法令文の中の不整合の検出

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1. 否定($\neg$)の不整合
2. or($\vee$)の不整合
3. 含意($\rightarrow$)の不整合

Conceptual conflict

cf. Hagiwara (JURIX '06)

$$guilty \wedge \neg guilty \vdash \perp$$
$$guilty \wedge innocent \vdash \perp$$
$$possible \wedge impossible \vdash \perp$$
$$human \wedge car \vdash \perp$$

Assumptive facts

$$P_1 \leftarrow Q_1, Q_2, Q_3.$$
$$\neg P_1 \leftarrow Q_2, Q_3, Q_4.$$

$$P_2 \leftarrow Q_1, Q_2, Q_3.$$
$$\neg P_2 \leftarrow Q_1, Q_2.$$

$$P_3 \leftarrow Q_1, Q_2, \neg Q_3.$$
$$\neg P_3 \leftarrow Q_1, Q_2, Q_3.$$

$$\forall x P_4 \leftarrow Q_1, Q_2, Q_3(x).$$
$$\neg P_4 \leftarrow Q_1, Q_2, Q_3(a).$$

Condito Sine Qua Non

"A caused B" if "if A had not happened, B would not have happened." (J. Glaser, 1858)

1. Lethal dose is 100mg.
2. A put 60mg.
3. B put 60mg.
4. If A had not put 60mg, C would not have died.
5. A is culpable for the death of C.

Condito Sine Qua Non – cont'd

1. Lethal dose is 100mg.
2. A put 120mg.
3. B put 120mg.
4. Even though A had not put 120mg, C would have died.
5. A is <u>not</u> culpable for the death of C??

Occam's razor

The more reasons are employed, the less plausible the result becomes.

$$\Downarrow$$

We need to find the minimal explanation. (Economy of reasoning)

Which is the minimal explation?

- A_{120mg}
- B_{120mg}
- $A_{120mg} \vee B_{120mg}$
- A_{120mg} or B_{120mg}

Minimal explanation

- A_{120mg} **implies** $A_{120mg} \vee B_{120mg}$.

Minimal explanation

- A_{120mg} implies $A_{120mg} \vee B_{120mg}$.
- If A_{120mg} caused C_{died}, then $A_{120mg} \vee B_{120mg}$ caused C_{died} ?

Minimal explanation

- A_{120mg} implies $A_{120mg} \vee B_{120mg}$.
- If A_{120mg} caused C_{died}, then $A_{120mg} \vee B_{120mg}$ caused C_{died} ?
- If yes, then $A_{120mg} \vee B_{120mg} \vee D_{120mg} \vee E_{120mg}$ caused C_{died}. The cause is obviously too weakened.

Formalization in Abduction

B **Background theory**

C **Set of facts**

O **Observation**

$$B \cup C \models O$$

In our case,

B **Known rules**

C **Possible causes**

O **Result**

$$\{A_{120mg} \supset C_{died}\} \cup \{A_{120mg}\} \models \{C_{died}\}$$

(i) C.S.Q.N. by Belief Revision

$T * P$: revision of T by P, is the set of maximal consistent subsets of $T \cup P$ including P.

Ex.

$\{\alpha \supset \beta, \alpha\} * \{\neg\beta\}$ is either $\{\alpha \supset \beta, \neg\beta\}$ or $\{\alpha, \neg\beta\}$

First approximation: for any S in $B \cup \{C * \{\neg A\}\}$, $S \not\models O$ (unless A, not O), then A is a cause of O.

$$\Downarrow$$

In order to entrench B, we revise the above as: for any S in $C * \{B \cup \{\neg A\}\}$, $S \not\models O$, then A is a cause of O. A is a critical cause if there is no A' such that $A' \models A$.

Example 1

$$\begin{cases} B = \{A_{120} \supset C_{died},\ B_{120} \supset C_{died}\} \\ C_1 = \{A_{120},\ B_{120}\} \\ O = C_{died} \end{cases}$$

A_{120} is <u>not</u> a cause.

$$\begin{cases} B \cup C_1 \models C_{died} \text{ and} \\ C_1 * (B \cup \{\neg A_{120}\}) \ni B \cup \{\neg A_{120},\ B_{120}\} \\ \text{entails } C_{died}. \end{cases}$$

$A_{120} \vee B_{120}$ is a <u>cause</u>.

$$\begin{cases} B \cup C_1 \models C_{died} \text{ and} \\ C_1 * (B \cup \{\neg(A_{120} \vee B_{120})\}) \ni \\ \qquad B \cup \{\neg A_{120} \wedge \neg B_{120}\} \\ \text{does } \underline{\text{not}} \text{ entail } C_{died}. \end{cases}$$

Furthermore, $A_{120} \vee B_{120}$ is a <u>critical cause</u>.

