Belief Consolidation for Description Logic

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The Aim Of The Research

Our motivation is based on the study about the check on consistency in municipal bylaw enacted by Toyama Prefecture in Japan (Hagiwara & Tojo 2006). Instead of corporeal documents, the prefecture office permitted the resident to submit the various kinds of application electrically in 2002. However, this new bylaw might be inconsistent with the old administrative procedure act, since digital documents are not considered as corporeal. Thus, there is a problem: According to the new act for administrative procedure with telecommunication, can we submit the digital documents instead of the corporeal documents?

In (Hagiwara & Tojo 2006), the method for automatic discordance detection was shown. In the study, inconsistency does not only arise from the logical one (A and $\neg A$), but also antonyms ('liquid' and 'solid', 'vice' and 'virtue', etc.). Whether two concepts are antonyms or not is decided by an ontology. However, the experimental result showed that there was no conflicting concepts except for several loops. That is, although we introduce the ontology, we can find few discrepancies in the law. We consider that the cause is not only due to a lack of the valid legal codes for detecting inconsistency, but also the poverty of the ontology. As shown in the study of LRI Core, predominant common-sense characters should be presupposed by all legal domain ontologies, but such a core ontology was not assumed in the domain ontology for the municipal bylaw. Therefore, we come to rewrite the ontology with some generic ontology like LRI Core.

Our main problem is whether there is inconsistency in the ontology or not. For example, we often consider that the following conditional sentences are valid.

example 1

1. *If a is a document, then a is corporeal.*
2. *If a is a digital document, then a is a document.*
3. *If a is a digital document, then a is not corporeal.*

When we suppose there is a digital document, we can conclude from 1 and 2 that it is corporeal, but we can also conclude from 3 that it is not corporeal. When that such sentences are included in our ontology, we want the ontology to be consistent. How do we solve the problem?

The Approach, Idea

To resolve the inconsistency, we use the idea of channel theoretic approach to reasoning with generics by (Cavedon 1995). Channel theory supposed by (Barwise & Seligman 1997) already has been applied to the research area of ontology, e.g., IF-Map is known as the study for ontology mapping with the channel theory (Kalfoglou & Schorlemmer 2003). Our purpose is to show that this approach is useful to solve the inconsistency in an ontology.

According to Cavedon (Cavedon 1995), the above problem arose from the lack of explicit background context. 1 is regarded as the following implicit background conditional sentence 1'.

1' If a is not a digital document, but a document, then a is corporeal.

However, we may doubt why we are able to assume such an implicit background condition. In Cavedon (Cavedon 1995), it is declared that *all background conditions are implicitly contained in the given set of regularities.* It means that 1' can be assumed when we suppose the conditional 1 and the following conditional 3. The consequents of 1 and 3 are obviously inconsistent. Thus, the negation of the antecedent of 3 , i.e., "a is not digital document", should be added to the antecedent of 1 for the exceptional case. Therefore, 1' is assumed from 1 and 3.

Note that this idea may not resolve inconsistency sufficiently. Suppose the following sentence.

4 If a is a document, then a is not corporeal.

The consequences of 1 and 4 are obviously inconsistent each other, but the negation of the antecedent of 4 cannot be added to the antecedent of 1, since the antecedent of 4 is same as that of 1. (Cavedon 1995) introduced Belnap's relevant logic (a paraconsistent logic), and hence, such an inconsistency did not have any trouble. However, we do not use any paraconsistent logic, because our ontologies are applied to the check on the inconsistency of legal rules written by XML files, and ontologies themselves should not include any inconsistency. So, in such a case, we will only eliminate the sentence 4, and resolve the inconsistency.

Progress of 2007

In (Suzuki 2007), we assumed that ontogy was divided into two types. One is for the application, while the other is for generic concept. The former is called a *domain ontology*, and the latter is called a *core ontology*. For example, LRI Core is famous for the legal core ontology. When we construct new ontology for an exceptional domain supported by some core ontology (e.g. LRI Core), we may encounter an inconsistency between the two ontologies unless revising the ontologies. To solve the problem, we used a channel theoretic approach to ontology (Kalfoglou & Schorlemmer 2003), and showed the procedure for the construction of some consistent background ontology, via which information flowed from the domain ontology to the core ontology, with an example for municipal bylaw constituted by Toyama Prefecture in Japan.

