Abstraction of programs in PML (Pointer Manipulation Language)

Koichi Takahashi
Yoshinori Tanabe
Toshifusa Sekizawa
Yoshifumi Yuasa
AIST

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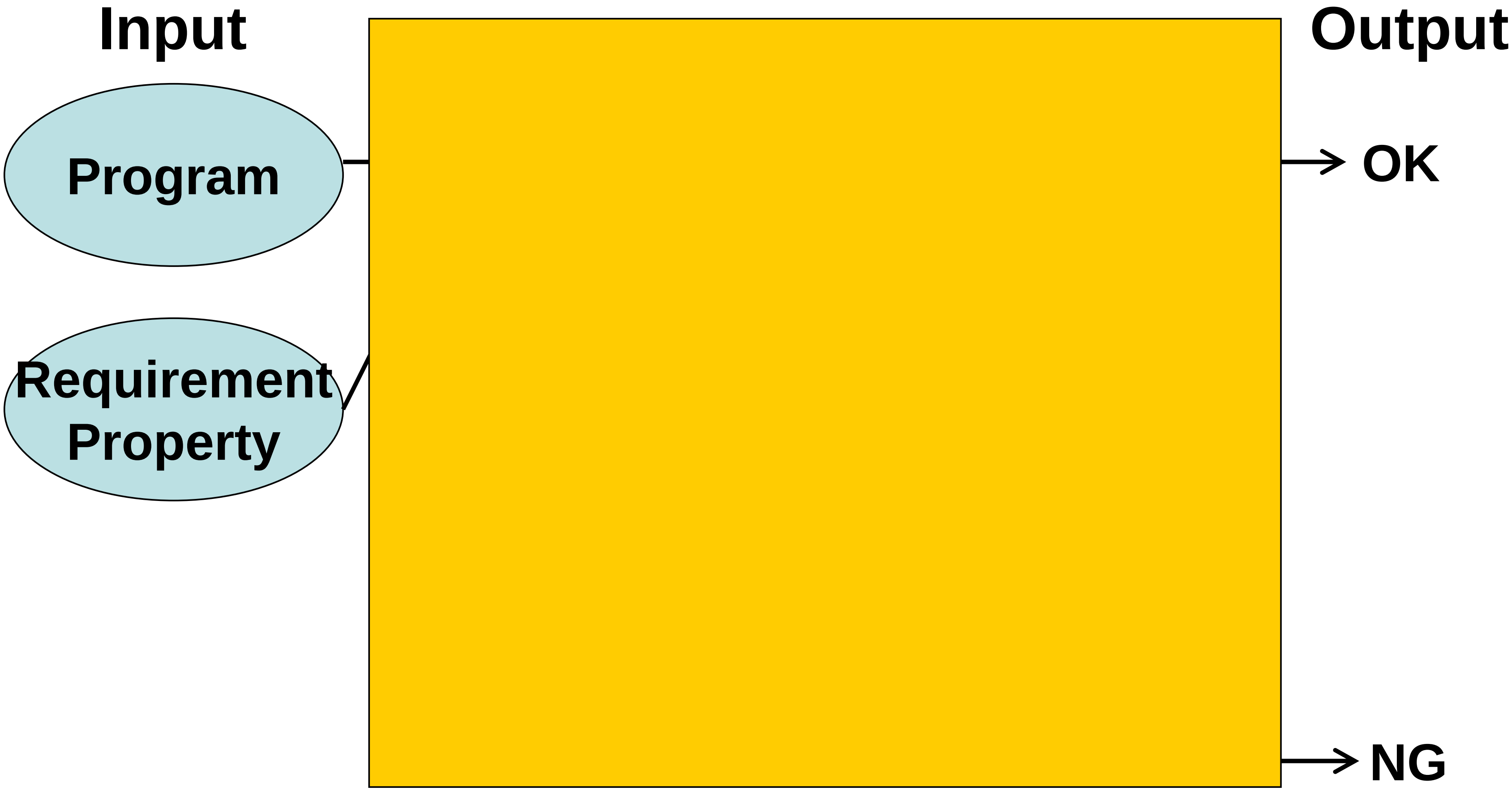
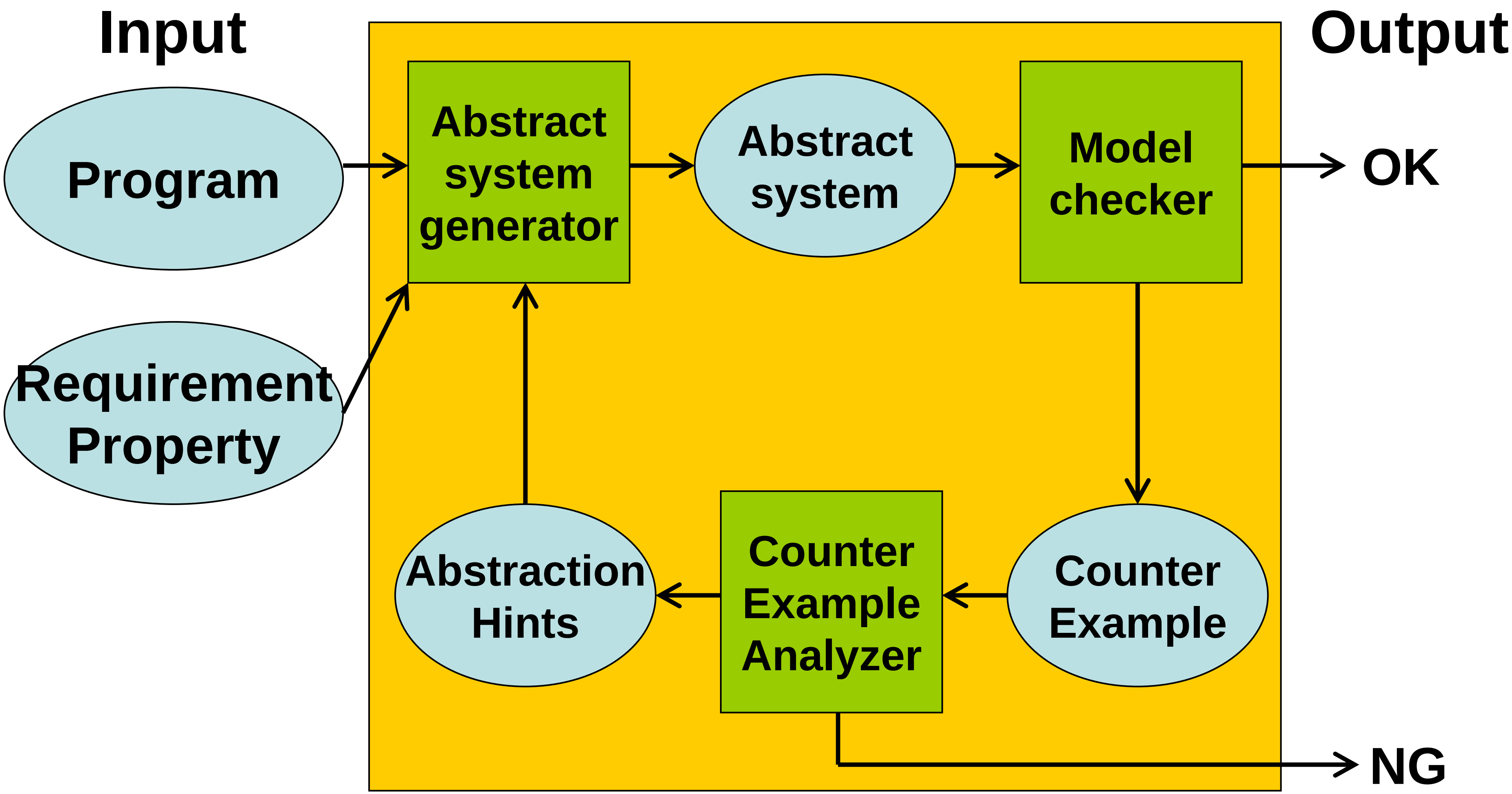
Overview

