Visibly Stack Automata

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Talk Outline

- Automata-theoretic based verification
  - Checking context-free specifications
  - Obstacles
- Visibly Pushdown Automata
  - Visibly Pushdown Languages
  - Determinization
- Visibly Stack Automata
  - Visibly Stack Languages
  - Determinization

Automata-theoretic based verification

- To verify if a software system satisfies a regular specification
  - System is modeled as a pushdown automaton M
  - Requirement is specified as a finite automaton S
- M |= S iff:
  - $L(M) \subseteq L(S)$
  - $L(M) \cap L(S)^C = \varnothing$
- Model checking problems are reduced to decision problems of formal languages
- Model checker: SPIN, MAGIC,..

Checking Context-free Specifications

- When S is a context-free specification
- Checking $M|= S$ becomes undecidable

- Obstacles
  - Context-free languages (CFL) are not closed under intersection, complementation
  - The inclusion problem of CFL is undecidable
- Goals: Find a class of non-regular languages
  - Enjoys closure properties, Inclusion problem is decidable
  - Robustness

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Pushdown Automata (PDA)

- Def. PDA $(P,\Sigma,\Gamma,\Delta,p_0,Z_0,F)$ where
  - P : finite control locations
  - Γ : finite stack alphabet
  - Σ : finite input alphabet
  - $\Delta \subseteq (P \times \Gamma \times (\Sigma \cup \{\varepsilon\}) \times (P \times \Gamma^*))$: transition
  - p_0 : initial control location
  - Z_0: initial stack symbol
  - $F \subseteq P$:final control locations
- Accepted ⇔ run reaches some control location in F

- PDA are not determinizable. PDA are closed under union, but not closed under intersection, complementation

Visibly Pushdown Automata (VPA)

- VPA was introduced by J. Alur and P.Madhusudan in 2004, [p.202-211, ACM-STOC's 04]
- Pushdown alphabet: partitioned into 3 disjoint sets

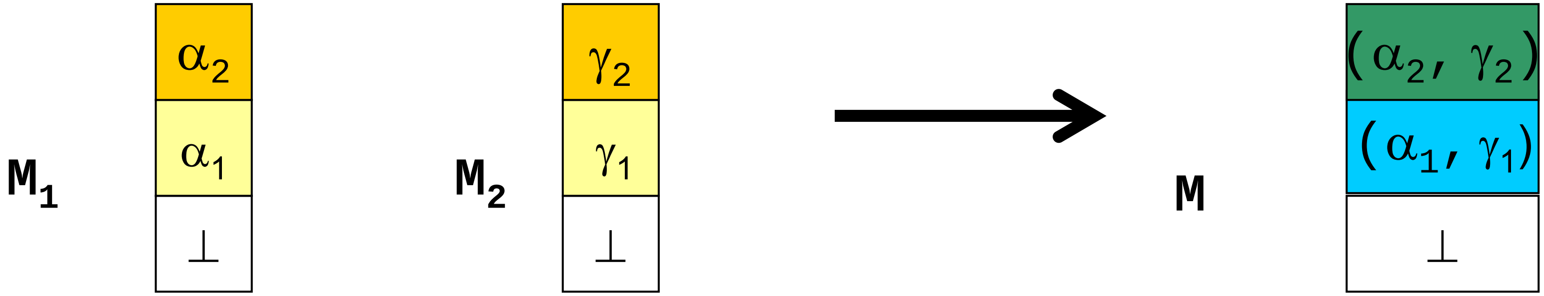
$$\Sigma = \Sigma_{push} \cup \Sigma_{pop} \cup \Sigma_{local}$$

- A *visibly pushdown automaton* over a pushdown alphabet Σ is a pushdown automaton that
  - pushes a symbol onto the stack on a symbol in Σ_{push}
  - pops the stack on a symbol in Σ_{pop}
  - cannot change the stack on a symbol in Σ_{local}

Key: Stack size at any time is determined by the input word but not control state or stack content

Visibly Pushdown Languages (VPL)

- A language L is a VPL over a pushdown alphabet Σ, if it is recognized by a VPA
- Examples
  - $L = \{ a^n b^n \mid n \geq 1, a \in \Sigma_{push}, b \in \Sigma_{pop} \}$
  - Every regular language *L* is a VPL
- VPLs are closed under:
  - Union:
  - Intersection: Product construction works !
  - Complementation (see later)

