Algorithms and Complexity for Puzzles

From classic to current & future issues in Theoretical Computer Science

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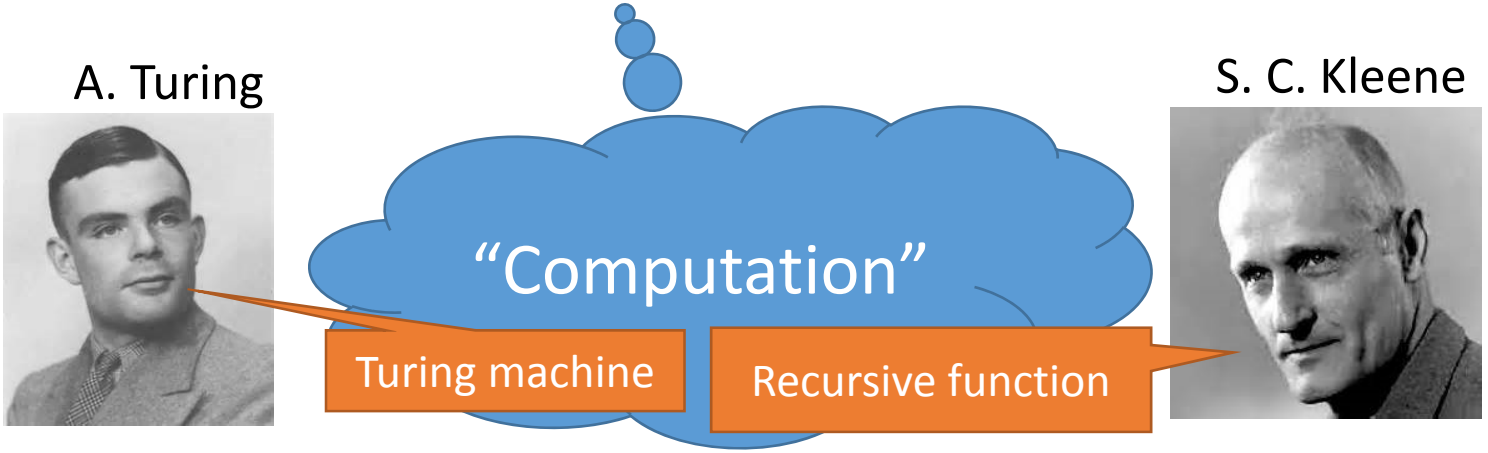
Computational Complexity v.s. Puzzles & Games

- What's "computation" could be… (1930s-1940s)



To consider "computation," what we need is

- Basic operations (=model of computation)
- How can we combine them (=algorithms)

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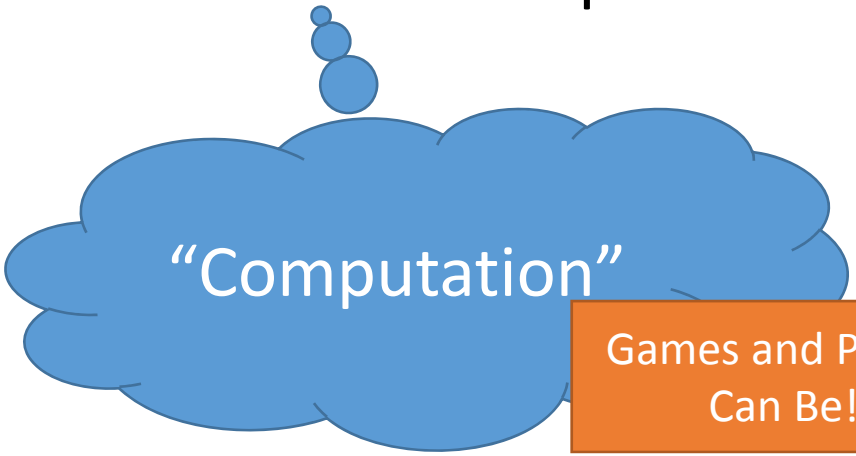
- What's "computation" could be… (1970s)