- Research Interest: Abstraction of graph transformation systems using modal logics.
  - Garbage Collection, Cellular Automata
- Automatic verification tool for pointer manipulation programs
  - Main issue: abstraction of heap
- Use of modal logic to describe heap
  - Seeds for predicate abstraction are described in modal formula
- Development of abstraction tool based on this idea

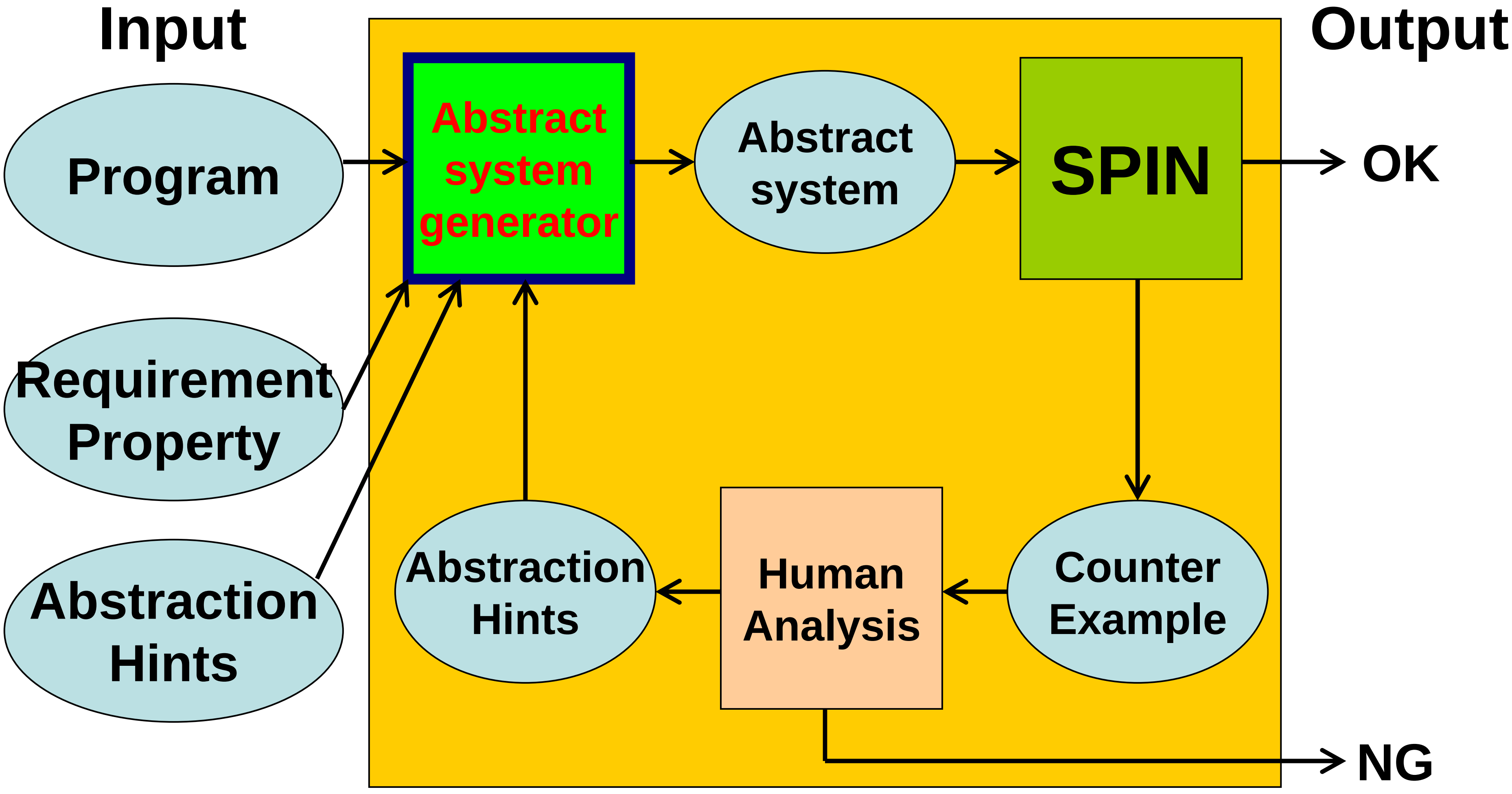
Whole picture



Whole picture



Current Development

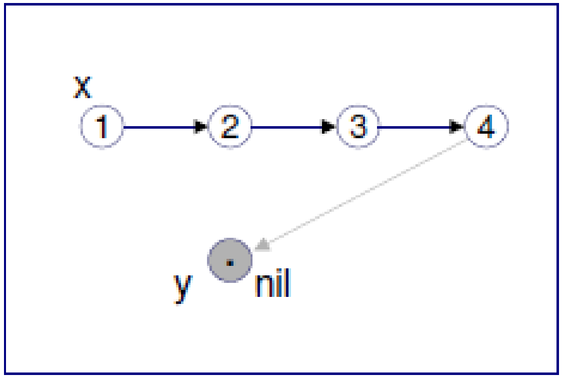


Idea

- Predicate Abstraction Framework
  - Most of tools developed in the early days handle properties on the value of variables as predicates used in abstraction
  - It was difficult to express properties on the shape of the heap of programs
- We use modal formulas as a method for abstracting heap structures
  - another idea: separation logic?