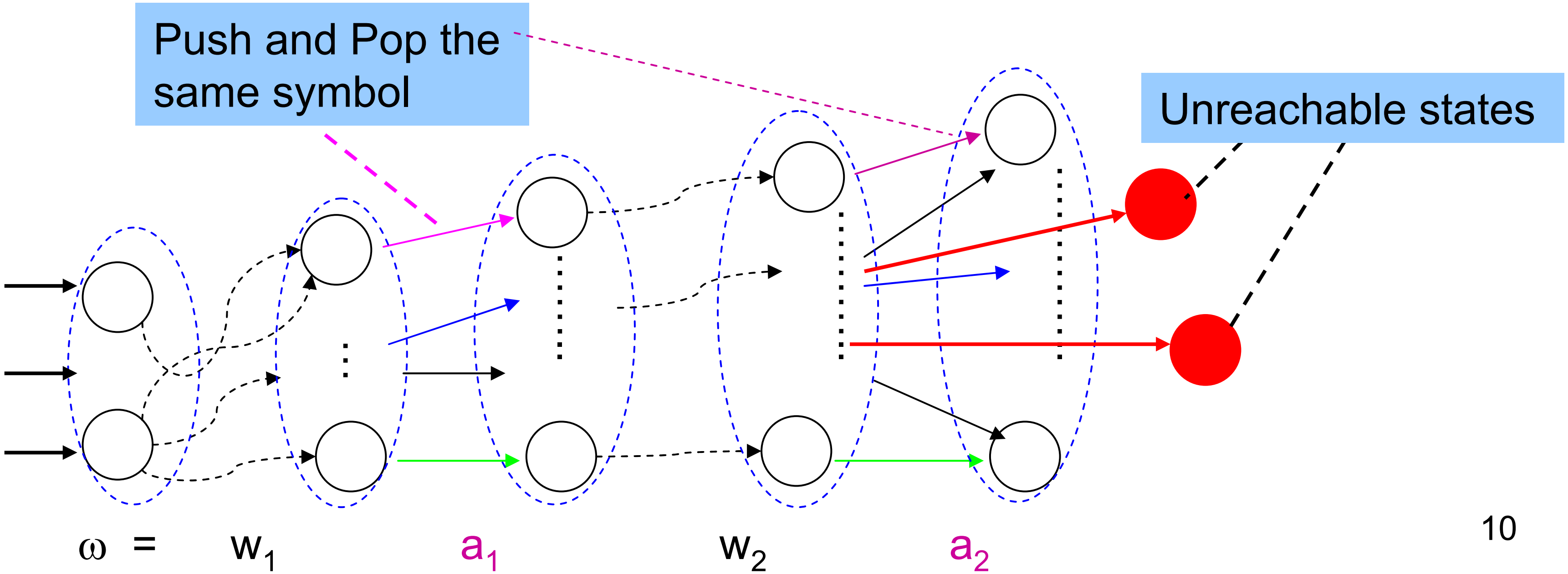


Decision Problems

- Given a nondeterministic *A* (n states), construct an equivalent deterministic *B* in size $O(2^{n^2})$.
- Emptiness: Decidable in polynomial-time (cubic)
- Language inclusion: $L(A) \subseteq L(B)$?
  - Determinize *B*, take its complement, take product with *A*, and test for emptiness
  - Exponential-time complete
- VPLs is a subclass of DCFLs (languages defined by deterministic PDAs)
  - DCFLs not closed under union, intersection
  - Equivalence problem for DCFLs decidable, but complex

Determinization: Key ideas

- Def. A word *u* is *well-matched* if
  - For each prefix u' of u, the number of Σ_{push} symbols in u' is at most the number of Σ_{pop} symbols in u'.
  - For each suffix u'' of u, the number of Σ_{pop} symbols in u'' is at most the number of Σ_{push} symbols in u''.



Determinization: Sketch of the construction

- Idea: a well-matched word preserves stack; thus regarded as internal transition (expressed as summaries S_i). Transitions by extra Σ_{push} symbols are postponed until corresponding Σ_{pop} symbols will be read.

