20-12:00	Miyabe	Degree of non-randomness and uniform Solovay reducibility
Lunch		
13:20-13:40	Iwanami	On systems between LK and LJ
13:40-14:00	Sakakibara	Tutte's Theorem in Reverse Mathematics
14:00-14:20	Yoshii	Game of determinacy in Second Order Arithmetic II
Break		
15:00-15:30	Izawa	Reverse Mathematics in Finite type Arithmetic
15:30-16:00	Sangu	Set-generated classes in Constructive Set Theory **CZF**
16:00-17:30	Informal session	

February 21

Keita Yokoyama

On Ramsey's theorem for pairs

Abstract: On Ramsey's theorem for pairs

Ramsey's theorem for pairs (RT^2_2) plays an important role in Reverse Mathematics as an intermediate axiom between RCA_0 and ACA_0. There are many theorems of combinatorics and model theory that are provable from RT^2_2. Thus, determining the exact strength of RT^2_2 is very important. It is well-known that RT^2_2 implies $\mathrm{B}\Sigma^0_2$. On the other hand, Cholak, Jockusch and Slaman showed that WKL_0+ RT^2_2+$\mathrm{I}\Sigma^0_2$ is a Π^1_1-conservative extension of RCA_0+$\mathrm{I}\Sigma^0_2$, *i.e.*, the first-order part of RT^2_2 is not stronger than $\mathrm{I}\Sigma^0_2$. Then, the question arises: is WKL_0+ RT^2_2 a Π^1_1-conservative extension of RCA_0+$\mathrm{B}\Sigma^0_2$? Recently, a partial answer to this question is given by Slaman, Chong and Yang. They showed that RCA_0+CAC is a Π^1_1-conservative extensions of RCA_0+ $\mathrm{B}\Sigma^0_2$. Here, COH is a combinatorial principles weaker than RT^2_2. Bovykin and Weiermann gave another approach. They showed that a certain finite version of Ramsey's theorem for pairs is exactly the same as the Π_2 consequences of RT^2_2. In this talk, I will summarize some recent developments on this topic, and consider the relation to the nonstandard method.

Sam Sanders

Reverse Mathematics & Nonstandard Analysis

Why some theorems are more equal than others

Abstract: Reverse Mathematics is a program in foundations of mathematics initiated by Friedman ([1,2]) and developed extensively by Simpson ([4]). Its aim is to determine which *minimal* axioms prove theorems of ordinary mathematics. Nonstandard Analysis plays an important role in this program ([3,5]). We consider Reverse Mathematics where equality is replaced by the predicate $\approx$, i.e. equality up to infinitesimals from Nonstandard Analysis. A 'copy' of Reverse Mathematics for WKL_0 and ACA_0 is obtained in a weak system of Nonstandard Analysis. We consider possible connections with constructive analysis and recursion theory. Our results also have implications for the Philosophy of Science. In particular, we show how the very nature of Mathematics in Physics implies that real numbers are not needed in Physics (cf. the famous Quine-Putnam indispensability argument).

REFERENCES

[1] Harvey Friedman, *Some systems of second order arithmetic and their use*, Proceedings of the International Congress of Mathematicians (Vancouver, B. C., 1974), Vol. 1, Canad. Math. Congress, Montreal, Que., 1975, pp. 235-242.

[2] Harvey Friedman, *Systems of second order arithmetic with restricted induction, I & II (Abstracts)*, Journal of Symbolic Logic 41 (1976), 557-559.

[3] H. Jerome Keisler, *Nonstandard arithmetic and reverse mathematics*, Bull. Symbolic Logic 12 (2006), no. 1, 100-125.

[4] Stephen G. Simpson, *Subsystems of second order arithmetic*, Perspectives in Mathematical Logic, Springer-Verlag, Berlin, 1999

[5] Kazuyuki Tanaka, *The self-embedding theorem of WKL0 and a non-standard method*, Ann. Pure Appl. Logic 84 (1997), no. 1, 41-49.

Tin Lok Wong

Omitting types theorem and reverse mathematics

Abstract: The omitting types theorem is very handy in building special models of arithmetic. On the one hand, as omitting types is essentially a kind of forcing, many theorems that can be proved using forcing can also be proved by omitting suitable types. On the other hand, omitting certain types in a purely model-theoretic context sometimes leads to the rediscovery of combinatorial principles that are of interest in reverse mathematics. I will demonstrate these with a selection of nice examples that originated from the work of Scott, Gaifman, Paris, Kirby, and others.