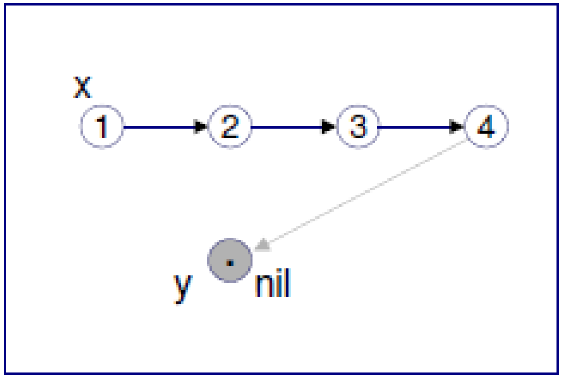
Model of Heap: Pointer Structure

- Heap consists of cells
- Each cell has a pointer and a value
  - to simplify explanation
- Pointer variables



Pointer Structure as Kripke Structure

- Pointer Structure can be seen as a Kripke structure
- Atomic propositions are values and variables



AP = {1,2,3,4,x,y,nil}

2CTL

$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathbf{E}_A\mathbf{X}\,\varphi \mid \mathbf{A}_A\mathbf{X}\,\varphi$
$\mid \mathbf{E}_A\mathbf{F}\,\varphi \mid \mathbf{A}_A\mathbf{F}\,\varphi \mid \mathbf{E}_A\mathbf{G}\,\varphi \mid \mathbf{A}_A\mathbf{G}\,\varphi$

where

p: atomic proposition,

$A \subseteq \mathsf{Mod}$: set of modality,

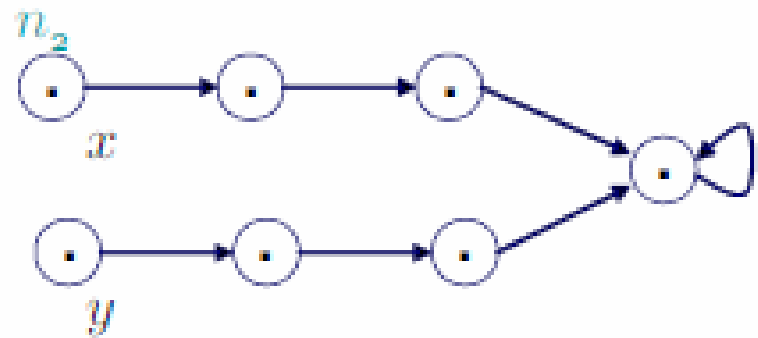
$\overline{a} \in \mathsf{Mod}$,

$\overline{\overline{a}} = a$ for $a \in \mathsf{Mod}$.

Properties

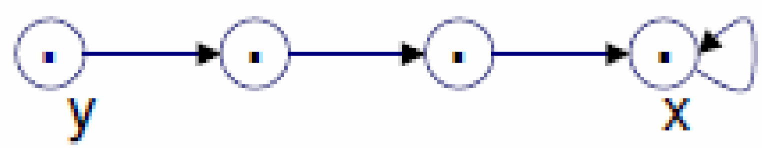
- Many properties of heap can be described
- Confluence

  $x \wedge \mathbf{E}_f\mathbf{F}\ \mathbf{E}_{\overline{f}}\mathbf{F}\ y$



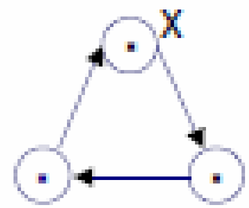
- Reachable
  - x is reachable from y

    $y \rightarrow \mathbf{EF}\ x$



- Loop
  - x is in loop

    $x \rightarrow \mathbf{EXEF}\ x$

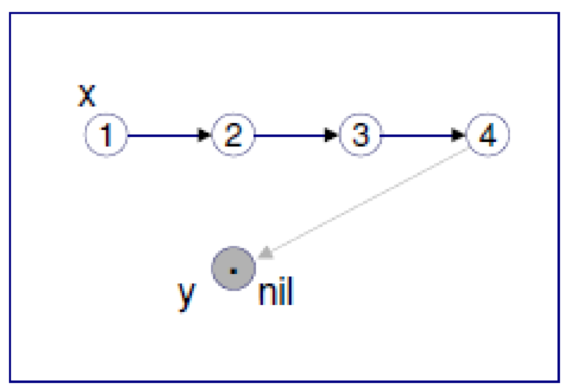


PML (pointer manipulation language)

- Target programs are written in PML
  - a tiny programming language manipulating heaps
- Statements are following:
  - **`x := y`**
  - **`x := y.next`**
  - **`x.next := y`**
  - **`x := new()`**
  - **`x.val := m`**
  - **`if (cond) goto`** ***`line`***
- Dynamic logic for PML?
  - ongoing

a PML program example

```
0: y := nil
1: if (x == nil) goto 7
2: t := y
3: y := x
4: x := x.next
5: y.next := t
6: goto 1
7: (end)
```



