Enumeration of Common Developments

Xu Dawei, Uehara Lab School of Information Science

JAIST

Introduction

Unfold the cube, we can get its "developments".



Big Open Problem:

Any convex polyhedron can be developed without overlapping by just cutting along its edges (?)

Open problem

<u>Theorem</u> [Biedl et al. 1999] Two polygons that can fold two boxes



<u>Theorem</u> [Abel, Demaine, Demaine, Matsui, Rote, Uehara 2011] For area 22, there are 2263 polygons that can fold to two boxes of size $1 \times 1 \times 5$ and $1 \times 2 \times 3$ (by exhaustive search). Among them, <u>only one</u> admits to fold a "box" of size $0 \times 1 \times 11$ of volume 0.



The enumerate approach

We enumerate all developments by super computer. It takes around 2 months to complete search.



<u>Theorem</u> [Dawei, Horiyama, Uehara 2015] By searching all developments of area 30, 1080 polygons fold boxes of size $1 \times 1 \times 7$ and $1 \times 3 \times 3$. Among 1080, there is 9 polygons that fold to a cube of size $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$, and <u>Only one</u> of 9, it has two ways of folding into the cube!





Future Algorithms with BDD

Development should be ...





- Cut-edges form a spanning tree
 - No cycles
 - All vertices are connected by cut-edges

A pseudo spanning tree

- No cycles
- 8 corners are connected by cut-edges
- Degrees of vertices:
 - Corners: 1 or more cut-edges
 - Other vertices: 0, 2, 3, 4