Ahmad Termimi Bin Ab Ghani

A New Network Game with Many Attackers and Defenders

Abstract: In [3], Mavronicolas et al. presented a network game as an undirected graph whose nodes are exposed to infection by attackers, and whose edges are protected by defender. In [4], we generalized their model so that we have many defenders instead of a single player. Then in [1], we introduced a new network game with the roles of players interchanged.

We obtained a graph-theoretic characterization of pure Nash equilibria of our new model. In this paper we focus on mixed strategies and study the complexity of finding a (mixed) Nash equilibria in the new game.

REFERENCES

[1] A.G. Ahmad Termimi, K. Tanaka. Network Games with Many Attackers and Defenders, To Appear in *Proceedings of RIMS Kôkyûroku*, Kyoto University.

[2] D. Blackwell. Infinite G_δ games with imperfect information. *Zastosowania Matematyki Applicationes Mathematicae*, Hugo Steinhaus Jubilee Volume **X**, p. 99-101, 1969.

[3] M. Mavronicholas, V. Papadopoulou, A. Philippou, P. Spirakis. *A Network Game with Attackers and Defender*, Algorithmica **51**: 315-341, 2000.

[4] MedYahya Ould-MedSalem, Y. Manoussakis, K. Tanaka. *A Game on Graphs with Many non-Centralized Defenders*, (EURO XXII) 22nd European Conference on Operations Research, July 8-11, Prague, 2007.

[5] D.B. West. *Introduction to Graph Theory*. Prentice Hall, 2nd edition. 2001.

Kojiro Higuchi

Weihrauch Lattice and Intuitionism

Abstract: In [1], the study of Weihrauch degrees is suggested to be an approach for classifying mathematical theorems according to their computational content and to their logical complexity. In this talk, we discuss on the relation between the Weihrauch lattice and intuitionism. The Weihrauch lattice is expected to be a semantics for a superintuitionistic logic as like Medvedev lattice. In [2], it turns out that the computable Weihrauch lattice is not but the continuous one is.

REFERENCES

[1] Vasco Brattka and Guido Gherardi, Effective Choice and Boundedness Principles in Computable Analysis, Journal of Symbolic Logic (to appear), 2010, arXiv:0905.4685.

[2] Kojiro Higuchi and Arno Pauly, The degree structure of Weihrauchreducibility, 2010, arXiv:1101.0112v1.

Takayuki Kihara

Degrees of difficulty of disjunctions

Abstract: In 2009, Cenzer, Weber, Wu, and the speaker introduced the notion of tree-immunity for closed subsets of Cantor space. Their method to prove some theorems on tree-immunity gives rise to a kind of "disjunction" under the limit-BHK interpretation of Limit Computable Mathematics (abbreviated LCM), a kind of constructive mathematics based on Learning Theory. This allows us to define various disjunctions as operations on the power set of Baire space. When disjunctive notions are represented as operations on subsets of Baire space, this enable us to compare "degrees of difficulty" of disjunctive notions. This allows us to formalize the intuition that the intuitionistic disjunction is somehow more difficult than the classical one. We then introduce the notion of learnability for subsets of Baire space. In this way, we can obtain a better understanding of the behavior of degrees of difficulty of disjunctive notions. Conversely, these disjunctive notions turn out to be useful for analyzing the Medvedev lattice of Π^0_1 subsets of Cantor space.

February 22

Ryo Kashima

Canonical models for some modal logics

Abstract: We establish the "canonical model theorem" for propositional modal logics with two modal operators, say X and Y, that behave as "Yp $\Leftrightarrow$ Xp & XXp & XXXp & ..." For example, (X, Y) = ("everyone knows", "it is common knowledge") in epistemic logic; and this fills a gap in the proof of the completeness theorem appears in a standard text book "Reasoning About Knowledge" by Fagin et al.

Gunnar Wilken

Derivation lengths classification of Goedel's T revisited

Abstract: Inspired by recent suggestions from William A. Howard on a manuscript providing a complete derivation lengths classification of the unrestricted lambda calculus version of Goedel's T (joint work with Andreas Weiermann), I will outline the technique of ordinal assignment and the collapsing procedure in order to obtain sharp upper bounds for the heights of reduction trees in T. Compared to the earlier manuscript the approach has become considerably more accessible.

