Proving Properties of Incremental Merkle Trees

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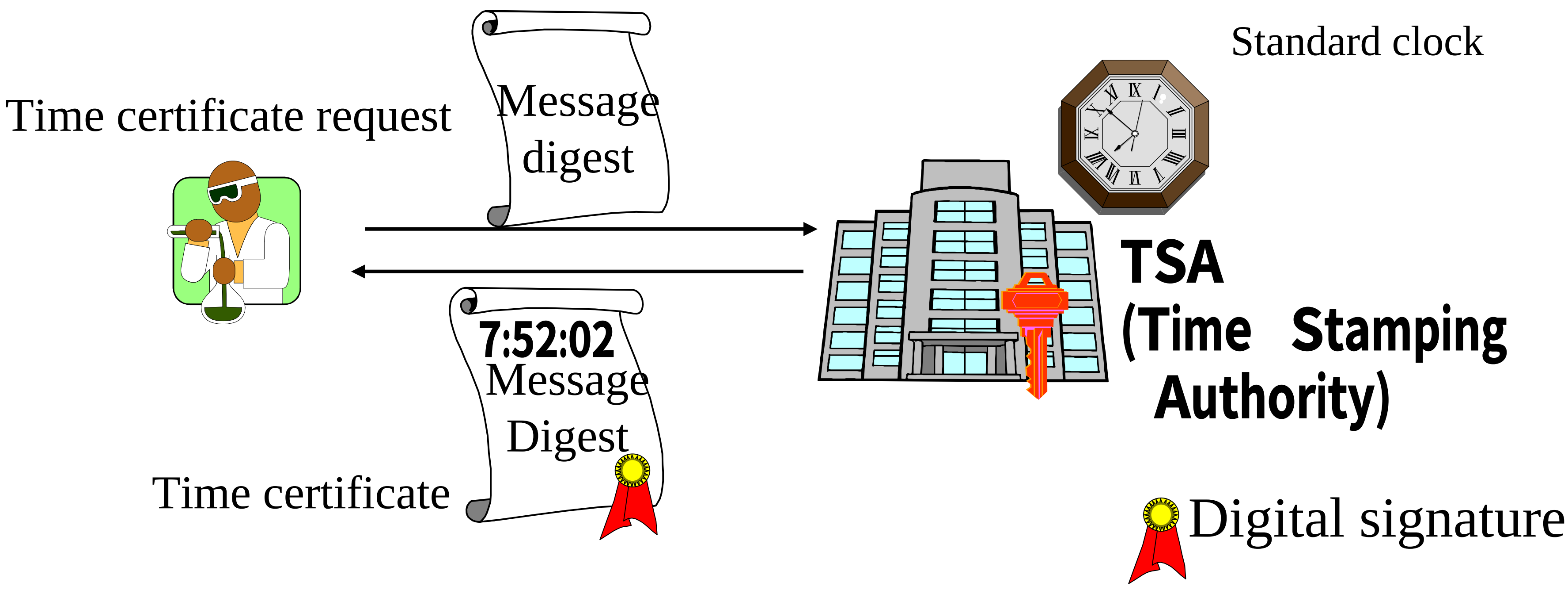
What is temporal authentication ?

Certificate an occurrence of an transaction at “*time*”

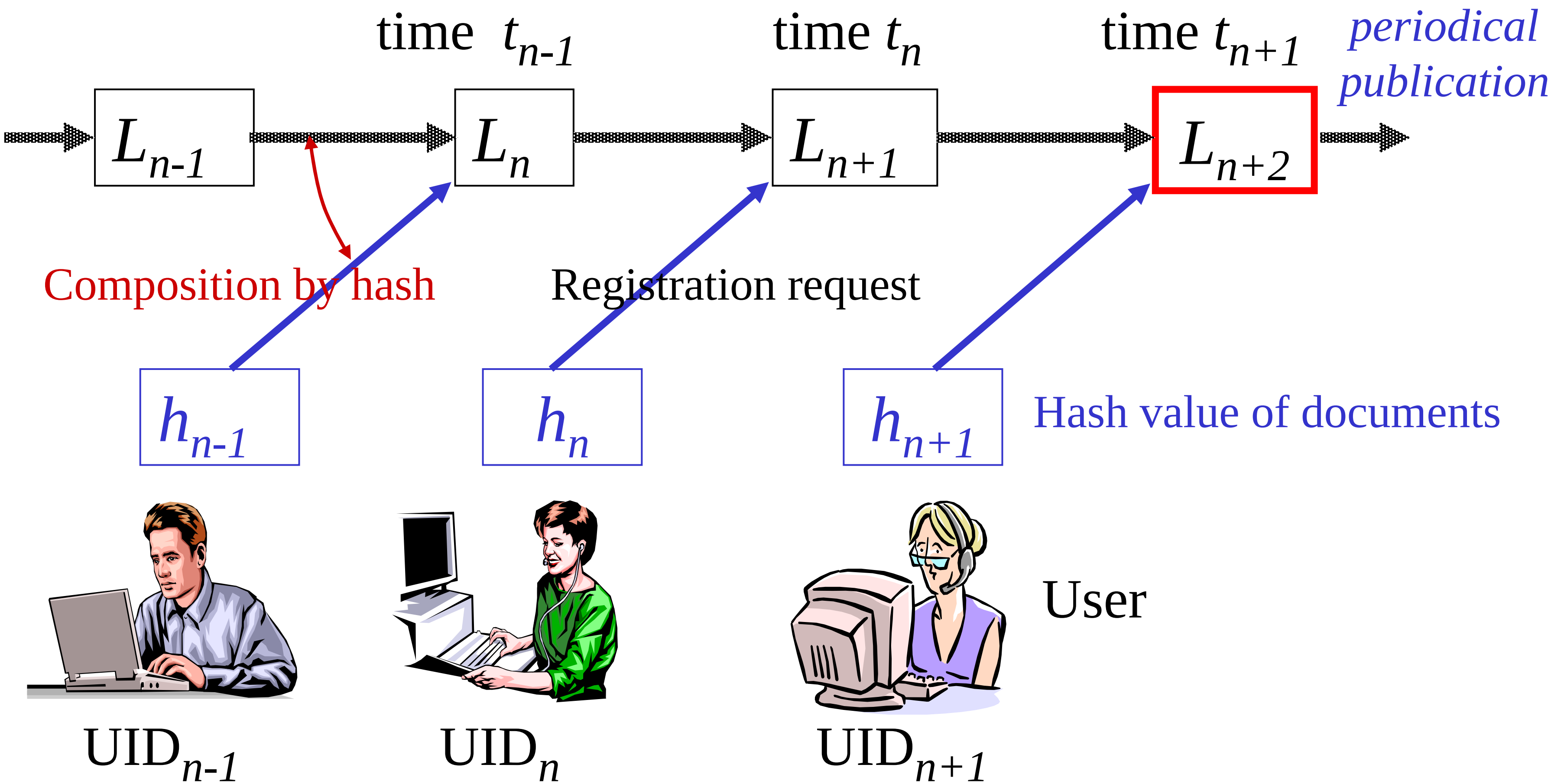
- Time stamp by digital signature (rfc-3161)
- Linking and publication by hash (ISO 18014-3)

