STABLE LEGAL KNOWLEDGE WITH REGARD TO CONTRADICTORY ARGUMENTS

Shingo Hagiwara
School of Information and Science
Japan Advanced Institute of Science and Technology
1-1 Asahidai, Nomi, Ishikawa 923-1292, Japan
email: `s-hagiwa,tojo@jaist.ac.jp`

1 The Aim of the Research

In this paper, we pay attention to the logical representation of codes. Then, the problem of detecting inconsistency becomes different from the conventional belief revision. Suppose the following pair of rules. $\Delta = \{\alpha \to \beta, \alpha \to \neg\beta\}$ Note that Δ is not inconsistent unless a fact α is supplied. Because the logical consequence of Δ together with α produces both of β and $\neg\beta$, $\Delta \cup \{\alpha\} \vdash \{\beta, \neg\beta\}$. Our motivation in this paper is to propose a method to detect such latent inconsistency of codes, that is superficially consistent in the logical point of view, supplying a *minimal* set of facts.

2 An Approach and an Idea

2.1 Minimal Inconsistent Set

Here, we give the definition of a minimal inconsistent set.

DEFINITION 1. *Minimal Inconsistent Set (MIS)*
Let Σ be sets of well-formed formulae. Γ is a minimal inconsistent set (MIS) in Σ iff $\Gamma \vdash \bot$ and $\forall\phi \in \Gamma[\Gamma \setminus \{\phi\} \nvdash \bot]$ for such $\Gamma \subseteq \Sigma$. Then, MI is such a function that $MI(\Sigma) = \{\Gamma | \Gamma$ is a MIS of $\Sigma\}$.

Now let us consider how the concept of MIS works. Prior to this, we need to define what an argument is.

DEFINITION 2. *Argument*
Let ϕ be formula and Φ be a set of formulae. $\langle\Phi, \phi\rangle$ is an argument iff $\Phi \vdash \phi$, $\forall\psi[\Phi \setminus \{\psi\} \nvdash \phi]$ and $\Phi \nvdash \bot$.

For the argument of $\langle\Phi, \phi\rangle$, ϕ is called a *conclusion* and Φ is called a *support*.

2.2 Invokers

We assume that the legal knowledge-base mainly consists of rules. However, we must replace those latent discrepancy: $\{\alpha \to \beta, \alpha \to \neg\beta\}$ for $\{\alpha \to \beta, \alpha \to \gamma\}$ with $(\beta \wedge \gamma) \vdash \bot$. However, unless α is supplied the situation is same; that is, the knowledge-base apparently is not inconsistent, and cannot be revealed to be contradictory. Because a legal rule has a structure of assumption–consequence relation, the consequence cannot emerge unless the assumption is not supplied. A written regulation 'If a person steals others' property, then the person is guilty' only produces some consequence when there is a fact that 'someone steals other's property.'

In the following, we consider an assumptive set called *invokers* that invoke such implication relations. Such an assumptive set must be added so as to expand the consequences. Suppose that a knowledge-base of $\{\alpha \to \beta, \beta \to \gamma\}$ is given. When we give β and δ to this knowledge, it would be expanded to: $\{\alpha \to \beta, \beta \to \gamma, \beta, \gamma, \delta\}$. The fact δ does not contribute to the inspection of the above rules while β does, though the rule '$\alpha \to \beta$' remains untouched. However, when we give only α to the knowledge, all the rules are invoked and produces all the consequences, as: $\{\alpha \to \beta, \beta \to \gamma, \alpha, \beta, \gamma\}$. Our next target is to determine the adequate set of the assumptive set, i.e., *invokers*. However, we must take care for the case of inconsistency. Generally speaking, classical logic assumes that from the inconsistency any formula could be deduced, and in such a case the knowledge-base would be explosive to contain the infinite number of propositions. Because our objective is to assess the adequacy of *invokers*, that is a minimal set of facts that invoke all the rules in a given knowledge-base, we need to suppress this explosion. For this purpose, we provisionally discard the above inference rule, i.e., we assume a paraconsistent logic . We define the function Cn^* and consequence relation $\vdash^*$ as follows.

DEFINITION 3. *Paraconsistent Consequence*
Let ϕ be an arbitrary formula. In the deduction by $\vdash^*$, $\frac{\bot}{\phi}$ does not hold even though for any Δ, $\Delta \vdash^* \bot$. Moreover, $Cn^*(\Delta) = \{\phi | \Delta \vdash^* \phi\}$.

Using DEFINITION 3, we proceed to the definition of invokers.

DEFINITION 4. *Invoker*
Let Δ be a set of formulae and ϕ be a formula. ϕ has an invoker of Δ if there exists such λ that $\Delta \nvdash^* \lambda$, $\{\phi\} \nvdash^* \lambda, \lambda \neq \bot$ and $\Delta \cup \{\phi\} \vdash^* \lambda$. Then, the function ivk is such a function that $\lambda \in ivk(\Delta)$.

In other words, when ϕ has an invoker with Δ, $Cn^*(\Delta \cup \{\phi\}) \setminus \{\bot\} \neq Cn^*(\Delta) \cup Cn^*(\{\phi\})$.

For a chaining of multiple rules: $\{\alpha \to \beta, \beta \to \gamma\}$, β invokes only one of the rules and produces γ while α invokes both of the rules and produces β and γ. Therefore, we can put an order between invokers.

DEFINITION 5. *Order of Invokers*
Let Δ is a set of formulae. Let ϕ, ψ be formulae. If $\phi \in ivk(\Delta)$, $\psi \in ivk(\Delta)$, $\Delta \cup \{\phi\} \vdash^* \psi$ and $\{\phi\} \nvdash^* \psi$, then, $\phi <_\Delta \psi$.