- Determinized VPA will consist of :
  - Control locations : { (S,R) | S: summary, R: reachables}
  - Stack alphabet : { (S,R,a) | S, R; $a \in \Sigma_{push}$}
  - The initial state (Id, P_{init}), where Id = {(q,q)| q$\in$ P}
  - Final states { (S,R) | $R \cap F \neq \varphi$},
  - where
    - R ($\subseteq$ P) = { all states reachable after a word w }
    - S ($\subseteq$ P×P) = { all summaries on a well-marched word w }
      (i.e., $(q,q') \in S$, if $(q,\perp)$ can reach to $(q',\perp)$).

Determinization: Sketch of transitions

- Let $w = w_1 c_1 w_2 c_2 \ldots c_n w_{n+1}$, where c_i's are in Σ_{push} , w_i's are well matched words, let a be the next input.
  - Stack is $(S_n, R_n, c_n) \ldots (S_1, R_1, c_1) \bot$
  - Control location is (S_{n+1}, R_{n+1}),
  - If $a \in \Sigma_{\text{local}}$: (S_{n+1}, R_{n+1}) is combined with transitions by a.
  - If $a \in \Sigma_{\text{push}}$: push $(S_{n+1}, R_{n+1}, c_{n+1})$ and control location is (id, { q' | reachable from $q \in R_{n+1}$ by a})
  - If $a \in \Sigma_{\text{pop}}$: let *Update* be combination of transitions by c_n (push $\gamma \in \Gamma$), S_{n+1}, and transitions by a (pop same γ). *NewS* is S_n combined with *Update*, and *NewR* is R_n combined with *Update* (R_{n+1} is discarded).
  - where
  - R_i ($\subseteq$ P) = Set of all states reachable after $w_1 c_1 \ldots w_i$
  - S_i ($\subseteq$ P×P) = Set of all summaries on w_i

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Stack Automata

- Stack automata: introduced by Ginsburg, Greibach and Harrison [JACM, No 2, Vol 14, pp.389-418, 1967]
- Stack automata = PDA + "read inside stack".
- More powerful than PDA. For instance, $\{a^n b^n c^n | n \geq 1\}$, $\{a^n b^{n^2} | n \geq 1\}$
- Def. Stack alphabet: $\Sigma = \Sigma_{push} \cup \Sigma_{pop} \cup \Sigma_{local} \cup \Sigma_{up} \cup \Sigma_{down}$

Visibly Stack Automata (VSA) (1/2)

- Def. A VSA A over stack alphabet Σ: A = $\langle P, P_{in}, \Gamma, \uparrow, \delta, F \rangle$
  - P : finite set of control locations
  - $P_{in} \subseteq P$: set of initial control locations
  - Γ : finite stack alphabet, special symbols $\perp$, T
  - $\uparrow$: stack pointer
  - $F \subseteq P$: set of final control locations
  - δ is a set of transitions $\langle \delta_{puah}, \delta_{pop}, \delta_{local}, \delta_{up}, \delta_{down} \rangle$,

$\delta_{push} \subseteq P \times \Sigma_{push} \times P \times \Gamma \backslash \{\perp, T\}$; $\delta_{pop} \subseteq P \times \Sigma_{pop} \times \Gamma \times P$;
$\delta_{local} \subseteq P \times \Sigma_{local} \times P$;$\delta_{down} \subseteq P \times \Sigma_{down} \times \Gamma \times P$; $\delta_{up} \subseteq P \times \Sigma_{up} \times \Gamma \times P$;

- $(q,a,\gamma,q') \in \delta_{up}$ $(\gamma \neq \perp, T)$ $\Leftrightarrow$ $(q,a,\gamma',q') \in \delta_{up}$ $(\gamma' \neq \gamma, \perp, T)$
- $(q,a,\gamma,q') \in \delta_{down}$ $(\gamma \neq \perp, T)$ $\Leftrightarrow$ $(q,a,\gamma',q') \in \delta_{down}$ $(\gamma' \neq \gamma, \perp, T)$

Visibly Stack Automata (2/2)