Time stamp by digital signature (rfc-3161)



Linking and publication by hash function (ISO18014-3)

Compose past time stamps by a hash function



We assume collision-resistance, one-way hash function.

Time stamp by digital signature v.s. Linking by hash

Time stamp by digital signature

First certificate

Pros:

- Relatively safe for intra-dishonesty by Hardware Secure Module.
- Fine precision (< sec).

Cons:

- Contamination of crypto system invalidates *all* certificates.
- Relatively short life span : 〜5years.

Linking and publication by hash function

Secondary certificate

Pros:

- Relies only on hardness of a hash function (e.g., SHA-1).
- Relatively long life span : 〜30years.

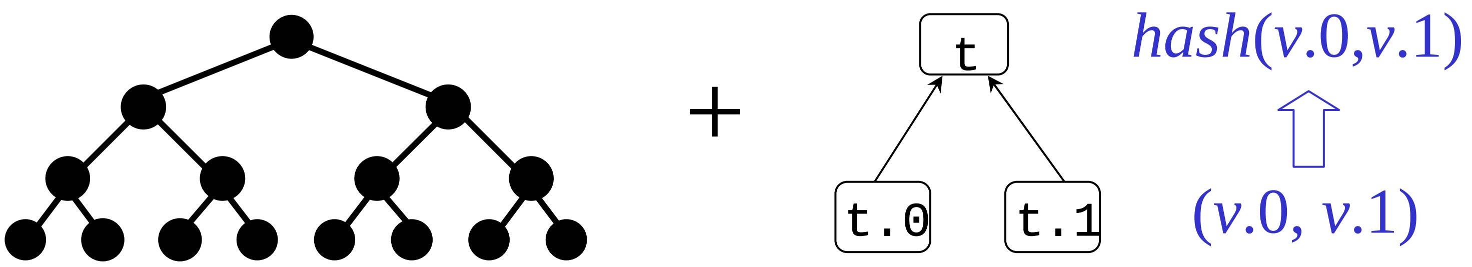
Cons:

- Hash function has no key; guarantee required for intra-dishonesty.
- During publication period, no auditor can check.

Merkle tree (Merkle 1979)

