Analysis and Control of Limit Cycle Walking Based on Linearized Equation of Motion

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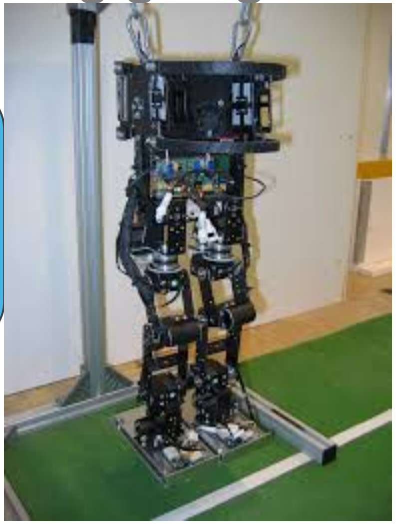
Introduction

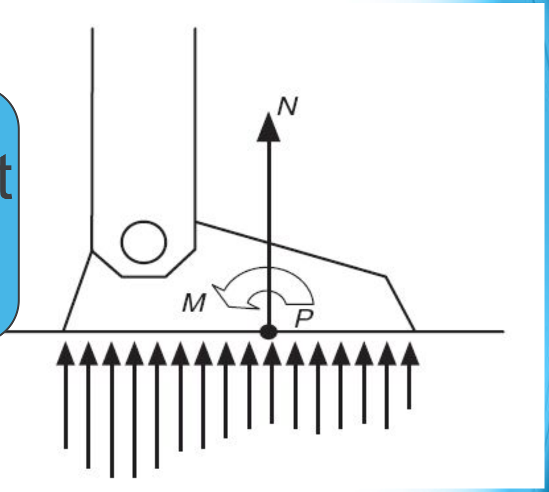
◆Artificial constraints

◆ **Stabilize the complex dynamic walking motion, however, limit the optimizing gait generation**

Static stability



Zero Moment Point (ZMP)



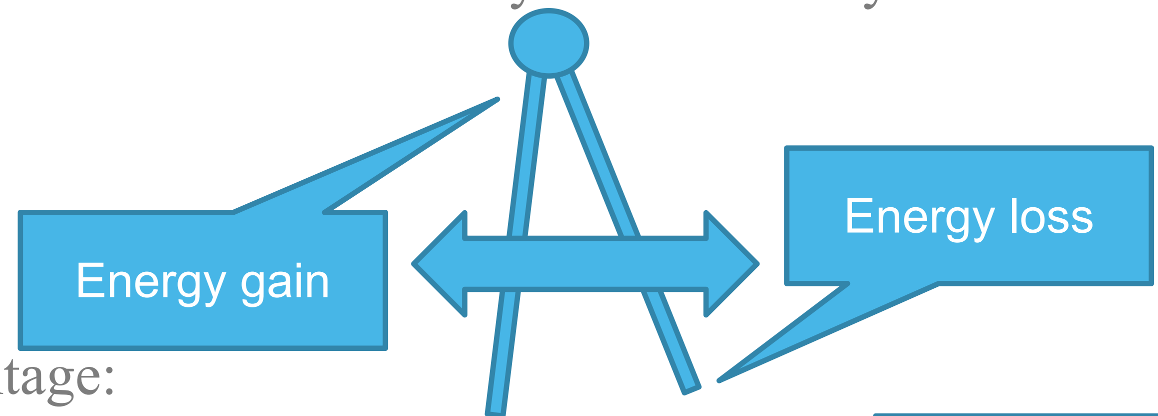
Efficiency, disturbance handling,
Cannot be compared
with human walking

Whether steady walking state can be generated?

Introduction

◆ Limit cycle walking (to release the constraints)

Limit cycle walking is a nominally periodic sequence of steps that is stable as a whole but not locally stable at every instant in time.



❖ Advantage:

- Energy efficiency (zero feedback gains)
- Versatility (less restrictive)

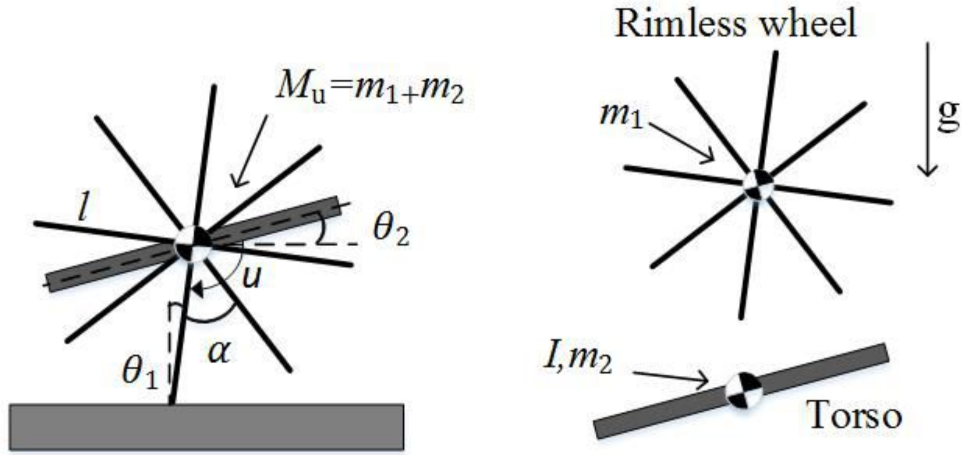
Solution: the feedback control system based on mathematical analysis