However, we did not show that there is some mathmatical postulates like the study of belief revision (Alchourrón, Gärdenfors, & Makinson 1985). Among other things, whether our operation satisfies the principle of minimal change (i.e., we should change the ontology as minimal as possible.) or not is our future subject.

Future direction

Instead of channel theory, we use the *description logic* (DL) and study the minimal change of an ontology, since our ontology for the municipal bylaw is described by OWL, a markup language for the semantic web (Horrocks *et al.* 2007), and DL is used for the undergrounding logic of OWL.

In the following discussion, our logic is assumed to be based on an extention of the famous DL $\mathcal{ALC}$. Therefore, our discussion can be applied to $\mathcal{SHIF}$ and $\mathcal{SHOIN}$, where $\mathcal{SHIF}$ and $\mathcal{SHOIN}$ are adopted by OWL DL and OWL Lite respectively.

Let *concepts* be formed by the following rule, where A is an *atomic concept* and R is a *role*:

$$C, D := A|\top|\bot|C \sqcap D|\neg C|\forall R.C$$

$C \sqcup D$ and $\exists R.C$ are abbreviations of $\neg(\neg C \sqcap \neg D)$ and $\neg\forall R.\neg C$ respectively. Sets of concepts are denoted by $\mathbb{A}$, $\mathbb{B}$, and so on. An *interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of a set $\Delta^{\mathcal{I}}$ (called *domain*) and a function $\cdot^{\mathcal{I}}$, which assigns to every atomic concept A a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ and to every role a binary relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. The function $\cdot^{\mathcal{I}}$ can be extended to concepts by the following inductive definitions:

- $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$.
- $\bot^{\mathcal{I}} = \emptyset$.
- $(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \backslash C^{\mathcal{I}}$.
- $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$.
- $(\forall R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} | \forall b.(a, b) \in R^{\mathcal{I}} \to b \in C^{\mathcal{I}}\}$.

A concept C is *satisfiable* iff there is an interpretation $\mathcal{I}$ such that $C^{\mathcal{I}} \neq \emptyset$. Such an interpretation is called a *model of* C. A concept D *subsumes* a concept C iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$. An *inclusion axiom* has the form $C \sqsubseteq D$, where C and D are concepts. $\mathbb{A} \sqsubseteq \mathbb{B}$ is the abbreviation of $\sqcap_{C \in \mathbb{A}} C \sqsubseteq \sqcup_{D \in \mathbb{B}} D$. Thus, $C \sqsubseteq D$ is considered as $\{C\} \sqsubseteq \{D\}$ A *terminology* is a finite set of inclusion axioms $\mathcal{T} = \{C_1 \sqsubseteq D_1, ..., C_n \sqsubseteq D_n\}$. A *well-formed terminology* is a terminology such that for any $C_i \sqsubseteq D_i, C_j \sqsubseteq D_j \in \mathcal{T}$, if $i \neq j$, then $C_i \neq C_j$. $\mathcal{T}$ is *satisfiable* iff there is an interpretation $\mathcal{I}$ such that $C_i^{\mathcal{I}} \subseteq D_i^{\mathcal{I}}$ for all $C_i \sqsubseteq D_i \in \mathcal{T}$. $\mathcal{T}$ is *consistent* iff there is an interpretation $\mathcal{I}$ such that $C_i^{\mathcal{I}} \subseteq D_i^{\mathcal{I}}$ and $C_i^{\mathcal{I}} \neq \emptyset$ for all $C_i \subseteq D_i \in \mathcal{T}$. Such an interpretation is called a *model of* $\mathcal{T}$. Every consistent terminology can be transformed into a consistent well-formed terminology, since $\mathcal{I}$ is a model of $\{C \sqsubset D_1, C \sqsubset D_2\}$ iff $\mathcal{I}$ is a model of $\{C \sqsubset D_1 \sqcap D_2\}$. Therefore, when we use the word 'terminology' in the following discussion, it means 'well-formed terminology'.

In the above definition, the differece of satisfiability and inconsistency is important. For example, $\mathcal{T} = \{penguin \sqsubseteq fly, penguin \sqsubseteq \neg fly\}$. is satisfiable, but not consistent. We consider that when there are some rules such that any instance does not satisfy antecedents of these rules, then these rules are nonsense. Thus, we revise Cavedon's method, because when a is known to be a document, but not known to be not a digital document, a satisfies the antecedent of 1, but does not satisfy the antecedent of 1'. However, the antecedent of the following rule is satisfied by a.