John Horton Conway

To consider "computation," what we need is

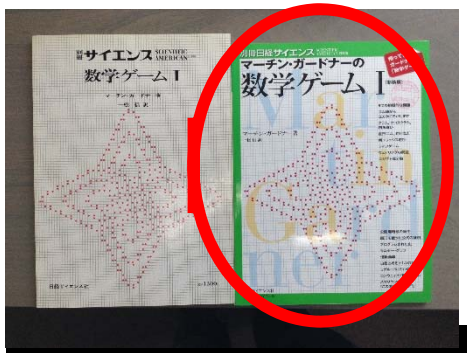
- Basic operations (=model of computation)
- How can we combine them (=algorithms)

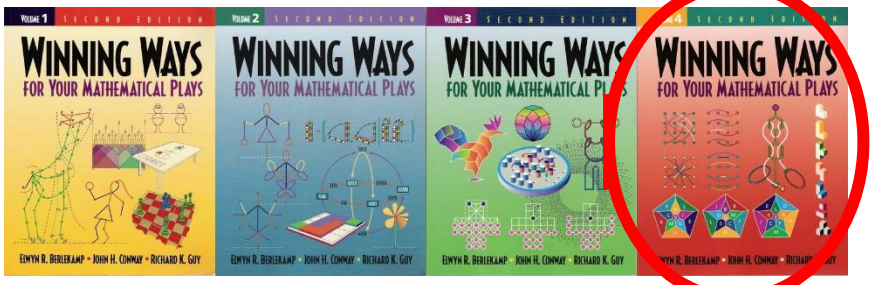
Computational Complexity v.s. Puzzles & Games

- What's "computation" could be… (1970s

  Conway's *Game of Life* (1970)

  - It is a kind of cellular automaton with quite simple rules.
  - It is "*Universal*", that is, it computes any function!
  - Some nice books:





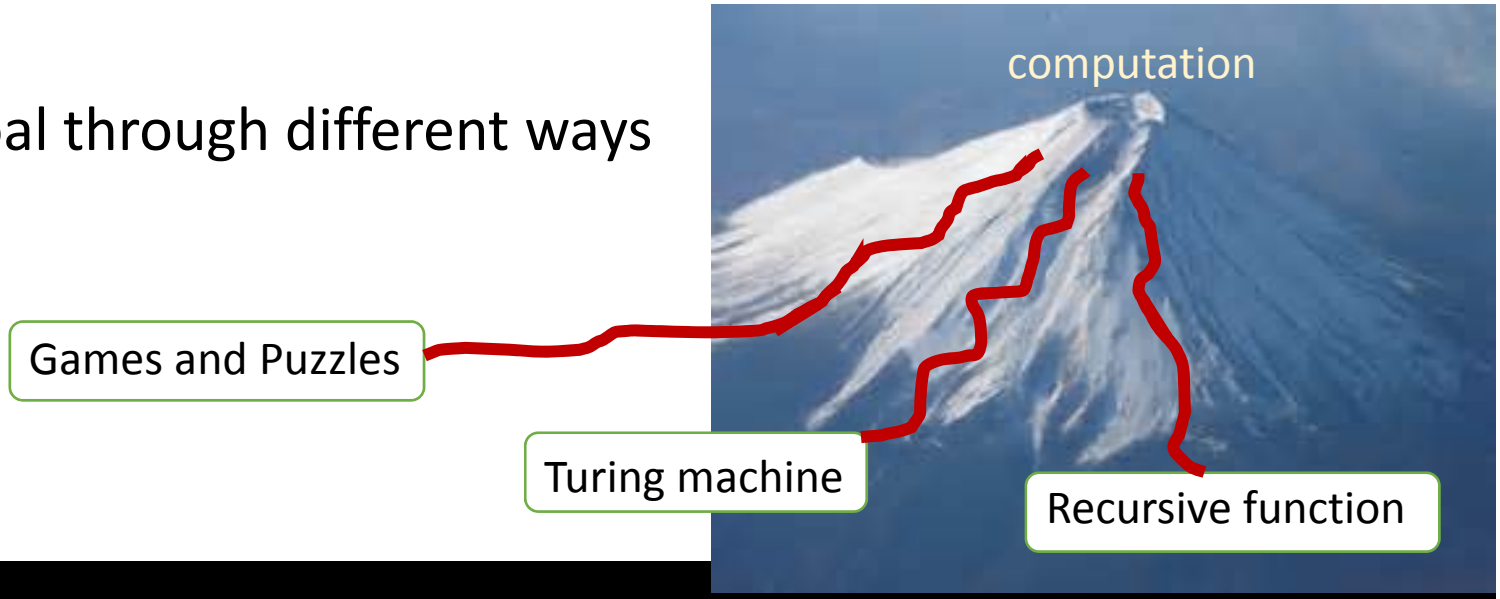
Simon J. Fraser, John "Horned" (Horton) Conway, *1975*