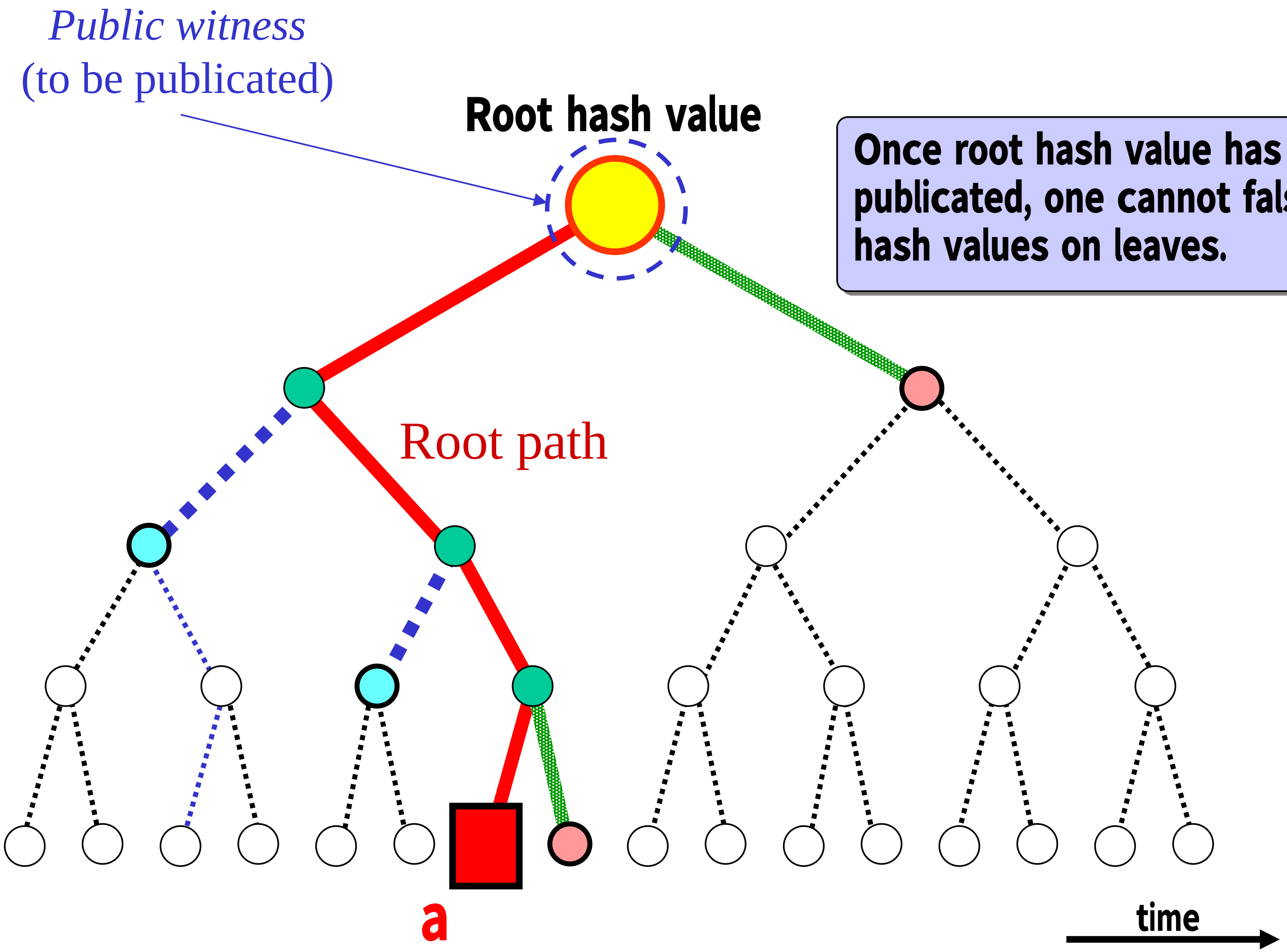
- *Merkle tree = Binary tree + hash function*

- Each node has its hash value, computed from a pair of hash values of its children.



We assume collision-resistance, one-way hash function.

Basic idea : Merkle tree (1)



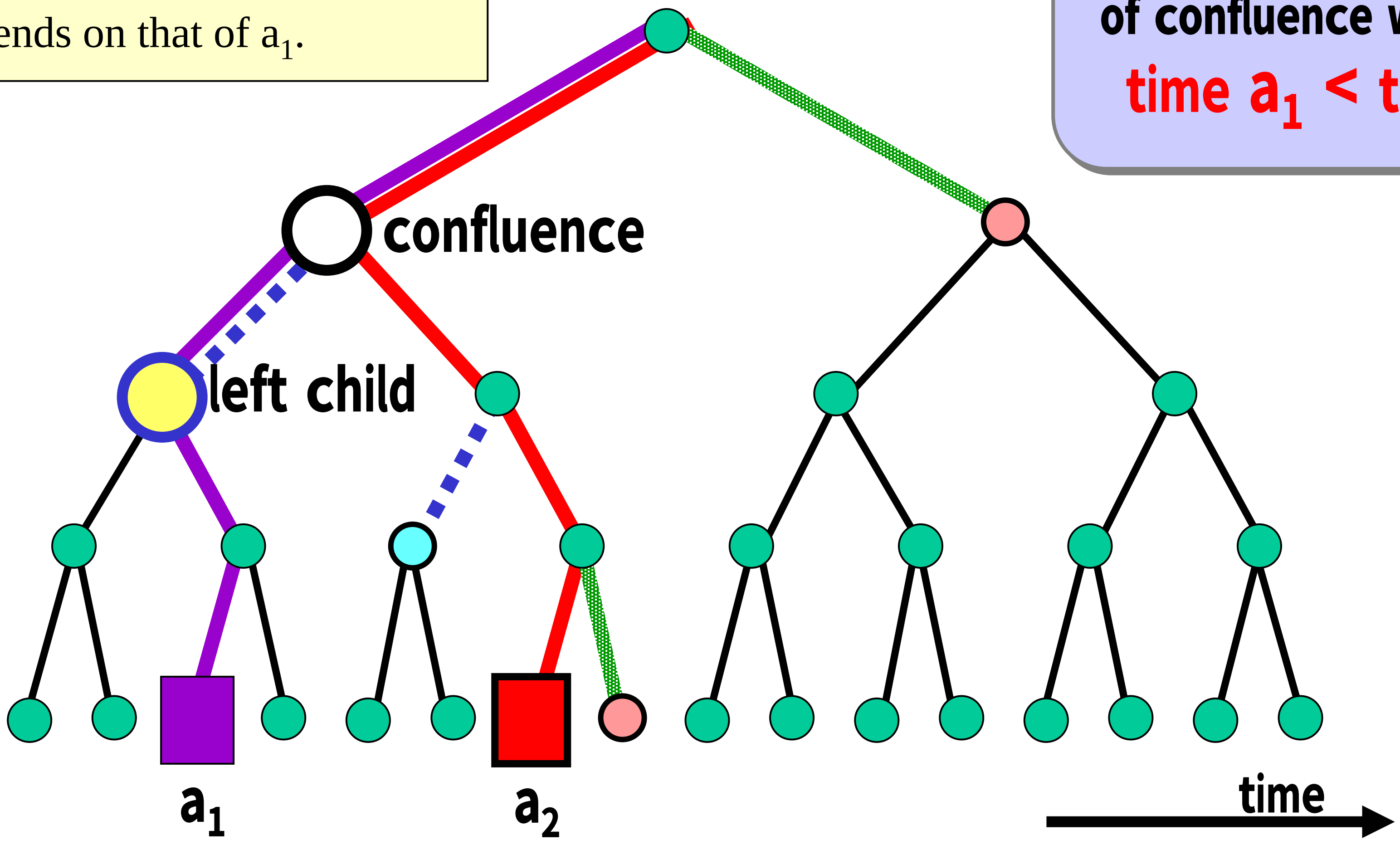
Once root hash value has been publicated, one cannot false hash values on leaves.