Example 2

$$\begin{cases} B = \{A_{120} \supset C_{died},\ B_{120} \supset C_{died}\} \\ C_2 = \{A_{120} \vee B_{120}\} \\ O = C_{died} \end{cases}$$

A_{120} is <u>not</u> a cause.

$$\begin{cases} B \cup C_2 \models C_{died} \text{ and} \\ C_2 * (B \cup \{\neg A_{120}\}) \ni B \cup \{\neg A_{120}, A_{120} \vee B_{120}\} \\ \text{entails } C_{died}. \end{cases}$$

$A_{120} \vee B_{120}$ is a <u>cause</u>.

$$\begin{cases} B \cup C_2 \models C_{died} \text{ and} \\ C_2 * (B \cup \{\neg(A_{120} \vee B_{120})\}) \ni \\ \qquad B \cup \{\neg A_{120} \wedge \neg B_{120}\} \\ \text{does } \underline{\text{not}} \text{ entail } C_{died}. \end{cases}$$

Furthermore, $A_{120} \vee B_{120}$ is a <u>critical cause</u>.

Problems of C.S.Q.N.

Example 1 shows:

- A_{120} is not a cause though A should be blamed.
- $A_{120} \vee B_{120}$ is a cause.
- $A_{120} \vee B_{120}$ is a critical cause.

Exmaple 2 shows:

- A_{120} is not a cause.
- $A_{120} \vee B_{120}$ is a cause.
- $A_{120} \vee B_{120}$ is a critical cause.
- But if A_{120} is a cause, $A_{120} \vee B_{120}$ is also a cause.

C.S.Q.N. does not satisfy Occam's razor.

(ii) Solution by Minimal Abduction

$$\begin{cases} B & \textbf{Background theory} \\ H & \textbf{Abducibles (a set of propositional formulae)} \\ O & \textbf{A propositional formula} \end{cases}$$

E $(\subseteq H)$ is an <u>explanation</u> iff

- $B \cup E \models O$ and $B \cup E \not\models \bot$.
- E is <u>minimal</u> if for any $E' \subset E$, $B \cup E' \not\models O$.

Example 1 – revisited –

When $H = \{A_{120},\ B_{120}\}$, the minimal explanations of O becomes $\{\{A_{120}\},\ \{B_{120}\}\}$. That is,

either A_{120} or B_{120} is the minimal cause.

In other words, there are two minimal explanations.

Example 2 – revisited –

When $H = \{A_{120} \vee B_{120}\}$, the minimal explanations of O becomes $\{\{A_{120} \vee B_{120}\}\}$. That is,

$A_{120} \vee B_{120}$ is the minimal cause.

Issue 1: Logical Implication and Causation

$$\begin{array}{ccccc} A_{120} \wedge B_{120} & \xrightarrow{implies} & A_{120} & \xrightarrow{implies} & A_{120} \vee B_{120} \\ causes \downarrow & & causes \downarrow & & causes \downarrow \\ C & & C & & C^* \end{array}$$

$(*\ but\ not\ minimal)$

In our case,

$$A_{120} \supset_{implies} (A_{120} \vee B_{120}) \supset_{causes} C_{died},$$

But

$$A_{120} \not\supset C_{died}.$$

cf. Deduction theorem: $\alpha \vdash \beta \iff \vdash (\alpha \supset \beta)$.

Issue 2: Scope of a predicate

(A or B) put 120mg

vs.

(A put 120mg) or (B put 120mg)

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$$K_A(\alpha \vee \beta) \not\supset K_A\alpha \vee K_B\beta$$

Issue 2: Scope of a predicate

(A or B) put 120mg

vs.

(A put 120mg) or (B put 120mg)

$A \not\supset A \vee B$ **(substructural logic)**

Summary

- Formalization of C.S.Q.N. to clarify its paradox.
- Minimal explanation, to distinguish between disjunction of causes and a disjunctive cause.

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