- Properties:
  - Stack has form $T\gamma_n \ldots \gamma_i \uparrow \gamma_{i-1} \ldots \gamma_1 \perp$
  - VSA can only push, pop when stack pointer at the top
  - When stack is empty, pop is read but not popped
  - Stack pointer cannot go beyond T or below $\perp$
  - When pointer is reading $\perp$ (T), down (up) is read but pointer does not move down (up)

- Def. A language L is a visibly stack language (VSL) if it is accepted by a VSA.
- Example: $L=\{a^n b^n c^n \mid n \geq 1,\ a \in \Sigma_{push}, b \in \Sigma_d, c \in \Sigma_u\}$
- VSL class is a proper extension of VPL class

Closure properties & Decision problems

- Similar to VPLs, VSLs are closed under:
  - Union, intersection, complementation
  - Determinizable (more complicated, see later)

- Emptiness for stack automata is decidable!

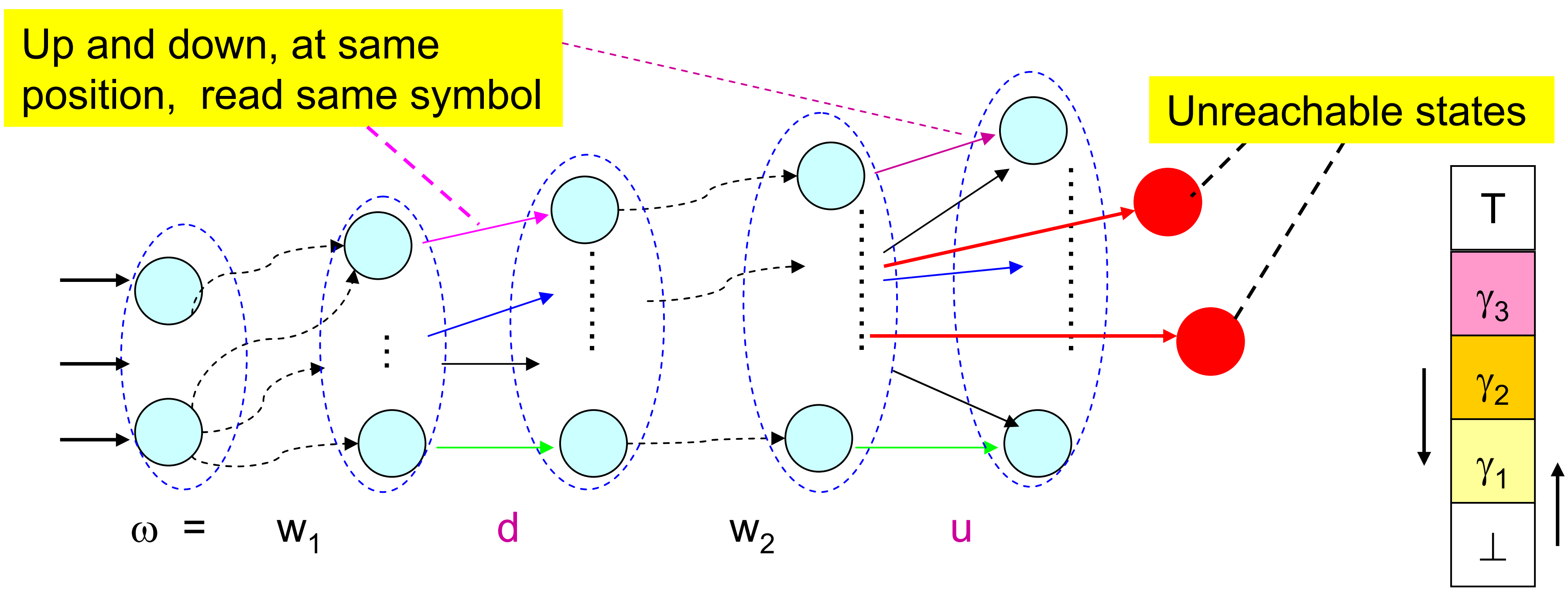
  D. Harel, Information and Computation, Vol.113, No.2, 278-299, 1994

- Inclusion problem is decidable for visibly stack languages.