0: y=nil

a PML program example

```
0: y := nil
1: if (x == nil) goto 7
2: t := y
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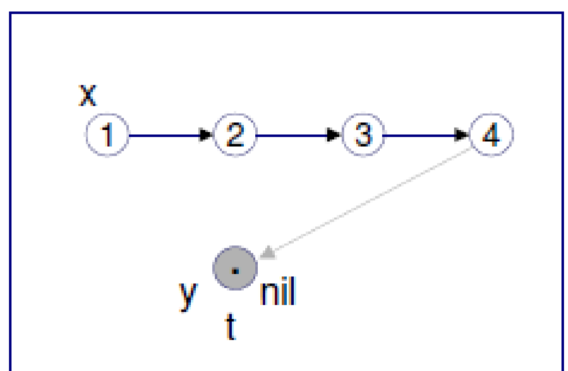
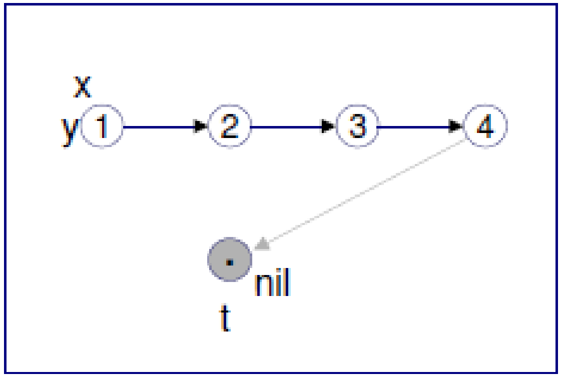


2: t=y

a PML program example

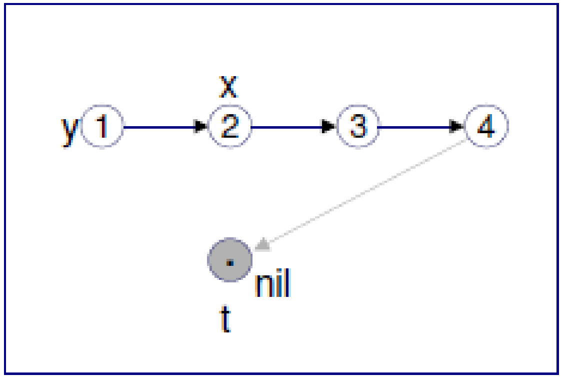
```
0: y := nil
1: if (x == nil) goto 7
2: t := y
3: y := x
4: x := x.next
5: y.next := t
6: goto 1
7: (end)
```



3: y=x

a PML program example

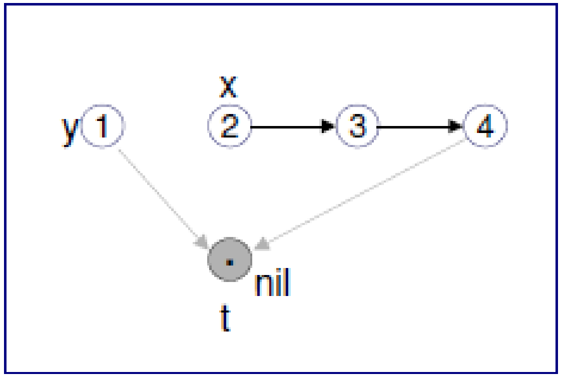
```
0: y := nil
1: if (x == nil) goto 7
2: t := y
3: y := x
4: x := x.next
5: y.next := t
6: goto 1
7: (end)
```



4: x=x.next

a PML program example

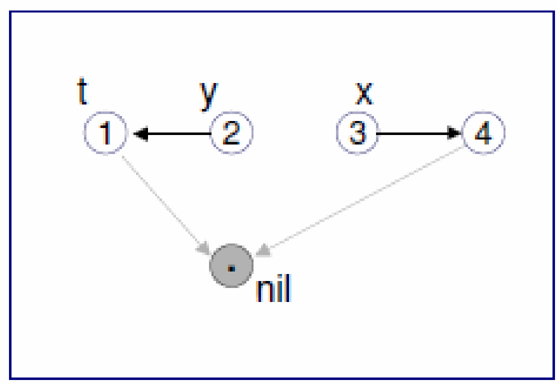
```
0: y := nil
1: if (x == nil) goto 7
2: t := y
3: y := x
4: x := x.next
5: y.next := t
6: goto 1
7: (end)
```



5: y.next=t

a PML program example

```
0: y := nil
1: if (x == nil) goto 7
2: t := y
3: y := x
4: x := x.next
5: y.next := t
6: goto 1
7: (end)
```



5: y.next=x

a PML program example

```
0: y := nil
1: if (x == nil) goto 7
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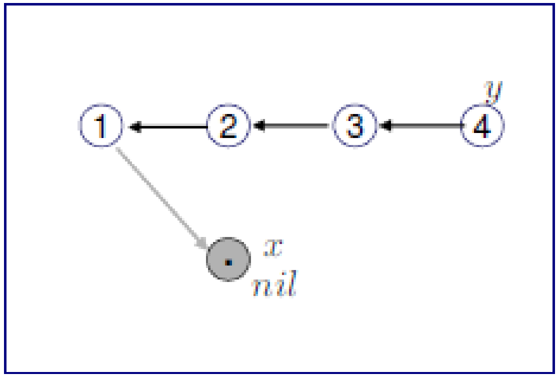


7: (end)

a verification example

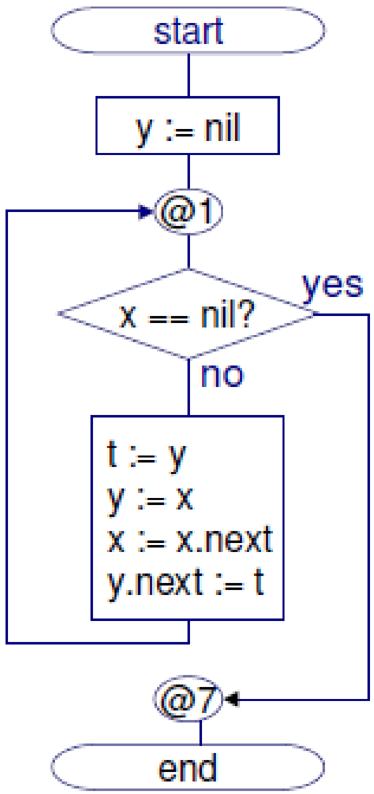
```
0: y := nil
1: if (x == nil) goto 7
2: t := y
3: y := x
4: x := x.next
5: y.next := t
6: goto 1
7: (end)
```