❖ Disadvantage:

- Zero feedback gains leads to a low ability of handle disturbances

Modeling

- Underactuated rimless wheel



- Linearized equations of motion

$$\begin{bmatrix} M_u l^2 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -M_u g l & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

- Collision

$$\dot{\theta}_{1(i)}^{+} = \dot{\theta}_{2(i)}^{+} = k_u \dot{\theta}_{1(i)}^{-}, k_u = \frac{M_u l^2 \cos\alpha + I}{M_u l^2 + I}$$

$$\theta_{1(i)}^{\pm} = \mp \frac{\alpha}{2}, \theta_{2(i)}^{+} = \theta_{2(i)}^{-} = 0$$

Underactuated Rimless Wheel

◆Control system

$$y = \boldsymbol{S}^{\mathrm{T}}\boldsymbol{\theta} = \theta_1 - \theta_2$$

$$\ddot{y} = \boldsymbol{S}^{\mathrm{T}}\ddot{\boldsymbol{\theta}} = \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}_0^{-1}\left(\boldsymbol{S}u - \boldsymbol{G}_0\boldsymbol{\theta}\right)$$

$$u = \frac{\ddot{y}_{\mathrm{d}}(t) + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}_0^{-1}\boldsymbol{G}_0\boldsymbol{\theta}}{\boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}_0^{-1}\boldsymbol{S}}$$

$$\ddot{y}_{\mathrm{d}}(t) = \begin{cases} \dfrac{4\alpha}{T_{\mathrm{set}}^2} & \left(0 \le t < \dfrac{T_{\mathrm{set}}}{2}\right) \\ -\dfrac{4\alpha}{T_{\mathrm{set}}^2} & \left(\dfrac{T_{\mathrm{set}}}{2} \le t < T_{\mathrm{set}}\right) \\ 0 & \left(t \ge T_{\mathrm{set}}\right) \end{cases}$$

$$\boldsymbol{M}_0\ddot{\boldsymbol{\theta}} + \boldsymbol{G}_0\boldsymbol{\theta} = \boldsymbol{S}u$$

When only the rimless wheel is considered

$$\ddot{\theta}_1 = \hat{\omega}^2\theta_1 + I_0\ddot{y}_{\mathrm{d}}(t), \quad \hat{\omega} = \sqrt{\frac{M_{\mathrm{u}}gl}{M_{\mathrm{u}}l^2 + I}}, I_0 = \frac{I}{M_{\mathrm{u}}l^2 + I}$$

Underactuated Rimless Wheel

- Linearized equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\theta_1\\ \dot{\theta}_1\end{bmatrix}=\begin{bmatrix}0 & 1\\ \hat{\omega}^2 & 0\end{bmatrix}\begin{bmatrix}\theta_1\\ \dot{\theta}_1\end{bmatrix}+\begin{bmatrix}0\\ I_0\end{bmatrix}\ddot{y}_\mathrm{d}(t)$$

$$\dot{\boldsymbol{x}}=\hat{\boldsymbol{A}}\boldsymbol{x}+\hat{\boldsymbol{B}}\ddot{y}_\mathrm{d}(t)$$

- As the solution of the steady states

$$\boldsymbol{x}_{i+1}^{-}=\mathrm{e}^{\hat{\boldsymbol{A}}T_i}\boldsymbol{x}_i^{+}+\int_{0^+}^{T_i}\mathrm{e}^{\hat{\boldsymbol{A}}(T_i-s)}\hat{\boldsymbol{B}}\ddot{y}_\mathrm{d}(s)\,\mathrm{d}s$$