1" If a is a document, then a is corporeal or a digital document.

That is to say, instead of the addition of the negation of the antecedent of 3 to the antecedent of 1, we will add the antecedent of 3 to the consequent of 1. Our method is justified by the fact that $document \sqcap \neg digital\_document \sqsubseteq corporeal$ is equivalent with $document \sqsubseteq corporeal \sqcup digital\_document$.

This method has a week point. For example, according to method, we must add the antecedent of 4 to the consequent of 1. Thus, we will acquire the following rule.

- If a is a document, then a is corporeal or a document.

However, this rule is a tautology. Therefore, we consider that a rule $A \sqsubseteq B$ such that $A \sqcap \neg B$ is unsatisfiable should be eliminated. We call such an inclusion axiom and a terminology that has the axiom *trivial*. In the following discussion, we will concetrate on nontrivial terminologies.

Thus, a variation of belief consolidation will be supposed. Belief consolidation is an operation that reject some sentences from an agent's knowledge base as minimal as possible, and make the knowledge base consistent (Hansson 1994). However, our belief consolidation of DL terminology does not only reject some inclusion axioms from a terminology, but also rewrite some consequents of the inclusion axioms in the terminology with the above method.

Before we formalize the consolidation, some of notations are introduced. The set of rewrited rule of $\mathbb{C} \sqsubseteq \mathbb{D}$ w.r.t. a terminology $\mathcal{T}$ is denoted by $\pi_{\mathcal{T}}(\mathbb{C} \sqsubseteq \mathbb{D})$ and defined as follows:

$$\mathbb{A} \sqsubseteq \mathbb{B} \in \pi_{\mathcal{T}}(\mathbb{C} \sqsubseteq \mathbb{D}) \text{ iff } \mathbb{A} = \mathbb{C} \text{ and } \mathbb{B} = \mathbb{D} \cup Ant.$$

where Ant is a subset of $\{C | C \sqsubseteq D \in \mathcal{T}\}$ and $\mathbb{A} \sqsubseteq \mathbb{B}$ must be nontrivial. We define the *terminological inclusion* $\Subset_{\mathcal{T}}$ w.r.t. a terminology $\mathcal{T}$ as follows.

$$\mathcal{S} \Subset_{\mathcal{T}} \mathcal{U} \text{ iff for any } \mathbb{A} \sqsubseteq \mathbb{B} \in \mathcal{S},$$
$$\text{for some } \mathbb{C} \sqsubseteq \mathbb{D} \in \mathcal{U}, \mathbb{A} \sqsubseteq \mathbb{B} \in \pi_{\mathcal{T}}(\mathbb{C} \sqsubseteq \mathbb{D}).$$

That is, $\mathcal{S}$ includes $\mathcal{U}$ iff for any rule in $\mathcal{S}$, there is some rule in $\mathcal{U}$ such that the cosequent consists of the consequent of the rule in $\mathcal{S}$ and the antecedents of rules in $\mathcal{T}$. Note that a model of $\mathcal{S}$ must be a model of $\mathcal{U}$, but a model of $\mathcal{U}$ may not be a model of $\mathcal{S}$. Besides, $\Subset_{\mathcal{T}}$ satisfies reflexivity, antisymmetricity and transitivity. In the following discussion, when $\mathcal{T}$ is obvious from the context, we utilize $\Subset$ instead of $\Subset_{\mathcal{T}}$.

Similarily, we define the *terminological intersection* $\Cap$ as follows.

$$\mathbb{A} \sqsubseteq \mathbb{B} \cup \mathbb{C} \in \mathcal{S} \Cap \mathcal{U} \text{ iff } \mathbb{A} \sqsubseteq \mathbb{B} \in \mathcal{S}$$

$$\text{and } \mathbb{A} \sqsubseteq \mathbb{C} \in \mathcal{U} \text{ and } \mathbb{A} \sqsubseteq \mathbb{B} \cup \mathbb{C} \text{ is nontrivial.}$$

Note that if $\mathcal{S} \Subset \mathcal{T}$ and $\mathcal{U} \Subset \mathcal{T}$, then $\mathcal{S} \Cap \mathcal{U} \Subset \mathcal{S}$ and $\mathcal{S} \Cap \mathcal{U} \Subset \mathcal{U}$.