Computational Complexity v.s. Puzzles & Games

Advantages of Puzzles & Games to consider "computation"

- Simple and Uniform (with reasonable model)
- That may extract the essence of the difficulty of some computation
- That may give us new aspect of some computation

⇒Same goal through different ways



Short Ads.

- In JAIST, we have "JAIST Gallery" that has around 10000 puzzles called *NOB's Puzzle Collection*!

I'm a director of this gallery!

NOB Yoshigahara
(1936-2004)

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Classic Results (1970s～1980s):

- Game to consider “computation”
  - Characterization by artificial game
  - *Pebble game* (though we have many variants)

    Input: Directed graph *G,* placement of “*pebbles*”

    Rule: Move pebbles along edges and remove some pebbles in certain rules

    Output: Determine if you can move a pebble to a goal
  - It is complete for some computational classes;
    - NLOG, P, NP, PSPACE, EXP

1 player/2 players
Number of pebbles
Acyclic or not

- References:
  - J. Hopcroft, W. Paul and L. Valiant. “On Time versus space,” *J. Assoc. Comput. Mach*. 1977
  - Richard J. Lipton and Robert E. Tarjan. “Applications of a Planar Separator Theorem,” *SIAM J. Comput*. 1980
  - Stephen Cook; Ravi Sethi. "Storage requirements for deterministic polynomial time recognizable languages". *Journal of Computer and System Sciences,* 1976.
  - Takumi Kasai; Akeo Adachi; Shigeki Iwata. "Classes of pebble games and complete problems". *SIAM Journal on Computing,* 1979.

More Classic Results (1980s〜):

- Puzzles to consider “computation”
  - Characterizations by *natural* games and puzzles
  - *Many puzzles and games*
    - *E.g.,* Geometry (しりとり), Solitaire, Crossword puzzle, Jigsaw puzzle (matching puzzle), UNO, Video games, Pencil puzzles, …

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So far … (1980s～2000s):

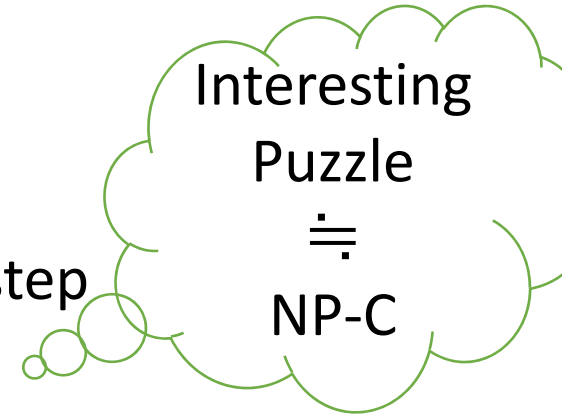
- We had tons of *X*-Complete problems;
  - NP-complete puzzles

    1 player, something decreases in each step
    - Tons of papers…
  - PSPACE-complete / EXP-complete games

    2 player version of these NP-complete problems
- They give some insight of these classes

  NP: 1 player, something decreases in each step.