Kenya Ueno

Proof Methods for Formula Size Lower Bounds: Classics and Frontier

Abstract: We consider a propositional Boolean formula, which is a binary tree with each leaf labeled by either a variable or a negated variable, and each internal node labeled by either AND or OR. Its size is measured by the total number of variables and negated variables. A super-polynomial formula size lower bound would separate NC1 and other complexity classed beyond it. In the case of NP, this is a weaker version of the P versus NP problem.

The most classical method by Khrapchenko gives a quadratic formula size lower bound for PARITY. Although its several extensions has been proposed, it is still hard to improve Khrapchenko's bound even for basic Boolean functions. In this talk, we will explain our recent work on linear programming based approaches improving formula size lower bounds after reviewing the

background and the classical method.

Masahiro Kumabe

TBA

Abstract:

Toshio Suzuki

No-free-lunch theorem and uniqueness of eigen-distribution of a game tree

joint work with Ryota Nakamura, Tokyo Metropolitan Univ.

Abstract: For a probability distribution d on the truth assignments to an AND-OR tree, Liu and Tanaka (Inform. Process. Lett., 2007) show that the followings are equivalent.

(LT1) d is an eigen-distribution, in other words, d achieves the equilibrium;

(LT2) d is a distribution on the 1-set (a set of assignments introduced by them) and expected computational cost of d does not depend on an algorithm;

(LT3) d is the uniform distribution on the 1-set.

We ask whether the above equivalences hold when algorithms are restricted to a given family closed under transposition of two sub-trees having a common parent node. In this setting, we show the followings.

(1) (LT1) and (LT2) are equivalent.

(2) (LT2) does not imply (LT3). We demonstrate a family of algorithms which has uncountably many eigen-distributions.

(3) With respect to the complement of the above family, (LT2) implies (LT3).

A variant of no-free-lunch theorem (NFLT) holds in our setting. Roughly speaking, NFLT (Walpert and MacReady, 1995) says that, averaged over all cost functions, all search algorithms give the same performance. NFLT shows the equivalence (1), but does not show the uniqueness of eigen-distribution.

Kohtaro Tadaki

A computational complexity-theoretic elaboration of weak truth-table reducibility

Abstract: The notion of weak truth-table reducibility plays an important role in recursion theory. In this talk, we introduce an elaboration of this notion, where a computable bound on the use function is explicitly specified. This elaboration enables us to deal with the notion of asymptotic behavior in a manner like in computational complexity theory, while staying in computability theory. We apply the elaboration to sets which appear in the statistical mechanical interpretation of algorithmic information theory. We demonstrate the power of the elaboration by revealing a critical phenomenon in the interpretation, which cannot be captured by the original notion of weak truth-table reducibility.

Yoshihiro Horihata

Ring of integers in second order arithmetic

Abstract: We develop basic theory of integers in weak second order arithmetic.

Taishi Kurahashi

Nonstandard model of arithmetic and QGL

Abstract: In 1976, Solovay proved the arithmetized completeness theorem for a propositional system GL of modal logic of provability. In 1984, Montagna proved that QGL which is a natural extension of GL to the predicate modal logic is not Kripke complete and that Solovay's theorem does not hold for QGL. Let Th(QGL), Fr(QGL) and PL(T) be the set of all theorems of QGL, the set of all sentences valid in all corresponding Kripke frames, and the set of all sentences provable in T for any arithmetical interpretation respectively. We prove that (1) PL(T) does not include Fr(QGL) for any Σ 1-sound recursively enumerable extension T of I Σ 1, (2) the intersection of PL(T) and Fr(QGL) does not coincide with Th(QGL) for any recursively enumerable extension T of I Σ 2 with the same language, and (3) the intersection of PL(T)'s for any recursively enumerable extension T of I Σ 1 with the same language does not characterize Th(QGL).

Ningning Peng

The distributional query complexity of unbalanced game trees

Abstract: The AND-OR tree is an extremely simple model to compute the read-once Boolean functions. In this talk, we investigate the distributional query complexity of some balanced or unbalanced game trees: T_2^k, $(T_2^-)^k$ etc. Here, we use the eigen-distribution arguments introduced by Liu and Tanaka [1], to obtain the worst distribution on assignments for its best deterministic algorithm and then to give the query complexity.