Now we define the set $\Downarrow \mathcal{T}$ of terminological maximal consistent subsets of $\mathcal{T}$ like the study of belief revision. $\mathcal{S} \in \Downarrow \mathcal{T}$ iff

1. $\mathcal{S} \Subset \mathcal{T}$,
2. $\mathcal{S}$ is consistent, and
3. If $\mathcal{S} \Subset \mathcal{U} \Subset \mathcal{T}$ and $\mathcal{U} \neq \mathcal{S}$, then $\mathcal{U}$ is not consistent.

That is, $\mathcal{S} \in \Downarrow \mathcal{T}$ iff $\mathcal{S}$ is a consistent terminology rewrited from $\mathcal{T}$ as minimal as possible. Therefore, $\Downarrow \mathcal{T}$ is considered as the set of the candidates for the output of the consolidation. However, there may be plural terminologies in $\mathcal{S} \in \Downarrow \mathcal{T}$. In order to decide the result of the consolidation, we will select some candidates from $\Downarrow \mathcal{T}$, i.e., we introduce *selection function* γ that chooses some best terminologies from $\Downarrow \mathcal{T}$. If $\Downarrow \mathcal{T}$ is nonempty, $\gamma(\Downarrow \mathcal{T})$ is equal to a nonempty subset of $\Downarrow \mathcal{T}$. Otherwise, $\gamma(\Downarrow \mathcal{T}) = \{\mathcal{T}\}$.

Now we define the *partial meet consolidation* for terminologies. Con is a partial meet consolidation for terminologies iff there is a selection function γ such that for any terminology $\mathcal{T}$,

$$Con(\mathcal{T}) = \Cap \gamma(\Downarrow \mathcal{T}).$$

What are the properties of the consolidation function? We introduce the postulates for the consolidation as follows.

- Con 1. $Con(\mathcal{T}) \Subset \mathcal{T}$.
- Con 2. $Con(\mathcal{T})$ is consistent.
- Con 3. If $Con(\mathcal{T}) = \mathcal{T}$, then $\mathcal{T}$ is consistent.
- Con 4. For any $\mathbb{A} \sqsubseteq \mathbb{B} \in \mathcal{T}$ and $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$ if $\mathbb{C} \sqsubseteq \mathbb{D} \in \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$ and $C \in \mathbb{D} \backslash \mathbb{B}$, then there is some $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$ such that
  - $\mathcal{T}'$ is consistent,
  - $\mathbb{X} \sqsubseteq \mathbb{Y} \in \mathcal{T}'$ for some $\mathbb{X} \sqsubseteq \mathbb{Y} \in \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$, and
  - $\mathcal{T}''$ is not consistent for any $\mathcal{T}''$ with $(\mathcal{T}' \backslash \{\mathbb{X} \sqsubseteq \mathbb{Y}\}) \cup \{\mathbb{X} \sqsubseteq (\mathbb{Y} \backslash \{C\})\} \Subset \mathcal{T}'' \Subset \mathcal{T}$.
- Con 5. For any $\mathbb{A} \sqsubseteq \mathbb{B} \in \mathcal{T}$, if there is no $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$ such that $\mathbb{C} \sqsubseteq \mathbb{D} \in \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$, then for any $\mathbb{C} \sqsubseteq \mathbb{D} \in \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$,
  - there is some consistent $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$ such that for any $\mathcal{T}''$ with $\mathcal{T}' \Subset \mathcal{T}'' \Subset \mathcal{T}$ and $\mathcal{T}' \neq \mathcal{T}''$, $\mathcal{T}''$ is not consistent and, for some $\mathbb{X} \sqsubseteq \mathbb{Y} \in \mathcal{T}'$, $\mathbb{X} \sqsubseteq \mathbb{Y} \in \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$ and $\mathbb{X} \sqsubseteq \mathbb{Y} \cup \mathbb{D}$ is trivial, or
  - there is some consistent $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$ such that for any $\mathcal{T}''$ with $\mathcal{T}' \cup \{\mathbb{C} \sqsubseteq \mathbb{D}\} \Subset \mathcal{T}'' \Subset \mathcal{T}$, $\mathcal{T}''$ is not consistent.