Basic idea: Merkle tree (2)

The hash value of the left child depends on that of a_1.

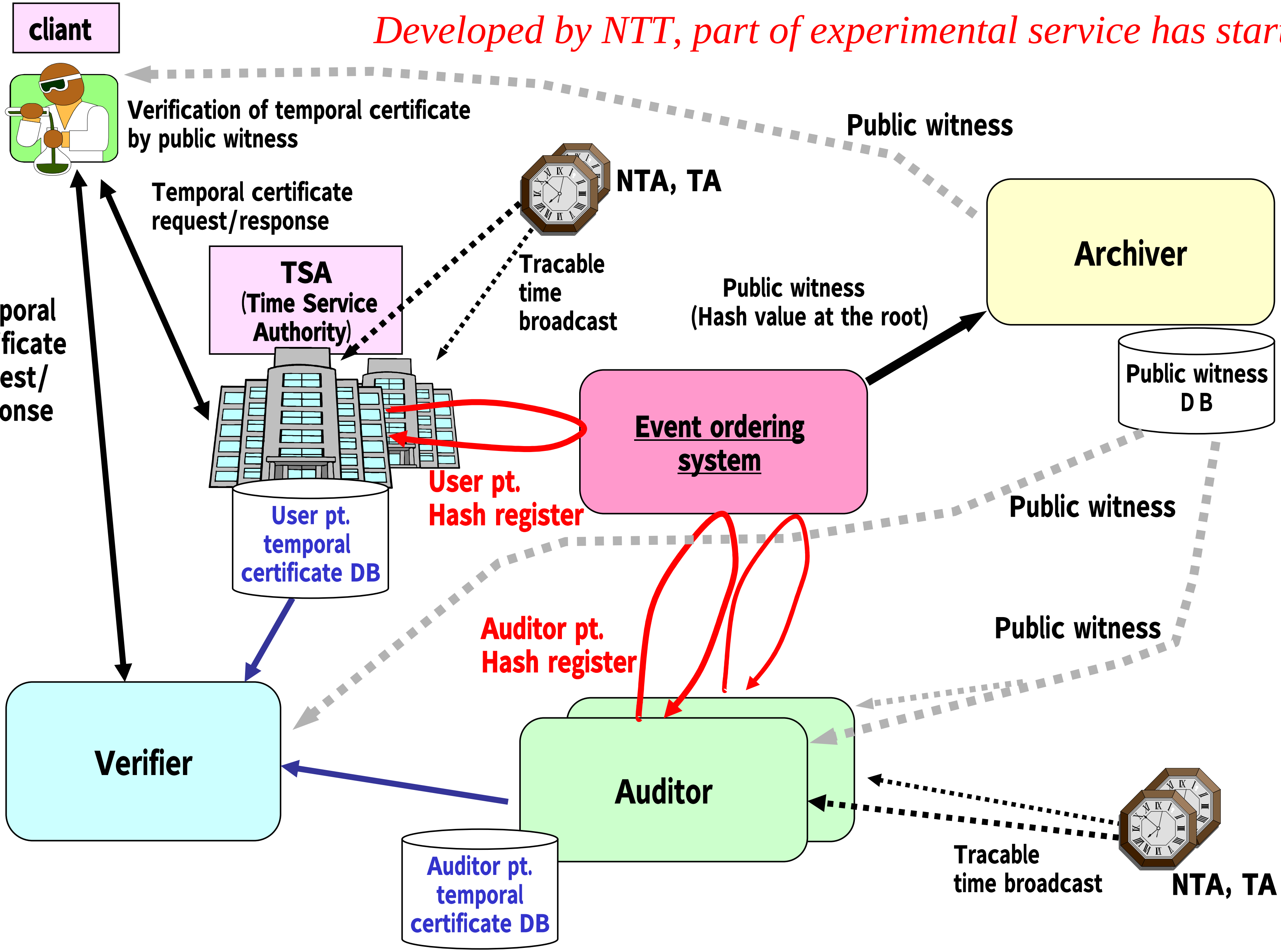
If the root path of a_1 contains the left child of confluence with a_2