- Verification statement:
  If a node is reachable from *x* at line 1,
  then the node is reachable from *y* at line 7.

$$Q_1 = x \rightarrow \mathbf{EF}\, u \qquad Q_2 = y \rightarrow \mathbf{EF}\, u$$

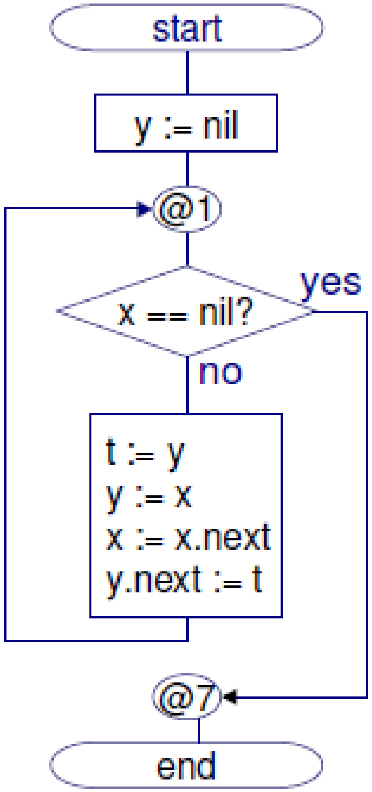
$$Q_1 \text{ holds at line 1} \Rightarrow Q_2 \text{ holds at line 7.}$$