Con 1 means that the consolidated terminology $Con(\mathcal{T})$ should not include an inclusion axiom, which is not rewrited from $\mathcal{T}$. From Con 2, the consolidated terminology $Con(\mathcal{T})$ must be consistent. When $Con(\mathcal{T})$ is not changed from $\mathcal{T}$, $\mathcal{T}$ already has been consistent (Con 3). Con 4 and Con 5 are the principles of minimal change. When an antecedent of a rule in $\mathcal{T}$ is added to a cosequent of a rule in $Con(\mathcal{T})$, there is some terminological superset of $Con(\mathcal{T})$ such that if the antecedent is deleted from the consequent, this superset becomes inconsistent (Con 4). When any rewrited rule from $\mathcal{T}$ is not in $Con(\mathcal{T})$, a trivial rule is generated from the rewrited rule or this rewrited rule is the cause of the inconsistency (Con5).

Now we can show the following theorem.

theorem 1 *Con is a partial meet consolidation iff Con satisfies the postulate Con 1 - Con 5.*

Proof. **Only-if Part.** Con 1 and Con 2 are obvious from the definition of $\Downarrow \mathcal{T}$. For Con 3, assume that $Con(\mathcal{T}) = \mathcal{T}$. That is, $\mathcal{T} = \Cap \gamma(\Downarrow \mathcal{T})$. Since $\emptyset$ is a consistent terminology, $\Downarrow \mathcal{T}$ is nonempty. Therefore, $\mathcal{T}$ is the intersection of consistent terminologies. Since the intersection of cosistent terminologies is also consistent, $\mathcal{T}$ is consistent.

For Con 4, assume that $\mathbb{A} \sqsubseteq \mathbb{B} \in \mathcal{T}$, $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$, $\mathbb{C} \sqsubseteq \mathbb{D} \in \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$ and $C \in \mathbb{D} \backslash \mathbb{B}$. Since $\mathbb{C} \sqsubseteq \mathbb{D} \in \gamma(\Downarrow \mathcal{T})$, there is some $\mathcal{T}' \in \Downarrow \mathcal{T}$ such that for some $\mathbb{X} \sqsubseteq \mathbb{Y} \in \mathcal{T}'$, $\mathbb{C} \sqsubseteq \mathbb{D} \in \pi_{\mathcal{T}}(\mathbb{X} \sqsubseteq \mathbb{Y})$ and $C \in \mathbb{Y}$. Obviously, $\mathcal{T}'$ is consistent. Since $\mathcal{T}' \Subset (\mathcal{T}' \backslash \{\mathbb{X} \sqsubseteq \mathbb{Y}\}) \cup \{\mathbb{X} \sqsubseteq (\mathbb{Y} \backslash C)\}$ and $\mathcal{T}' \neq (\mathcal{T}' \backslash \{\mathbb{X} \sqsubseteq \mathbb{Y}\}) \cup \{\mathbb{X} \sqsubseteq (\mathbb{Y} \backslash C)\}$, $\mathcal{T}''$ is not consistent for any $\mathcal{T}''$ with $(\mathcal{T}' \backslash \{\mathbb{X} \sqsubseteq \mathbb{Y}\}) \cup \{\mathbb{X} \sqsubseteq (\mathbb{Y} \backslash C) \Subset \mathcal{T}'' \Subset \mathcal{T}$. Con 4 is shown.