time a_1 < time a_2

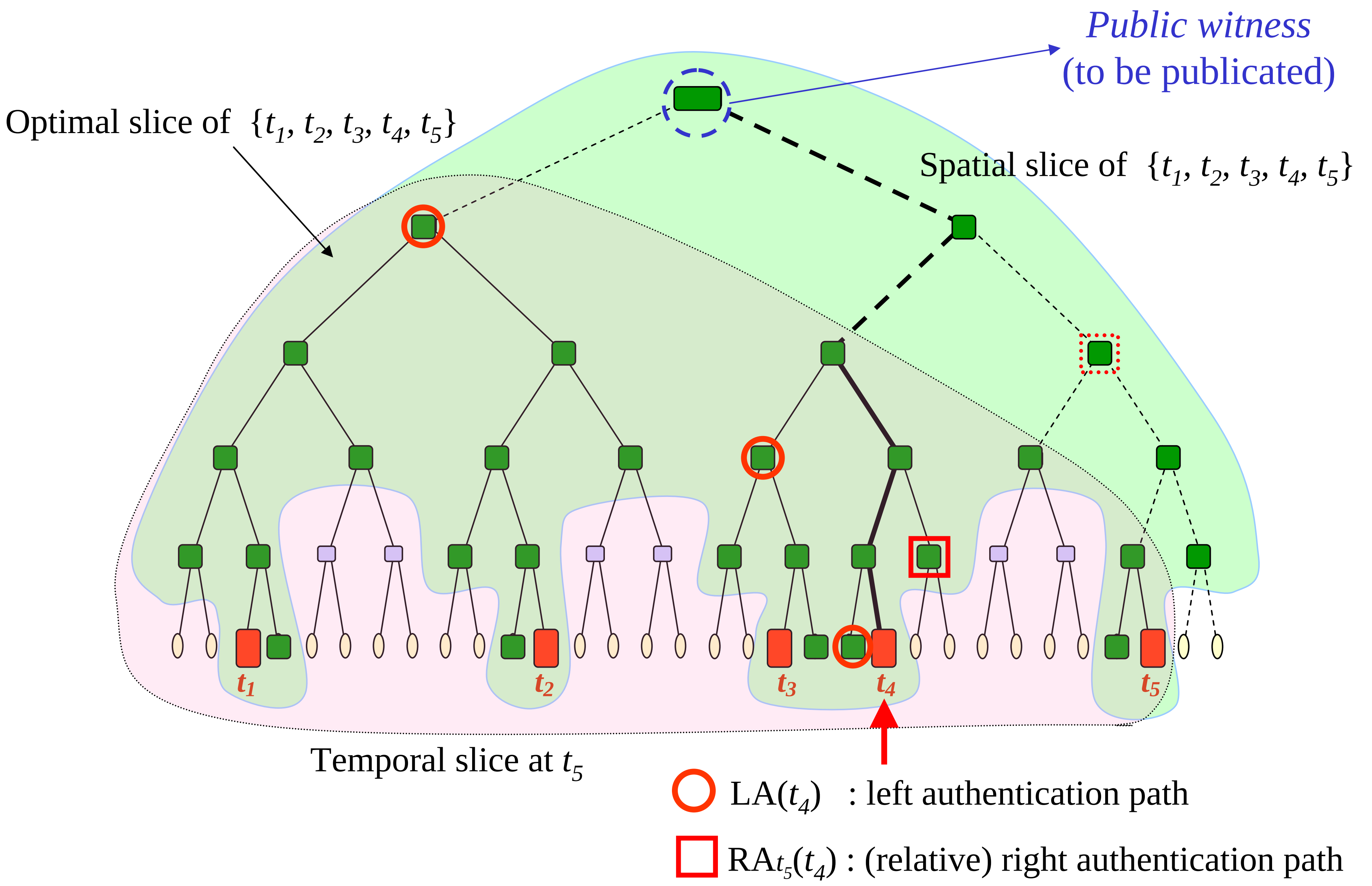


Reference model of event ordering system based on Incremental Merkle trees

Developed by NTT, part of experimental service has started



Incremental Merkle trees construction for registration requests at t_1, t_2, t_3, t_4, t_5



Protocol between users and an event-ordering system

- We assume that each user will register reasonably frequent.

- Assume a user registers at $t_1, t_2, \ldots, t_n$ and receives:
  - $(\phi, LA(t_1) \cup \{t_1\})$ at t_1.
  - $(RA_{t_i}(t_{i-1}), LA(t_i) \cup \{t_i\})$ at t_i with $0 < i \leqq n$.