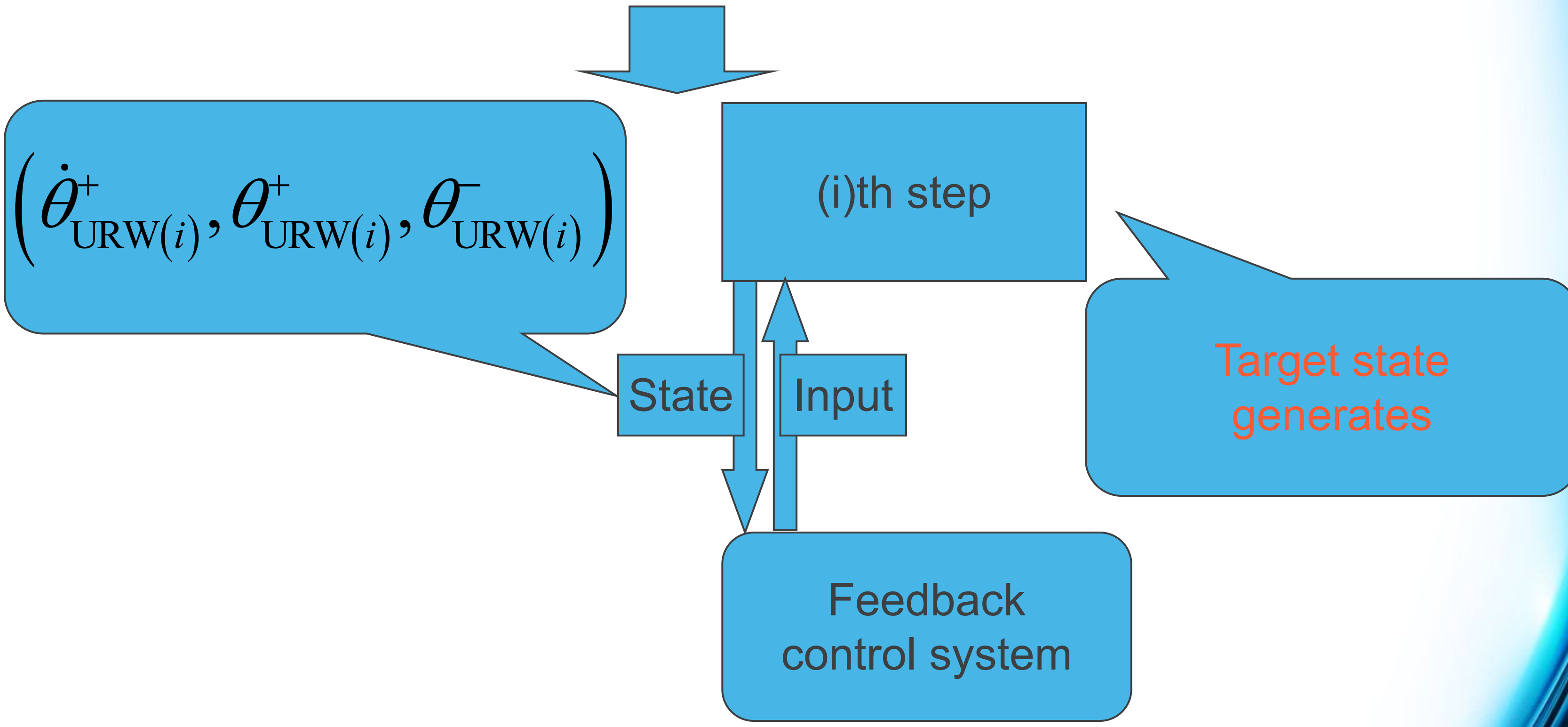
$$\begin{bmatrix}\theta_{1(i+1)}^{-}\\ \dot{\theta}_{1(i+1)}^{-}\end{bmatrix}$$

$$=\mathrm{e}^{\hat{\boldsymbol{A}}T_i}\boldsymbol{x}_\mathrm{i}^{+}+\int_{0^+}^{\frac{T_\mathrm{set}}{2}}\mathrm{e}^{\hat{\boldsymbol{A}}(T_i-s)}\hat{\boldsymbol{B}}\frac{4\alpha}{T_\mathrm{set}^2}\mathrm{d}s-\int_{\frac{T_\mathrm{set}}{2}}^{T_\mathrm{set}}\mathrm{e}^{\hat{\boldsymbol{A}}(T_i-s)}\hat{\boldsymbol{B}}\frac{4\alpha}{T_\mathrm{set}^2}\mathrm{d}s$$

Feed-forward Control on URW

◆Feed-forward control on level/uneven ground

- ◆For the level ground, $\theta^{+}_{\mathrm{URW}(i)}, \theta^{-}_{\mathrm{URW}(i)}$ are constant.
- ◆For the uneven ground, if the state of surface is known,



Feed-forward Control on URW

◆Feed-forward control on uneven ground

- **Boundary condition**

Boundary condition

$$F_{\mathrm{URW}}\left(T_{\mathrm{set}(i)},\dot{\theta}^{+}_{\mathrm{URW}(i)},\theta^{+}_{\mathrm{URW}(i)},\theta^{-}_{\mathrm{URW}(i)}\right)=\theta^{-}_{i+1}-\theta^{-}_{\mathrm{URW}(i)}$$

$$F_{\mathrm{URW}}\left(T_{\mathrm{set}(i)}\right)=0$$

Ridders' Method

$$T_{\mathrm{set}(i)}$$