For Con 5, assume that $\mathbb{A} \sqsubseteq \mathbb{B} \in \mathcal{T}$ and there is no $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$ such that $\mathbb{C} \sqsubseteq \mathbb{D} \in \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$. Suppose that $\mathbb{C} \sqsubseteq \mathbb{D} \in \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$, where there is no consistent $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$ such that for any $\mathcal{T}''$ with $\mathcal{T}' \sqsubseteq \mathbb{D}\} \Subset \mathcal{T}'' \Subset \mathcal{T}$ and $\mathcal{T}' \neq \mathcal{T}''$, $\mathcal{T}''$ is not consistent and, for some $\mathbb{X} \sqsubseteq \mathbb{Y} \in \mathcal{T}'$, $\mathbb{X} \sqsubseteq \mathbb{Y} \in \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$ and $\mathbb{X} \sqsubseteq \mathbb{Y} \cup \mathbb{D}$ is trivial. We must show that there is some consistent $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$ such that for any $\mathcal{T}''$ with $\mathcal{T}' \cup \{\mathbb{C} \sqsubseteq \mathbb{D}\} \Subset \mathcal{T}'' \Subset \mathcal{T}''$, $\mathcal{T}''$ is not consistent. For cotradiction, assume that there is no a consistent $\mathcal{T}'$. Then, for any $\mathcal{T}' \in \gamma(\Downarrow \mathcal{T})$, $\mathcal{T}' \cup \{\mathbb{C} \sqsubseteq \mathbb{D}\}$ is consistent. Thus, for any $\mathcal{T}' \in \gamma(\Downarrow \mathcal{T})$, there is some $\mathbb{X} \sqsubseteq \mathbb{Y} \in \mathcal{T}'$ such that $\mathbb{C} \sqsubseteq \mathbb{D} \in \pi_{\mathcal{T}}(\mathbb{X} \sqsubseteq \mathbb{Y})$. Thus, there is some $\mathbb{X} \sqsubseteq \mathbb{Y} \in Con(\mathcal{T})$ such that $\mathbb{C} \sqsubseteq \mathbb{D} \in \pi_{\mathcal{T}}(\mathbb{X} \sqsubseteq \mathbb{Y})$. Obviously, $\mathbb{X} \sqsubseteq \mathbb{Y} \in \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$. However, it contradicts that there is no $\mathbb{X} \sqsubseteq \mathbb{Y} \in Con(\mathcal{T})$ such that $\mathbb{X} \sqsubseteq \mathbb{Y} \in \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$. Therefore, there is some consistent $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$ such that for any $\mathcal{T}''$ with $\mathcal{T}' \cup \{\mathbb{C} \sqsubseteq \mathbb{D}\} \Subset \mathcal{T}'' \Subset \mathcal{T}$, $\mathcal{T}''$ is not consistent.

If Part. We define the selection function γ as follows.

$$\gamma(\Downarrow \mathcal{T}) = \begin{array}{ll} \{\mathcal{T}' \in \Downarrow \mathcal{T} | \mathcal{T} \Subset \mathcal{T}'\} & \text{if } \Downarrow \mathcal{T} \text{ is nonempty.} \\ \{\mathcal{T}\} & \text{otherwise.} \end{array}$$

At first, we will prove that if $\Downarrow \mathcal{T}$ is nonempty, then $\gamma(\Downarrow \mathcal{T})$ is a nonempty subset of $\Downarrow \mathcal{T}$. Suppose that $\Downarrow \mathcal{T}$ is nonempty.

From the definition, $\gamma(\Downarrow \mathcal{T})$ is a subset of $\Downarrow \mathcal{T}$. In order to show that $\gamma(\Downarrow \mathcal{T})$ is nonempty, we will prove that there is some $\mathcal{T}' \in \Downarrow \mathcal{T}$ such that $\mathcal{T} \Subset \mathcal{T}'$.

In the case of $\mathcal{T} = Con(\mathcal{T})$, $\mathcal{T}$ is consistent by Con 3. From the definition of $\Downarrow \mathcal{T}$, $\mathcal{T} \in \Downarrow \mathcal{T}$. Since $\mathcal{T} \Subset \mathcal{T}$, the proof is shown.

Suppose that $\mathcal{T} \neq Con(\mathcal{T})$. By Con 1, there is some $\mathbb{C} \sqsubseteq \mathbb{D} \in \mathcal{T}$ such that $\mathbb{C} \sqsubseteq \mathbb{D} \notin \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$ for any $\mathbb{A} \sqsubseteq \mathbb{B} \in Con(\mathcal{T})$. When there is some $\mathbb{A} \sqsubseteq \mathbb{B} \in Con(\mathcal{T})$ such that $\mathbb{A} \sqsubseteq \mathbb{B} \in \pi_{\mathcal{T}}(\mathbb{C} \sqsubseteq \mathbb{D})$, we can conclude that there is some $C \in \mathbb{D} \backslash \mathbb{B}$, and by Con 4, there is some cosistent $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$, i.e., there is some maximal cosistent $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$. Suppose that there is no $\mathbb{A} \sqsubseteq \mathbb{B} \in Con(\mathcal{T})$ such that $\mathbb{A} \sqsubseteq \mathbb{B} \in \pi_{\mathcal{T}}(\mathbb{C} \sqsubseteq \mathbb{D})$. By Con 5, there is some cosistent $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$, i.e., there is some maximal cosistent $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$. Thus, there is some $\mathcal{T}' \in \Downarrow \mathcal{T}$ such that $\mathcal{T} \Subset \mathcal{T}'$.