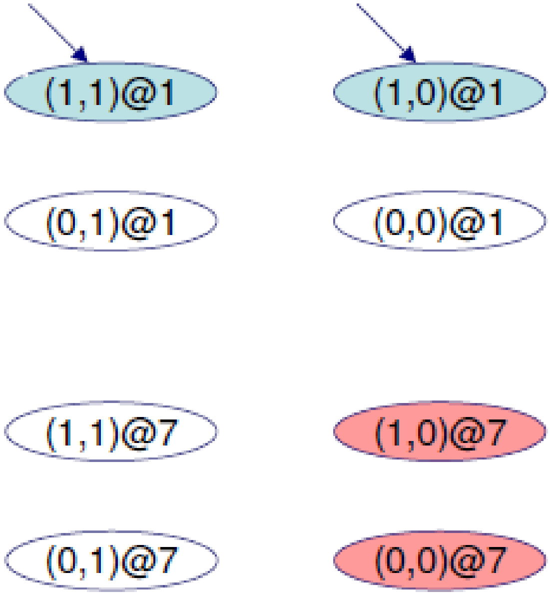
a verification example



$$Q_1 = x \to \mathbf{EF}\, u \qquad Q_2 = y \to \mathbf{EF}\, u$$

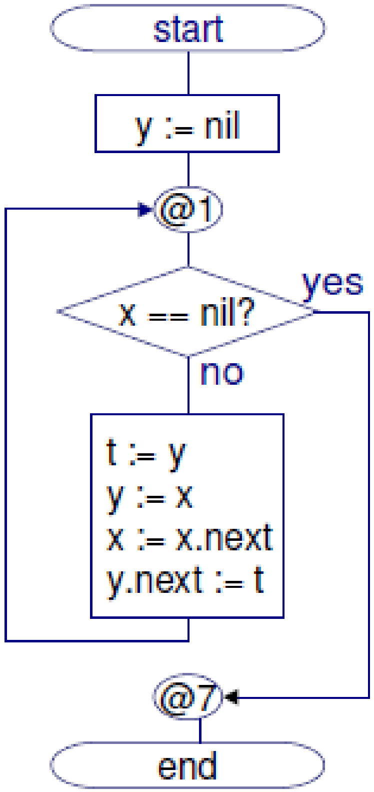
a verification example

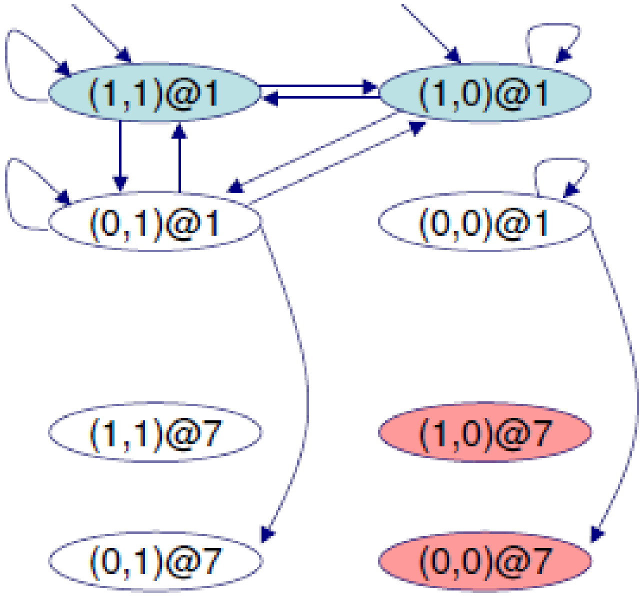




$$Q_1 = x \rightarrow \mathbf{EF}\ u \qquad Q_2 = y \rightarrow \mathbf{EF}\ u$$

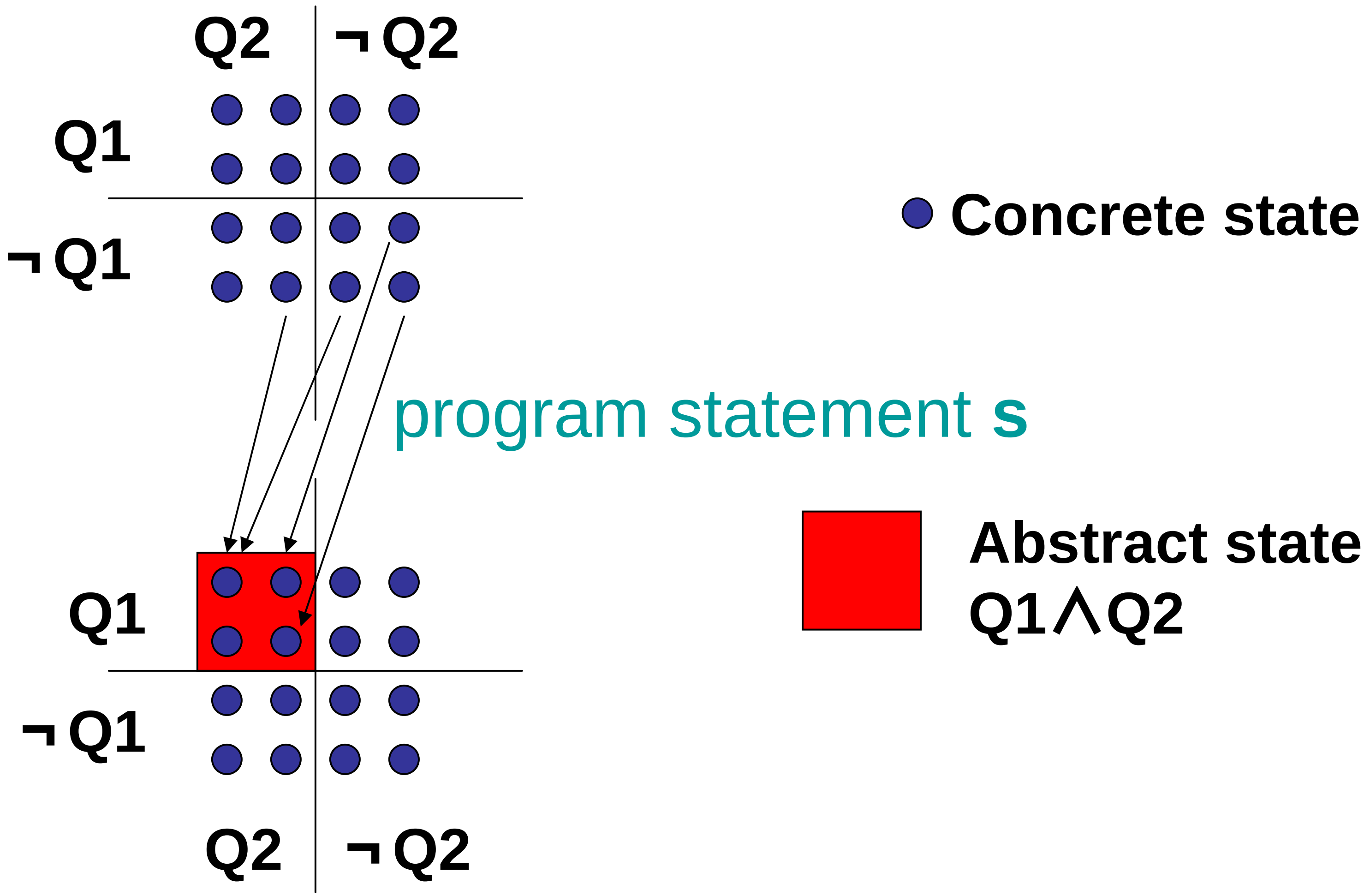
a verification example



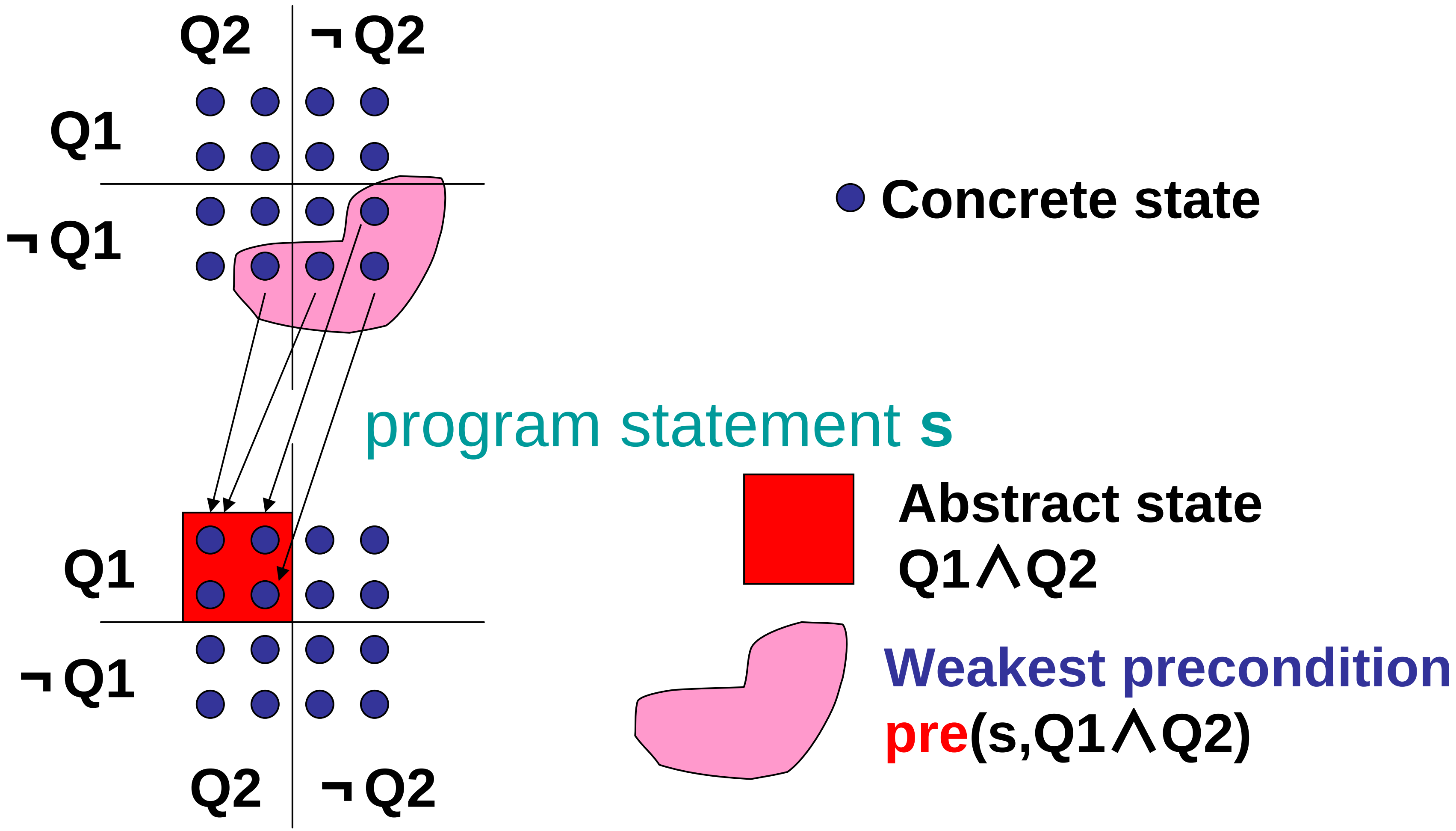


$$Q_1 = x \rightarrow \mathbf{EF}\, u \qquad Q_2 = y \rightarrow \mathbf{EF}\, u$$