Size O(n)

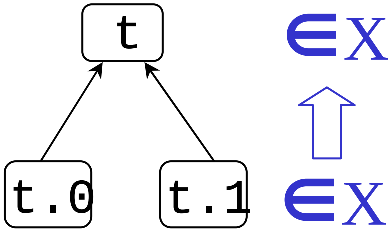
- We denote $LS(t_i) = LA(t_i) \cup \{t_i\}$

$$LSR_{t_{i+1}}(t_i) = LS(t_i) \cup RA_{t_{i+1}}(t_i)$$

Incremental scheme for Optimal Slice replication

- **Def.** Closure Cls(X) is the minimum set such that
  - X ⊆ Cls(X)
  - t.0 (left child), t.1 (right child) ∈ Cls(X)

    ⇒ t ∈ Cls(X)



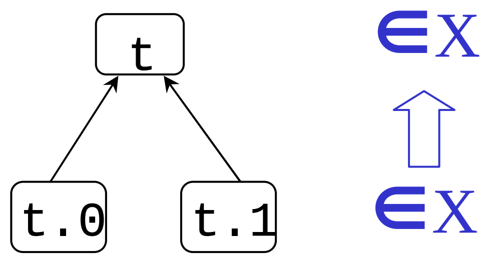
- **Th 1.** OptimalSlice($\{t_1, t_2, \ldots, t_n\}$)

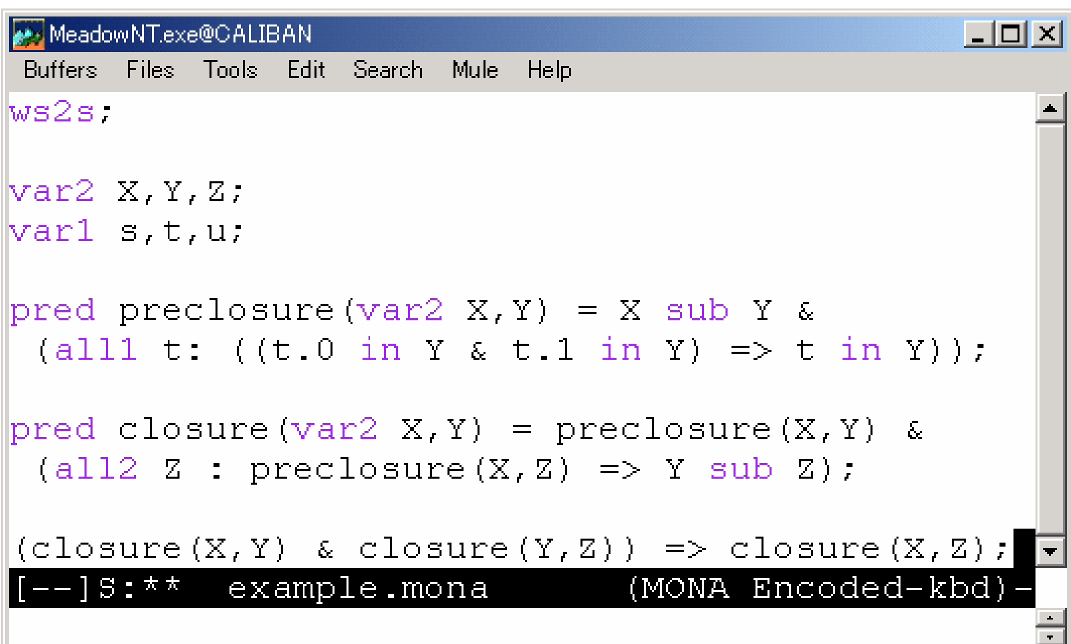
  $= (\bigcup_{1 \leqq i < n} \mathrm{Cls}(\mathrm{LSR}_{t_{i+1}}(t_i))) \cup \mathrm{Cls}(\mathrm{LS}(t_n))$

  Described in WS2S

  (incomparable(A) & opt_slice(A,X) &
  LSRclosure_union(A,Y)) => X = Y;

MONA:WS2S satisfiablity checker






```
ws2s;

var2 X,Y,Z;
var1 s,t,u;

pred preclosure(var2 X,Y) = X sub Y &
 (all1 t: ((t.0 in Y & t.1 in Y) => t in Y));

pred closure(var2 X,Y) = preclosure(X,Y) &
 (all2 Z : preclosure(X,Z) => Y sub Z);

(closure(X,Y) & closure(Y,Z)) => closure(X,Z);
```


```
$ mona example.mona
MONA v1.4-5 for WS1S/WS2S
Copyright (C) 1997-2002 BRICS

PARSING
Time: 00:00:00.05

CODE GENERATION
DAG hits: 34, nodes: 34
Time: 00:00:00.03

REDUCTION
Projections removed: 0 (of 4)
Products removed: 1 (of 16)
Other nodes removed: 0 (of 13)
DAG nodes after reduction: 32
Time: 00:00:00.02

AUTOMATON CONSTRUCTION
100% completed

Time: 00:00:00.22

Automaton has 12 states and 36 BDD nodes

ANALYSIS
Formula is valid
```

Trick 1 : Generalized Merkle tree

- MONA cannot describe that :

  “*a binary tree has the same depth*”