Next, we will show $Con(\mathcal{T}) = \Cap\gamma(\Downarrow \mathcal{T})$. From the definition of γ, $Con(\mathcal{T}) \Subset \Cap\gamma(\Downarrow \mathcal{T})$ is obvious. Therefore, it suffices to show $\Cap\gamma(\Downarrow \mathcal{T}) \Subset Con(\mathcal{T})$. Suppose that $\mathbb{A} \sqsubseteq \mathbb{B} \in \Cap\gamma(\Downarrow \mathcal{T})$. We want to show that there is some $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$ such that $\mathbb{A} \sqsubseteq \mathbb{B} \in \pi_{\mathcal{T}}(\mathbb{C} \sqsubseteq \mathbb{D})$.

For contradiction, suppose that there is no such a $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$. Then, for any $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$, $\mathbb{C} \neq \mathbb{A}$ or $\mathbb{D} \subseteq \mathbb{B}$, i.e., there is some $C \in \mathbb{D} \backslash \mathbb{B}$.

Suppose that for any $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$, $\mathbb{C} \neq \mathbb{A}$. From $\Cap\gamma(\Downarrow \mathcal{T}) \Subset \mathcal{T}$, there is some $\mathbb{X} \sqsubseteq \mathbb{Y} \in \mathcal{T}$ such that $\mathbb{A} \sqsubseteq \mathbb{B} \in \pi_{\mathcal{T}}(\mathbb{X} \sqsubseteq \mathbb{Y})$. However, for any $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$, $\mathbb{C} \sqsubseteq \mathbb{D} \notin \pi_{\mathcal{T}}(\mathbb{X} \sqsubseteq \mathbb{Y})$. By Con 5, for any $\mathbb{C} \sqsubseteq \mathbb{D} \in \pi_{\mathcal{T}}(\mathbb{X} \sqsubseteq \mathbb{Y})$, (i) there is some consistent $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$ such that for any $\mathcal{T}''$ with $\mathcal{T}' \sqsubseteq \mathbb{D}\} \Subset \mathcal{T}'' \Subset \mathcal{T}$ and $\mathcal{T}' \neq \mathcal{T}''$, $\mathcal{T}''$ is not consistent and, for some $\mathbb{E} \sqsubseteq \mathbb{F} \in \mathcal{T}'$, $\mathbb{E} \sqsubseteq \mathbb{F} \in \pi_{\mathcal{T}}(\mathbb{X} \sqsubseteq \mathbb{Y})$ and $\mathbb{E} \sqsubseteq \mathbb{F} \cup \mathbb{D}$ is trivial, or (ii) there is some consistent $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$ such that for any $\mathcal{T}''$ with $\mathcal{T}' \cup \{\mathbb{C} \sqsubseteq \mathbb{D}\} \Subset \mathcal{T}'' \Subset \mathcal{T}''$, $\mathcal{T}''$ is not consistent. However, (i) and (ii) are inconsistent with $\mathbb{A} \sqsubseteq \mathbb{B} \in \Cap\gamma(\Downarrow \mathcal{T})$. Thus, for some $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$, $\mathbb{C} = \mathbb{A}$.