  PSPACE: 2 players (…alternating Turing Machine)
- We needed some general model for them…

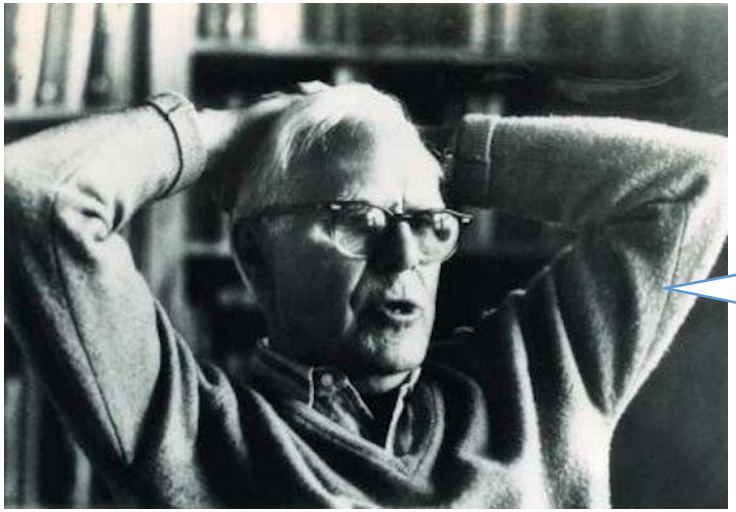


So far … (1980s～2000s):

Still unsolved

- Sliding Block puzzles like
  “Daddy Puzzle”,
  “Sokoban”

Martin Gardner said that…

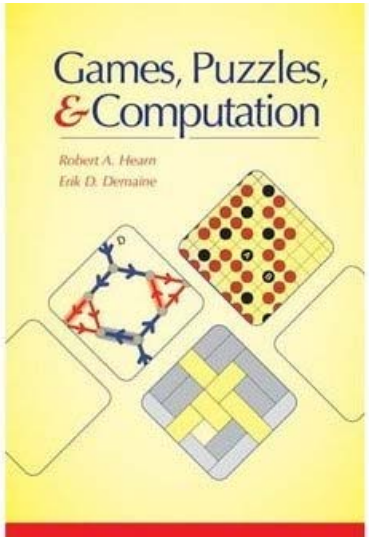


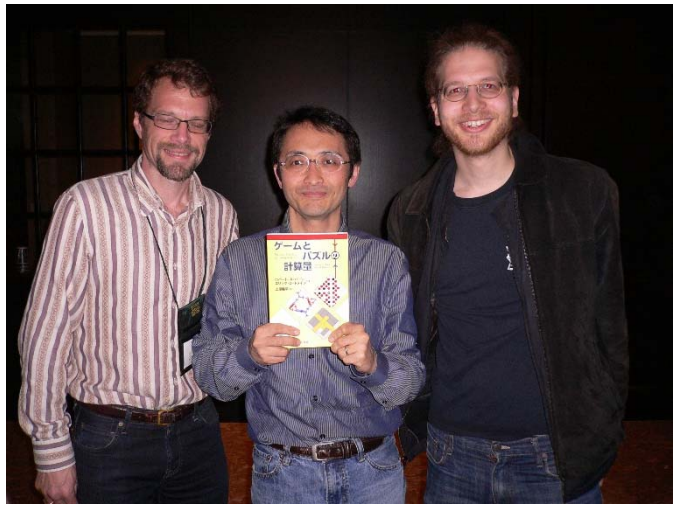
“These puzzles are very much in want of a theory”
Scientific American 210 (1964)

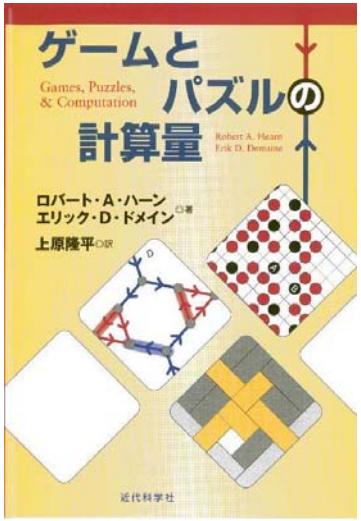
… 40 years later,

Modern Results (2010s～):

- New framework to consider “computation”
  - “Constraint Logic” by Bob Hearn and Erik D. Demaine
  - *characterizing games (2player) and puzzles (1player)*
    - *That can model many previous known games and puzzles,*
    - *And solves the open problems including sliding block type puzzles.*







Modern Results (2010s〜):

- New framework to consider “computation”
  - “Constraint Logic” by Bob Hearn and Erik D. Demaine
  - *Roughly, it is a game on a graph*