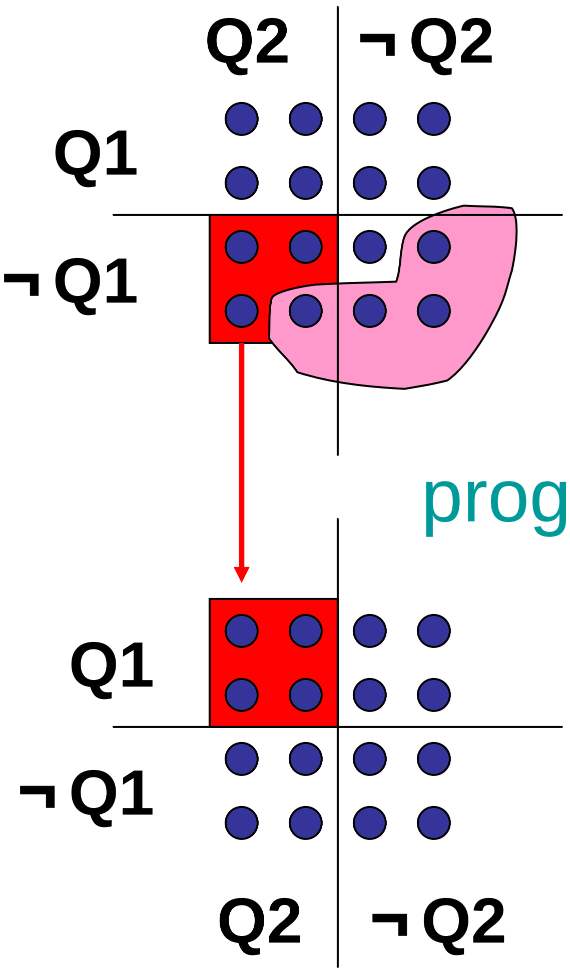
Compute abstract transition



Compute abstract transition



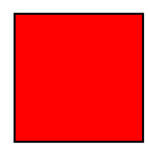
Compute abstract transition



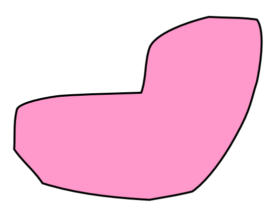
Abstract transition ⇔
Intersection ⇒
sat(¬ Q1∧Q2∧pre(s,Q1∧Q2)) = 1

Concrete state

program statement **s**



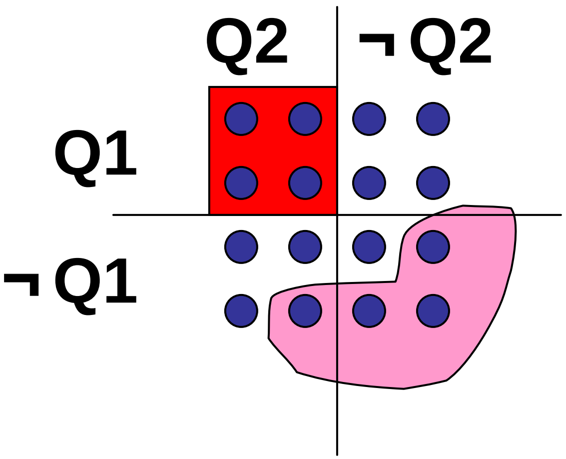
Abstract state
Q1∧Q2

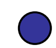


Weakest precondition
pre(s,Q1∧Q2)

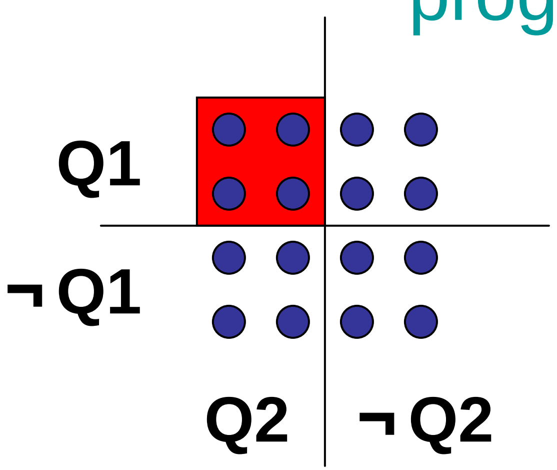
Compute abstract transition

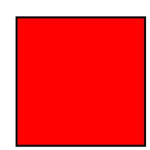
No abstract transition ⇔
Disjoint ←
sat($Q1 \wedge Q2 \wedge pre(s, Q1 \wedge Q2)$) = 0



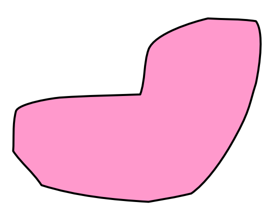
 **Concrete state**

program statement **s**





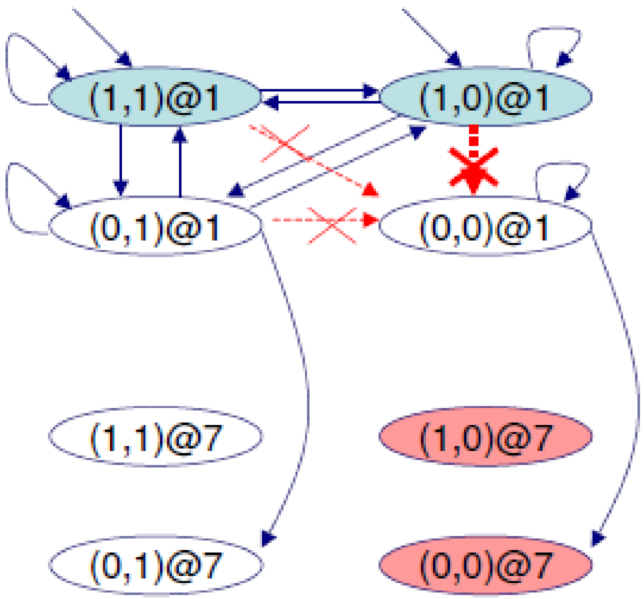
Abstract state
$Q1 \wedge Q2$



Weakest precondition
$pre(s, Q1 \wedge Q2)$

Our case

- Weakest precondition of 2CTL for PML
- Sat checker for 2CTL



$$Q_1 = \forall(x \rightarrow \mathbf{EF}\ u)$$
$$\neg Q_2 = \forall(y \rightarrow \neg\mathbf{EF}\ u)$$