Suppose that $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$ and $\mathbb{C} = \mathbb{A}$. Besides, suppose that there is some $C \in \mathbb{D} \backslash \mathbb{B}$. From Con 1, there is some $\mathbb{E} \sqsubseteq \mathbb{F} \in \mathcal{T}$ such that $\mathbb{C} \sqsubseteq \mathbb{D} \in \pi_{\mathcal{T}}(\mathbb{E} \sqsubseteq \mathbb{F})$. Since $\mathbb{A} \sqsubseteq \mathbb{B} \in \pi_{\mathcal{T}}(\mathbb{E} \sqsubseteq \mathbb{F})$, $C \in \mathbb{D} \backslash \mathbb{F}$. By Con 4, there is some $\mathcal{T}'$ with $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$ such that $\mathcal{T}'$ is consistent, $\mathbb{X} \sqsubseteq \mathbb{Y} \in \mathcal{T}'$ for some $\mathbb{X} \sqsubseteq \mathbb{Y} \in \pi_{\mathcal{T}}(\mathbb{A} \sqsubseteq \mathbb{B})$, and $\mathcal{T}''$ is not consistent for any $\mathcal{T}''$ with $(\mathcal{T}' \backslash \{\mathbb{X} \sqsubseteq \mathbb{Y}\}) \cup \{\mathbb{X} \sqsubseteq (\mathbb{Y} \backslash \{C\})\} \Subset \mathcal{T}'' \Subset \mathcal{T}$. Since $\mathcal{T}'$ is consistent and $Con(\mathcal{T}) \Subset \mathcal{T}' \Subset \mathcal{T}$, there is some $\mathcal{S} \in \Downarrow \mathcal{T}$ such that $\mathcal{T}' \Subset \mathcal{S}$. Then, for some $\mathbb{W} \sqsubseteq \mathbb{Z} \in \mathcal{S}$, $\mathbb{W} \sqsubseteq \mathbb{Z} \in \pi_{\mathcal{T}}(\mathbb{E} \sqsubseteq \mathbb{F})$. Since $\mathcal{S}$ is consistent, we can conclude that $(\mathcal{T}' \backslash \{\mathbb{X} \sqsubseteq \mathbb{Y}\}) \cup \{\mathbb{X} \sqsubseteq (\mathbb{Y} \backslash \{C\})\} \not\Subset \mathcal{S}$. From this conclusion and $\mathcal{T}' \Subset \mathcal{S}$, $C \in \mathbb{F}$. From the definition of $\Cap\gamma(\mathcal{T})$, $C \in \mathbb{B}$. It contradicts with $C \in \mathbb{D} \backslash \mathbb{B}$. Therefore, there is no $C \in \mathbb{D} \backslash \mathbb{B}$.

Thus, for some $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$, $\mathbb{C} = \mathbb{A}$ and there is no $C \in \mathbb{D} \backslash \mathbb{B}$. That is, there is some $\mathbb{C} \sqsubseteq \mathbb{D} \in Con(\mathcal{T})$ such that $\mathbb{A} \sqsubseteq \mathbb{B} \in \pi_{\mathcal{T}}(\mathbb{C} \sqsubseteq \mathbb{D})$. □

Our next step will be to show that there is an implementation of the partial meet consolidation. For example, we will introduce some total order $\leq$ over a terminology $\mathcal{T}$. Then, an equivalent relation $\sim$ is defined as follows.

$$\mathbb{A} \sqsubseteq \mathbb{B} \sim \mathbb{C} \sqsubseteq \mathbb{D} \text{ iff } \mathbb{A} \sqsubseteq \mathbb{B} \leq \mathbb{C} \sqsubseteq \mathbb{D} \text{ iff } \mathbb{C} \sqsubseteq \mathbb{D} \leq \mathbb{A} \sqsubseteq \mathbb{B}$$

The corresponding equivalent classes are denoted by $\overline{\mathbb{A} \sqsubseteq \mathbb{B}}$. The set of equivalent classes is denoted by $\overline{\mathcal{T}}$. That is, $\leq$ is considered as a total order over $\overline{\mathcal{T}}$. Now the *prioritized set of maximal consistent subsets* $\downarrow \mathcal{T}$ *of* $\mathcal{T}$ is defined such that $\mathcal{S} \in \downarrow \mathcal{T}$ iff

1. $\mathcal{S} = \bigcup_{\overline{\mathbb{A} \sqsubseteq \mathbb{B}} \in \overline{\mathcal{T}}} \mathcal{T}_{\overline{\mathbb{A} \sqsubseteq \mathbb{B}}}$
2. for all $\overline{\mathbb{A} \sqsubseteq \mathbb{B}} \in \overline{\mathcal{T}}$, $\mathcal{T}_{\overline{\mathbb{A} \sqsubseteq \mathbb{B}}} \Subset_{\mathcal{T}} \overline{\mathcal{T}}$, and
3. for all $\overline{\mathbb{A} \sqsubseteq \mathbb{B}} \in \overline{\mathcal{T}}$, $\mathcal{T}_{\overline{\mathbb{A} \sqsubseteq \mathbb{B}}}$ is maximal w.r.t. terminological inclusion among the subsets of $\overline{\mathbb{A} \sqsubseteq \mathbb{B}}$ such that $\bigcup_{\mathbb{C} \sqsubseteq \mathbb{D} \geq \mathbb{A} \sqsubseteq \mathbb{B}} \mathcal{T}_{\overline{\mathbb{C} \sqsubseteq \mathbb{D}}}$ is consistent.

Our future aim is to show that there is some selection function γ such that

$$\gamma(\Downarrow \mathcal{T}) = \downarrow \mathcal{T}.$$

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