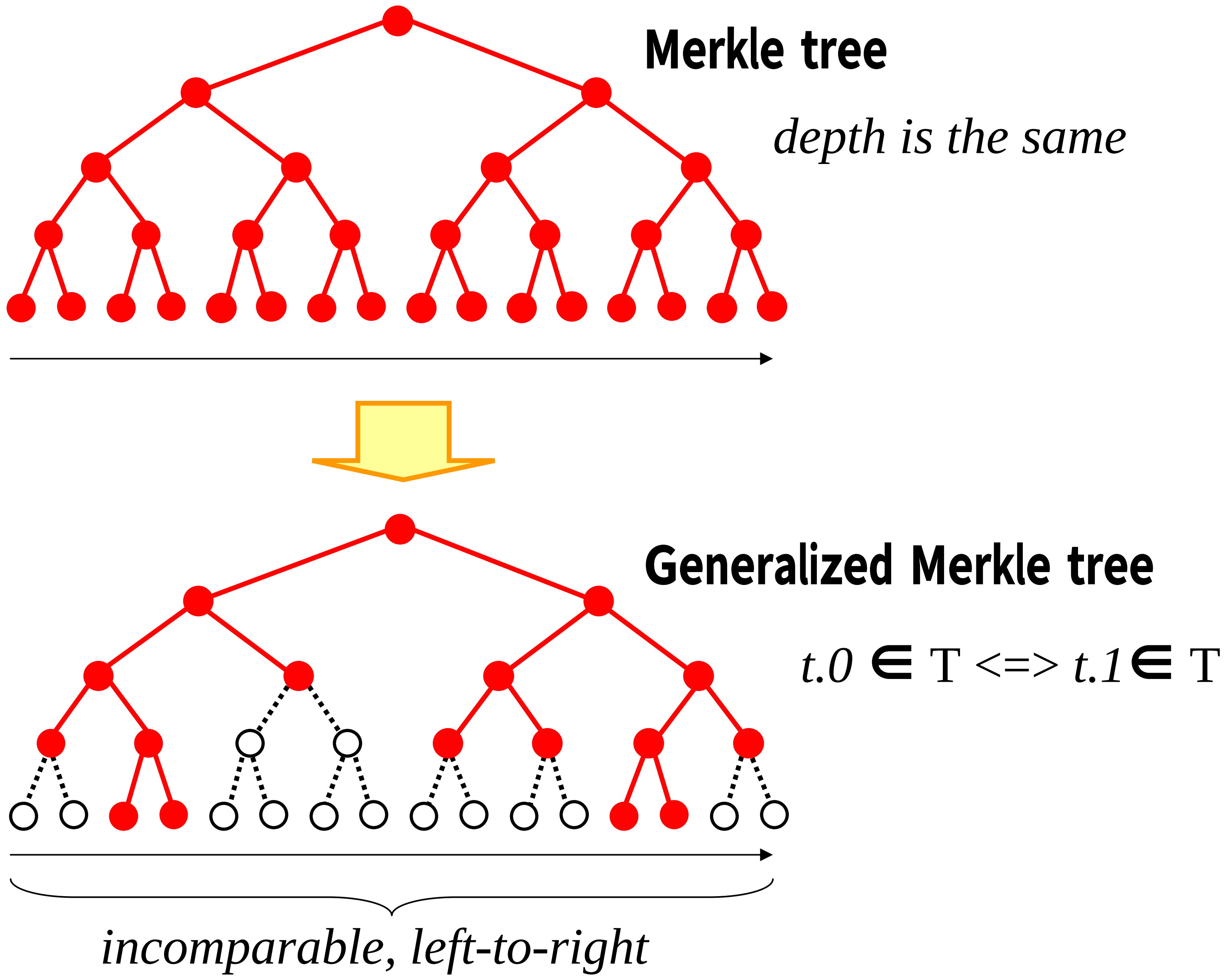
  (i.e., *each root path has the same length*)

- We have been implicitly assuming that :

  “*a Merkle tree has the same depth*”

- We relax “*the same depth*” to just “*don’t care depth*”.