    Input: Directed graph *G*, each edge has weight and direction

    Rule: Each vertex is *balanced*, an operation is flipping an edge

    Output: Determine if you can flip some specified edge
  - Relatively higher classes:

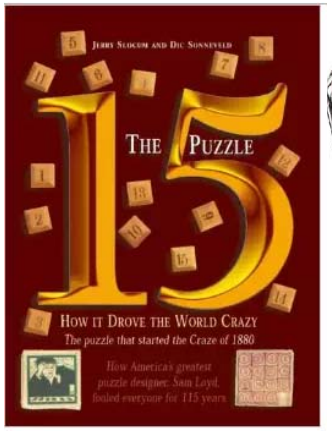
#flips of an edge

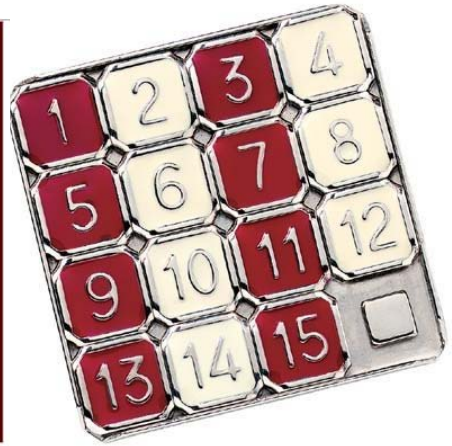
	0 player	1 player	2 player	Team, imperfect information
Unbounded	PSPACE	PSPACE	EXPTIME	RE (undecidable)
Bounded	P	NP	PSPACE	NEXPTIME

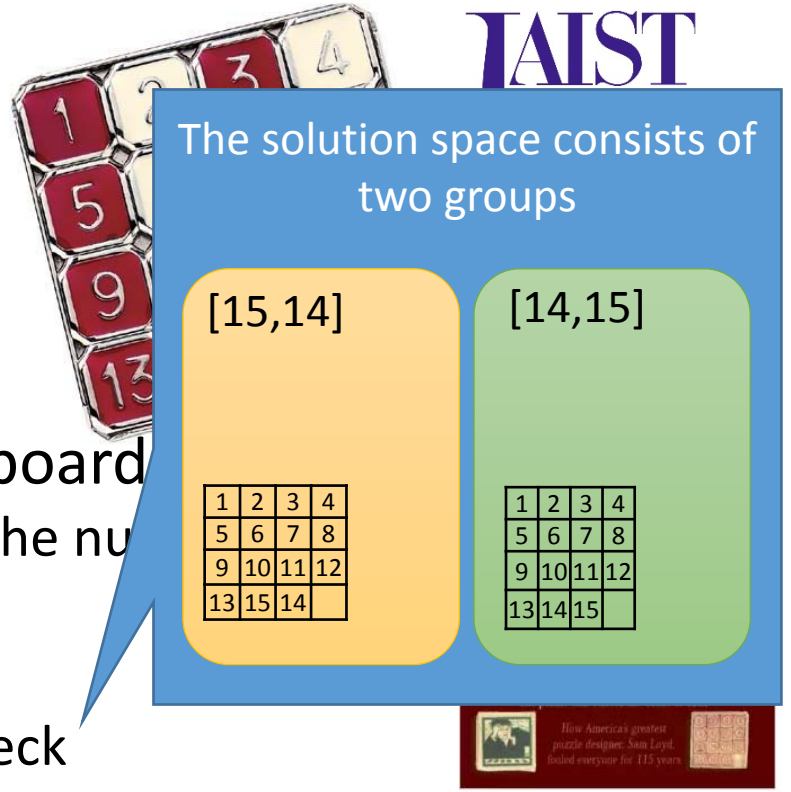
Some remarkable puzzles...

- Finally solved
  - Sliding Block puzzles are PSPACE-complete.
  - Unlike other NP-complete problems, it can recover the same state many times… that property makes them to be PSPACE-complete?
- It reminds us a classic puzzle solved in 1990s,,,
  - 15 puzzle
  - It has a long and funny stories;
    see “The 15 Puzzle Book”
    by Jerry Slocum, 2006.

Top puzzle collector in the world…







Some remarkable puzzles...