REFERENCES

[1] Liu C.G., and Tanaka K.: Eigen-Distribution on Random Assignments for Game Trees. Information Processing Letters, 104(2): 73-77, (2007).

[2] Saks M., Wigderson A.: Probabilistic Boolean Decision Trees and the Complexity of Evaluating Game Trees. In Proceedings of 27th Annual IEEE Symposium on Foundations of Computer science(FOCS). (1986) 29-38

[3] Yao, A. C.-C.: Probabilistic Computations: Towards a Unified Measure of Complexity. In Proceedings of 18th Annual IEEE Symposium on Foundations of Computer science(FOCS). (1977) 222-227

February 23

Toshiyasu Arai

Quick cut-elimination in intuitionistic logic calculi

Abstract: We show that cut inferences are quickly eliminable in intuitionistic logic calculi, and the elimination is applied to problems on intuitionistic arithmetics.

Satoru Kuroda

Circuits and Bounded Arithmetic for LOGDCFL

Abstract: We consider the problem of constructing a bounded arithmetic theory for the class LOGDCFL. We use the proof tree size characterization of the class and introduce a function algebra and several two-sort systems. As a corollary, we show that the witnessing problem of LOGDCFL certificate is feasible within the class, which can be seen as an analogue of the result by Gottlob et.al. for LOGCFL.

Kenshi Miyabe

Degree of non-randomness and uniform Solovay reducibility

Abstract: Each characterization of Martin-Löf randomness intuitively says that a sequence is ML-random iff a degree of non-randomness in it is finite. We introduce a degree of non-randomness using a test concept. The degree implies a notion of an optimal ML-test, which is different from a universal test. It is known that the measure of each component of a universal test is Solovay complete. The sequence of measures of an optimal ML-test is very random in the following sense. Solovay reducibility is a measure of relative randomness between two reals. We introduce uniform Solovay reducibility, which is a measure of relative randomness between two sequences of reals. Finally we prove that a sequence is uniform Solovay complete iff it is the sequence of measures of an optimal ML-test. The proof is much more simple than that of a universal test. So optimality is a more natural notion than universality.

Shu Iwanami

On systems between LK and LJ

Abstract: In Classical sequent calculus LK, any number of formulas is allowed to appear on the right-hand side. In contrast, in Intuitionistic sequent calculus LJ, the number of formulas in all rules on this side is restricted to one. We study sequent systems such that some of the rules are restricted.

Taku Sakakibara

Tutte's Theorem in Reverse Mathematics

Abstract: We study the proof-theoretic strength of theorems on graph theory. In particular, we consider a perfect matching, that is, a matching which includes all vertices on a given graph. We prove that Tutte's theorem for countable graph (A graph G has a perfect matching if and only if for any matching M and for any vertex $v \in V(M) \setminus V$ there exists an M-augmenting path starting at v) is equivalent to ACA_0.

Keisuke Yoshii

Game of determinacy in Second Order Arithmetic II

Abstract: The axiom of Σ^1_1 inductive definition, denoted by Σ^1_1-ID_0, was first formalized by K.Tanaka in second order arithmetic to capture the logical strength of Σ^0_2 determinacy. After that, M.Medsalem, K.Mashiko, K.Tanaka and others have considered stronger axioms of inductive definition. We here present a new axiom $[\Sigma^1_1]^2$-IDTR_0 and prove that it is equivalent to $\Delta((\Sigma^0_2)_3)$ determinacy.

Shohei Izawa

Reverse Mathematics in Finite type Arithmetic

Abstract: We consider, from the view point of reverse mathematics, a framework that approach abstract mathematics in finite type arithmetic. We prove some of proof theoretic strength relations between axioms that be seems important in the framework. We also introduce some results in reverse mathematics.

Yasushi Sangu

Set-generated classes in Constructive Set Theory CZF

Abstract: A formal system the constructive Zermelo-Fraenkel set theory **CZF**, was founded by Peter Aczel. **CZF** does not have the Power Set Axiom and therefore the notion of set-generated class was introduced in **CZF**. In this talk, We introduce a set-generated class, the class of Σ-closed sets, and its applications.