Note that the order of invokers functions similarly as the logical implication but it does not exactly correspond to that. If $\phi <_\Delta \psi$, then both of $\phi, \psi \in ivk(\Delta)$ are presupposed. Therefore, for $\Delta = \{\phi \to \psi\}$, since $\psi \notin ivk(\Delta)$, we should not claim that $\phi <_\Delta \psi$. Also note that, for $\Delta = \{\phi\}$, $(\phi \to \psi) \in ivk(\Delta)$, because $\Delta \cup \{\phi \to \psi\} \vdash^* \psi$ while $\Delta \nvdash^* \psi$ and $\{\phi \to \psi\} \nvdash^* \psi$.

The order of invokers is transitive, i.e., $\phi <_\Delta \psi$, $\psi <_\Delta \chi$, then $\phi <_\Delta \chi$. but is not reflexive. $\phi \not<_\Delta \phi$.

Now, we can define the minimal assumptive sets by using above definitions.

DEFINITION 6. *Assumptive Set*
Let Δ is a set of formulae and ϕ be an invoker of Δ, respectively. For any other invoker ψ of Δ, if $\phi <_\Delta \psi$ then we call ϕ the assumptive set. Then, $mivk$ is such a function that $\phi \in mivk(\Delta)$

2.3 Stable Knowledge

In this section, we apply various notions defined thus far in terms of legal reasoning. When we call a legal knowledge-base, it consists of multiple rules as well as facts.[1] A legal knowledge-base is expanded given a minimal invoker set(the assumptive set), and as a result, the whole set may become inconsistent, from which we can retrieve MIS's by the function MI.

2.4 Stable and Semi-stable rules

Although an inconsistent knowledge is not tractable as it is, it is not practical for us to rewrite whole of the code. When a new amendment is added to a code, it is worth finding which part should be kept untouched and which part should be revised. As MIS is a source of inconsistency which we limited the size minimal, we may claim that such MIS should be a target of revision. On the contrary, we can fix a reliable part of the code, that is rather indifferent to the inconsistency in terms of MIS's. Here, we propose a *stable* part that is such independent part of the inconsistency, and after that, we consider *semi-stable* part.

Hereafter we use the word *knowledge* that may include the inconsistency, as the expanded legal knowledge-base with invokers.

DEFINITION 7. *Stable Knowledge Set*
Let Σ be a knowledge. Then $St(\Sigma) \subseteq \Sigma$ is a stable knowledge of Σ iff $St(\Sigma) = \Sigma \setminus \bigcup_{\Theta \in MI(\Sigma)} \Theta$.

Because the rules and the facts in $St(\Sigma)$ do not concern the chaining for the inconsistency, we can preserve $St(\Sigma)$ as a robust area for the revision of the code. The knowledge dose not relate to inconsistency completely. Therefore, when we decide the knowledge which should be kept, this knowledge can be a criterion.

Next, we consider a weaker notion of stableness. We regard that a rule of knowledge, the consequence of which can be an antecedent of a rule in $St(\Sigma)$, can be rather reliable though in some cases the rule may contribute to the inconsistency. We call such rules *semi-stable*.

DEFINITION 8. *Semi-stable Knowledge*
Let Σ be a knowledge. For a rule in Σ, if its consequence is identical to an antecedent of some rule in $St(\Sigma)$, the rule is called semi-stable. The whole set of semi-stable rules together with $St(\Sigma)$ is called a semi-stable knowledge of Σ.

Because a semi-stable rule only exists in a MIS, and a MIS consists of two contradictory arguments, the rule must be a part of an argument that forms the inconsistency. Thus, if $\Theta \in MI(\Sigma)$ consists of two arguments $\langle \Psi, \psi \rangle$ and $\langle \Psi', \psi' \rangle$ where $\psi \wedge \psi' \vdash^* \perp$ and $\Psi \cup \Psi' = \Theta$, and if a subargument $\Phi \subseteq \Psi$ supports an invoker of $St(\Sigma)$, i.e., $\phi \in ivk(St(\Sigma))$ for such a $\langle \Phi, \phi \rangle$, Φ would be a part of semi-stable knowledge.

3 future direction

At this stage in our method, the property of invokers and the order among them are not fully investigated. The main reason is that we only defined Modus Pones for the '$\to$' and other inference rules are left undefined. If we apply relevant logic, such an interesting deduction as $\alpha \wedge \beta \nvdash \alpha \to \beta$ c be adopted.

The relevant logic is preferable, compared with the intuitionistic logic, multi-valued logics, truth-maintenance system (TMS), and others, because we disregard the contraposition which unnecessarily increases the number of inference rules. Although the intuitionistic logic excludes one direction of contrapositions, $\{\alpha \to \beta, \alpha \to \neg\beta, \neg\alpha \to \gamma, \neg\alpha \to \neg\gamma\}$ still holds. Moreover, multi-valued logic is to improve the expressiveness of a proposition, the purpose of which is different from ours. And TMS could also find an inconsistency, but the system cannot find which knowledge to be corrected. Also, if we consider paraconsistent logic, we need to consider what '$\perp$' infers further, admitting the criticism that the paraconsistant logic might just ignore the inconsistency . Even though, we are still hopeful in that such modern logics would bring more practical inference methods into the legal reasoning.

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- Stable Legal Knowledge with Regard to Contradictory Arguments, AIA2006 on IASTED
- 矛盾した法的知識集合からの極小矛盾集合の抽出, JSAI2005
- Model Updating by Comunication Channel, CLIMA06 on AAMAS(under submission)

[1] Actually, a fact is a kind of rule whose antecedent is $\top$; in code, several definitions may be included and we regard them as facts.