- The 15 Puzzle

  It is easy to generalized to n × n board

  Input: Two arrangements ***s*** and ***t*** of the nu

  Goal: Slide a panel from ***s*** to ***t***

  Output: …

  Yes/No: Linear time by parity check

  If Yes, find *any* sequence of arrangements: $O(n^2)$ time

  Furthermore, output *any* sequence: $O(n^3)$ time

  However, find a shortest sequence: NP-complete!!

- Reference:
  - Daniel Ratner and Manfred Warmuth. “The (n^2-1)-Puzzle and Related Relocation Problems,” *J. of Symbolic Computation*, 1990.

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Recent and Future Results (2010s～):

- *New concept* of problems to consider "complexity" inspired by these puzzles:

  Reconfiguration Problems

  Input: Problem P, two feasible solutions S_1 and S_2

  Operation: Simple rule for modification of a solution

Decision	Problem 1: Determine if S_1 can be transformed to S_2
Find	Problem 2: Find any sequence of solutions joining S_1 and S_2
Shortest	Problem 3: Find a shortest sequence between S_1 and S_2

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Recent and Future Results (2010s～):

- *New concept* of problems to consider "complexity" inspired by these games/puzzles:

Reconfiguration Problems

	Sliding Block puzzle		(n^2-1) puzzle
Decision	PSPACE-complete		Linear
Find	(PSPACE-complete)	⇔	Poly-time
Shortest	(PSPACE-complete)		NP-complete

Many problems should be here
And they give some new insight of the classes

Recent and Future Results (2010s～):

- Not game-like results for reconfiguration problems:
  - SAT: “Decision problem” is PSPACE-complete

    Reference:

    P. Gopalan, P.G. Kolaitis, E.N. Maneva, C.H. Papadimitriou, “The connectivity of Boolean satisfiability: computational and structural dichotomies,” *SIAM J. Comput*. 2009.
  - IS, Clique, Vertex Cover, Set Cover, IP: “Decision problem” is PSPACE-complete

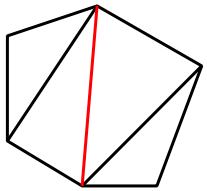
    Reference:

    T. Ito, E. D. Demaine, N. J. A. Harvey, C. H. Papadimitriou, M. Sideri, R. Uehara, and Y. Uno: On the Complexity of Reconfiguration Problems, *Theoretical Computer Science*, 2010.

In my measure, “Sliding-block puzzle type”

Recent and Future Results (2010s～):

- Bit game-like result for reconfiguration problems:
  - Famous open problem in Computational Geometry

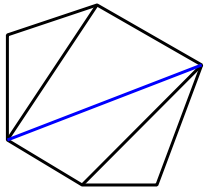


Input: Simple polygon, two triangulations T_1, T_2

Operation: “flip” one diagonal

Known: Every T_1 is flippable to T_2 in $O(n^2)$ flippings

Question: Find a shortest flipping



Result: It is NP-complete!!

It was open 40 years like sliding block puzzle…

Reference:

O. Aichholzer, W. Mulzer, A. Pilz, Flip distance between triangulations of a simple polygon is NP-complete, ESA 2013.

In my measure, “(n^2-1) puzzle type”

Recent and Future Results (2010s～):

- Not game-like, but something remarkable:
  - SAT: Trichotomy for the classes P, NP, and PSPACE from the viewpoint of "Shortest problem"

    Reference:

    A. E. Mouawad, N. Nishimura, V. Pathak and V. Raman: Shortest Reconfiguration Paths in the Solution Space of Boolean Formulas, *ICALP 2015*, 2015/7/8.

In my measure, this one may be the first example between "Sliding-block puzzle type" and "(n^2-1) puzzle type".

Summary and Future work

- Games and Puzzles give us a new insight about “computation”
- Some new problems are not yet well-settled.
  - Reconfiguration problem, especially, (n^2-1) puzzle type problem.
  - We need new model that characterizes the classes P, NP, PSPACE, (EXP) in this manner.

Conway’s Life Game

Pebble game

Real games/puzzles

Constraint Logic

Games based on “Reconfiguration”

These games are very much in want of a theory!