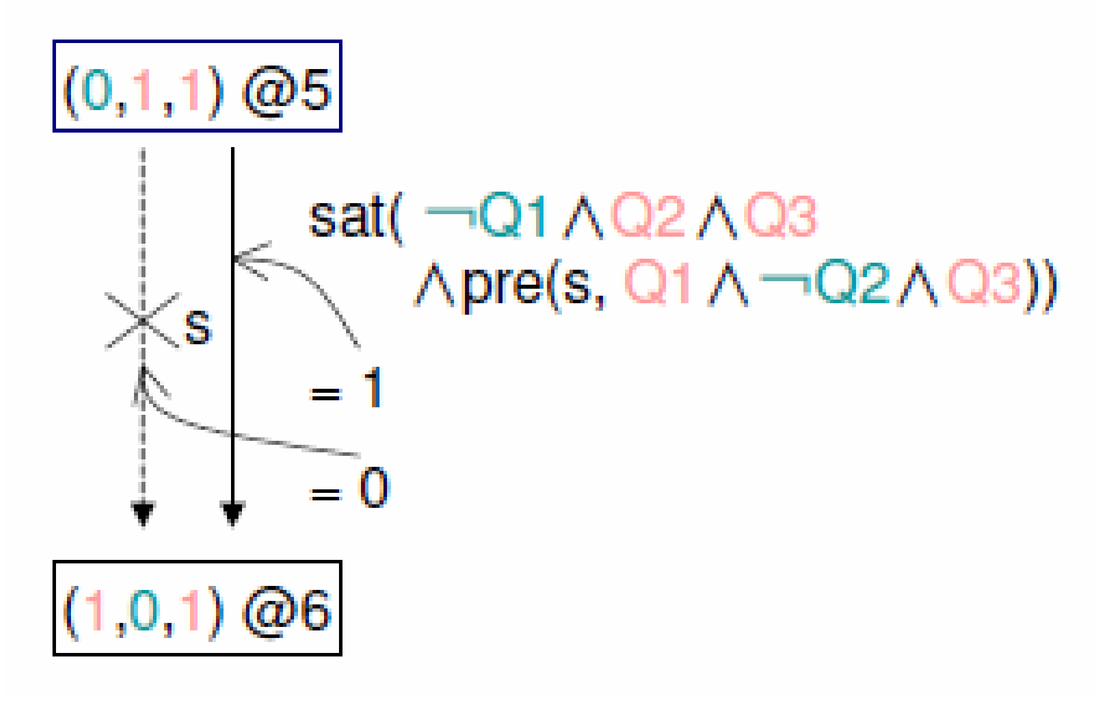
$$\mathsf{pre}(s, \neg Q_1) =$$
$$\forall(x \rightarrow \mathbf{EF}\ \mathbf{EX}x) \wedge \forall(u \rightarrow \neg(x \vee \mathbf{E}\ (\neg x\ \mathbf{U}\ (\neg x \wedge (\mathbf{EX}x \vee y)))))$$
$$\vee\ \ \forall(x \rightarrow \neg\mathbf{EF}\ \mathbf{EX}x) \wedge \forall(u \rightarrow \neg\mathbf{EF}\ \mathbf{EX}x)$$

$$\mathsf{pre}(s, \neg Q_2) = \forall(u \rightarrow \neg(x \vee \mathbf{E}\ (\neg x\ \mathbf{U}\ (\neg x \wedge y))))$$

$(s = $"`t:=y; y:=x; x:=x.next; y.next:=t`"$)$

Compute abstract transition

- Precondition and
- Satisfiability checking



Precondition

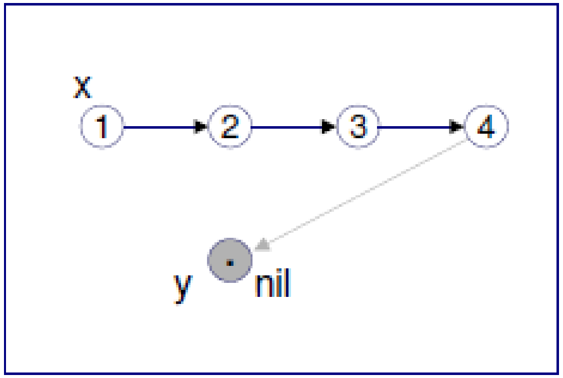
- We restrict 2CTL to p-formula for simplification
  - It is enough to describe important properties
- We have calculated weakest precondition of p-formula for each PML statement
  - weakest precondition of p-formula is p-formula
  - We are now implementing

Satisifiability check

- Usual: **ϕ** is sat ⇔ there exists a Kripke structure K s.t. K |= **ϕ**
  - this checking is too rough
  - previous verification example does not work

Pointer Structure as Kripke Structure

- Pointer Structure can be seen as a Kripke structure
- Atomic propositions are values and variables



AP = {1,2,3,4,x,y,nil}

- **Variable property holds at most one node**
- **A node has at most one next node**

Satisifiability check

- Usual: $\boldsymbol{\phi}$ is sat $\Leftrightarrow$ there exists a Kripke structure K s.t. K $\models$ $\boldsymbol{\phi}$
- Our modification: $\boldsymbol{\phi}$ is sat $\Leftrightarrow$ there exists a Pointer structure P s.t. P $\models$ $\boldsymbol{\phi}$
  - more accurate
    - previous verification example works
  - We are now implementing
    - BDD

Current Development