Determinization: Key ideas

- Matching condition between push and pop symbols

- Read the same symbol, whenever pointer goes up, or goes down at the same position



Determinization: Some definitions

- Def. A word u is an *up-down* segment if:
  1. For each prefix *u′* of *u*, the number of Σ_{down} symbols in *u′* is at most the number of Σ_{up} symbols in *u′*.
  2. For each suffix *u″* of *u*, the number of Σ_{up} symbols in *u″* is at most the number of Σ_{down} symbols in *u″*.
  3. There is no push, pop symbols in u
- Def. A word u is *well-matched* if:
  1. For each prefix *u′* of *u*, the number of Σ_{push} symbols in *u′* is at most the number of Σ_{pop} symbols in *u′*.
  2. For each suffix *u″* of *u*, the number of Σ_{pop} symbols in *u″* is at most the number of Σ_{push} symbols in *u″*.
  3. For each up (down) symbol *a* of u, *a* must belongs to an *up-down* segment *ud*, *ud* is a subword of u.

Determinization: Sketch of the construction

- Determinized VSA will consist of :
  - Control locations : { (S,R) | S: summary, R: reachables}
  - Stack alphabet : { (S,R,a) | S, R; $a \in \Sigma_{push}$}
  - The initial state (Id, P_{init}), where $Id = \{(q,q) \mid q \in P\}$
  - Final states $\{ (S,R) \mid R \cap F \neq \varphi \}$,

  – where
  – R ($\subseteq$ P) = {all states reachable after a word w}
  – S ($\subseteq$ P×P) = {all summaries on a well-marched word w}
    (i.e., $(q,q') \in S$, if $(q, T\uparrow\perp)$ can reach to $(q', T\uparrow\perp)$).

Determinization: Sketch of transitions

- Let $w = w_1c_1w_2c_2 \ldots c_nw_{n+1}$, where c_i's are in Σ_{push}, w_i's are well matched words, let a be the next input.
  - Stack is $T\uparrow(S_n,R_n,c_n)\ldots(S_1,R_1,c_1)\perp$
  - Control location is (S_{n+1},R_{n+1}),
  - If $a \in \Sigma_{local}$: (S_{n+1},R_{n+1}) is updated using subset construction with transitions by a.
  - If a $\in \Sigma_{down}$: (S_{n+1},R_{n+1}) is updated with transition by a. Stack now is $T(S_n,R_n,c_n)\uparrow\ldots(S_1,R_1,c_1)\perp$
  - If a $\in \Sigma_{up}$: (S_{n+1},R_{n+1}) is updated with transitions by a, the pointer cannot go up. Stack stays unchanged, $T\uparrow(S_n,R_n,c_n)\ldots(S_1,R_1,c_1)\perp$

  - where
  - $R_i (\subseteq \mathrm{P})$ = Set of all states reachable after $w_1c_1\ldots w_i$
  - $S_i (\subseteq \mathrm{P}\times\mathrm{P})$ = Set of all summaries on w_i

Determinization: Sketch of transitions

- If $a \in \Sigma_{push}$: push $(S_{n+1}, R_{n+1}, c_{n+1})$ and control location is (id, { q' | reachable from $q \in R_{n+1}$ by a}). Stack now is $T\uparrow(S_{n+1},R_{n+1},c_{n+1})(S_n,R_n,c_n)\ldots(S_1,R_1,c_1)\perp$

- If $a \in \Sigma_{pop}$: let *Update* be combination of transitions by c_n (push $\gamma \in \Gamma$), S_{n+1}, and transitions by a (pop same γ). *NewS* is S_n combined with *Update*, and *NewR* is R_n combined with *Update* (R_{n+1} is discarded). Stack now is $T\uparrow(S_{n-1},R_{n-1},c_{n-1})\ldots(S_1,R_1,c_1)\perp$
  - where
  - R_i ($\subseteq$ P) = Set of all states reachable after $w_1 c_1 \ldots w_i$
  - S_i ($\subseteq$ P×P) = Set of all summaries on w_i

Conclusion

- Proposed the class of visibly stack languages recognized by visibly stack automata
- To our knowledge, to date, VSLs is the largest class which enjoys closure properties. All the decision problems are decidable for VSA.
- Infinite words:
  - Visibly Büchi pushdown automata [AM04]
  - Visibly Büchi stack automata
  - Closure properties, not determinizable, but language inclusion is still decidable!

Thank for your attention !