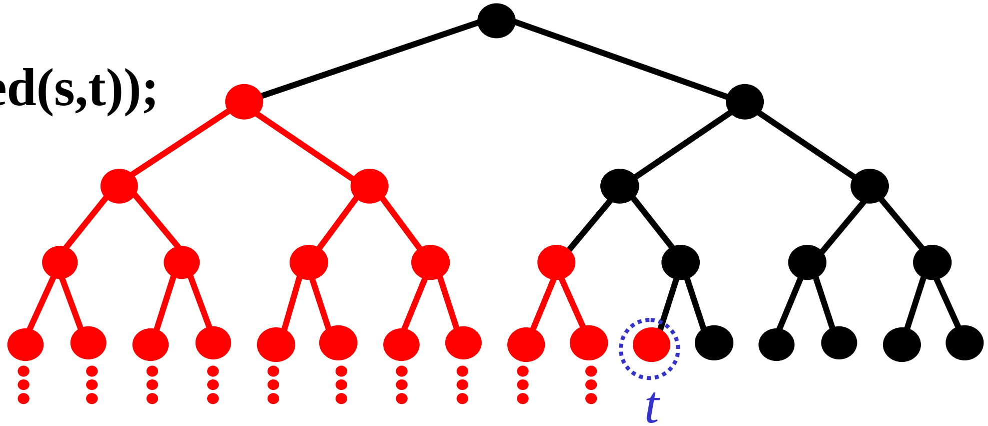
Generalized Merkle tree example



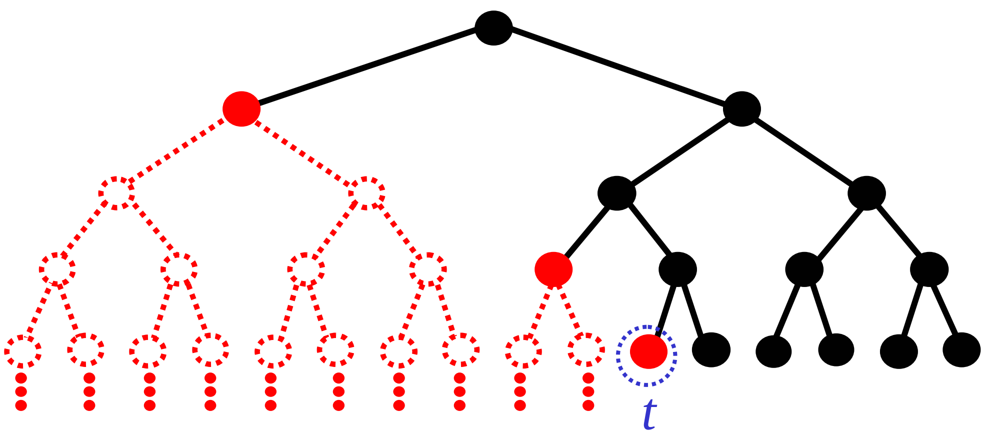
Trick 2: *Temporal slice* in WS2S

- First attempt : "*becomes an infinite set.*"

all1 s: (s in X <=> defined(s,t));

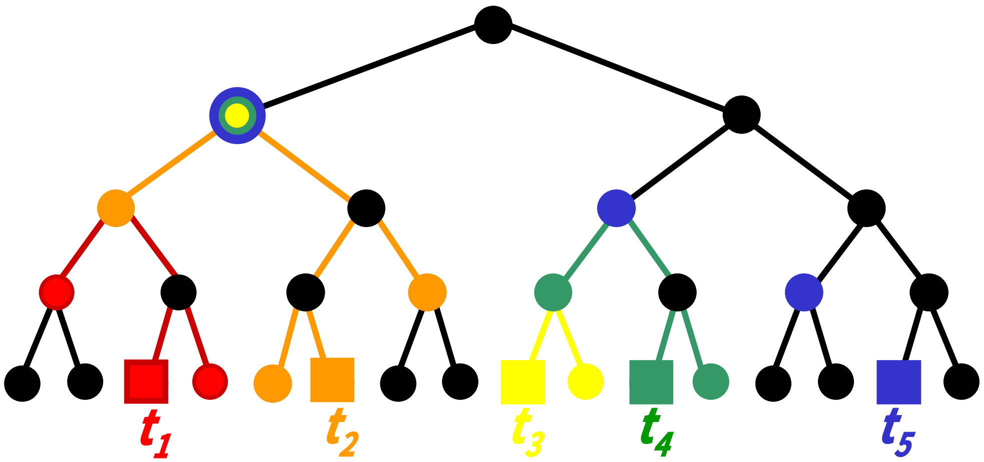


- Second attempt : "*temporal slice as its roots*."



Second challenge: Sanity Check

- Hash values are computed from different $\mathrm{LSR}_{t_{i+1}}(t_i)$'s. If multiple computations at each node coincide (i.e., consistent), *it suggests no internal-failures*.



Consistency

- Let (U_i, α_i) such that

  U_i : a set of incomparable nodes,

  α_i : labeling function on U_i

  (extended $\alpha_i(\mathrm{t}) = hash(\alpha_i(t.0), \alpha_i(t.1))$ on $\mathrm{Cls}(U_i)$)

- **Def.** $\{(U_i, \alpha_i)\}$ is *weakly consistent* if

  $\alpha_i(t) = \alpha_j(t)$ for each $t \in \mathrm{Cls}(U_i) \cap \mathrm{Cls}(U_j)$

- **Def.** $\{(U_i, \alpha_i)\}$ is *consistent* if α is well-defined for

$$\alpha(t) = \begin{cases} \alpha_i(t) & \text{when} \quad t \in U_i \\ hash(\alpha(t.0), \alpha(t.1)) & \text{when} \quad \neg\, t \in \mathrm{leaves}(\bigcup U_i) \end{cases}$$

Correctness of incremental sanity check

Cannot be described in WS2S

- **Th 2.** *If* $\{ (LSR_{t_{i+1}}(t_i), \alpha_i), (LS(t_{i+1}), \alpha_{i+1}) \}$ *is weakly consistent for each i with* $1 \leqq i < n$, $\{(LSR_{t_{i+1}}(t_i), \alpha_i) \mid 1 \leqq i < n\} \cup \{(LS(t_n), \alpha_n)\}$ *are consistent.*

Checked by large-scale experiment, but has not been proved !

- **Key Lemma**. Let $i+1 \leqq k \leqq j$. Then, $Cls(LSR_{t_{i+1}}(t_i)) \cap Cls(LSR_{t_{j+1}}(t_j)) \subseteq Cls(LS(t_k))$

Described in WS2S

(lefter(s,t) & (t = u | lefter(t,u)) & (u = v | lefter(u,v)) & (v = w | lefter(v,w)) & LSRclosure(s,t,X) & LSclosure(u,Y) & LSRclosure(v,w,Z)) => X inter Z sub Y;

Conclusion

- Case study of proving new properties of an event-ordering system developed by NTT, using MONA.
- Once clarified, they are not difficult; but when finding the *first* proofs (there are pitfalls), MONA assists very well.
- Found bug in “*where*”-sentence in WS2S mode of MONA:-)
- *Future work*: combine MONA & Isabelle/HOL, e.g., Th.2 (I have been saying this; but recent my focus is on Math…)

Thank you