What are Polynomial Constraints?

Q: Find the real values of $x$, $y$ such that:

$$y^2 + y - x^2y + 1 < 0 \text{ and } x^2 + y^2 - 4 < 0$$

A: $x = 0.687783209694$, $y = 1.875$

$$f > 0 \text{ is } \begin{cases} \text{satisfiable (SAT) if there exist values for variables } x,y, \ldots \\ \text{unsatisfiable (UNSAT) otherwise} \end{cases}$$
Many applications in Software Verification

- Automatic termination proving.
- Round-off and overflow errors analysis.
- Invariant generation.
- Test case generation.
Approaches for Polynomial Constraints

- QE-CAD: complete but DEXPTIME complexity.
- Bit-blasting: suffers with high number of variables or high degree of polynomials.
- Linearization: suffers with high degree of polynomials.
- Virtual substitution: needs root formulas of polynomial degree $\leq 4$
- ICP: uses Interval arithmetic (IA) and suffers with touching cases.

$$raSAT = ICP + \text{testing}$$
Interval Arithmetic

• $xy$
• $x \in [-2, 4]$
• $y \in [-1, 5]$

Interval Arithmetic

$xy \in [-10, 20]$

- e.g., $f \in [1, 10] \rightarrow f > 0$ is IA-VALID $\Rightarrow$ SAT
- e.g., $f \in [-10, -1] \rightarrow f > 0$ is IA-UNSAT $\Rightarrow$ UNSAT
- e.g., $f \in [-12, 10] \rightarrow f > 0$ is IA-SAT $\Rightarrow$ UNKNOWN

estimate ranges of polynomials
Testing

find values for variables that satisfy the constraint

- $xy > 1$
- $x \in [-2, 4]$
- $y \in [-1, 5]$

$x y > 1$ is SAT with $x = 1.2, y = 2$

Randomly generated with the guide of IA results
raSAT loop

\[ x \in [l, h] \leftrightarrow x \in [l, m] \lor x \in [m, h] \text{ with } l \leq m \leq h \]
## raSAT Performance

- **iSAT3, dREAL: ICP-based solvers**

<table>
<thead>
<tr>
<th>Benchmarks</th>
<th>raSAT</th>
<th>iSAT3 with bounds [-1000, 100]</th>
<th>dREAL</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Solved No.</td>
<td>Time(s)</td>
<td>Solved No.</td>
</tr>
<tr>
<td>Matrix-1(SAT)</td>
<td>25</td>
<td>414.99</td>
<td>11</td>
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<tr>
<td>Matrix-1(UNSAT)</td>
<td>2</td>
<td>0.01</td>
<td>3</td>
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<td>Matrix-25(SAT)</td>
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<td>3</td>
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<td>Matrix-25(UNSAT)</td>
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<td>0.38</td>
<td>12</td>
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<tr>
<td>Meti-Tarski(SAT)</td>
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<td>419.25</td>
<td>2916</td>
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<tr>
<td>Meti-Tarski(UNSAT)</td>
<td>1052</td>
<td>821.85</td>
<td>1225</td>
</tr>
</tbody>
</table>

* means $\delta - SAT$ and $\delta - SAT \leftrightarrow SAT$
raSAT Performance

Participated in SMT competition 2015.

► 3\textsuperscript{rd} among 6 in QF_NRA - category for reals
► 2\textsuperscript{nd} among 7 in QF_NIA - category for integers
► Overall, ranked 8\textsuperscript{th} among 19 solvers.