Simulation Theorems in Multi-valued Modal μ -Calculus

Koki Nishizawa (AIST/CVS)
Yukiyoshi Kameyama (Univ. of Tsukuba)
Yoshiki Kinoshita (AIST/CVS)

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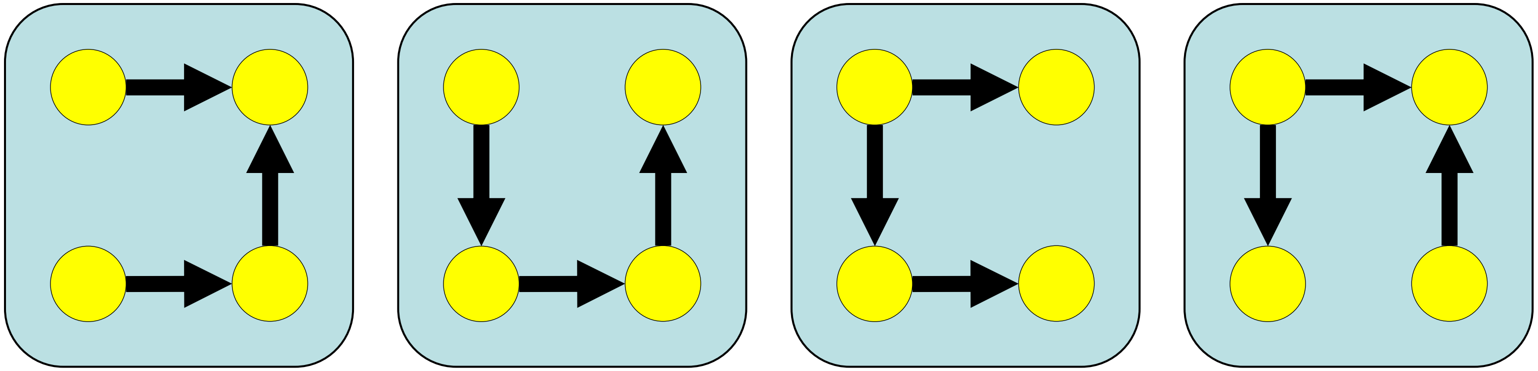
Motivation

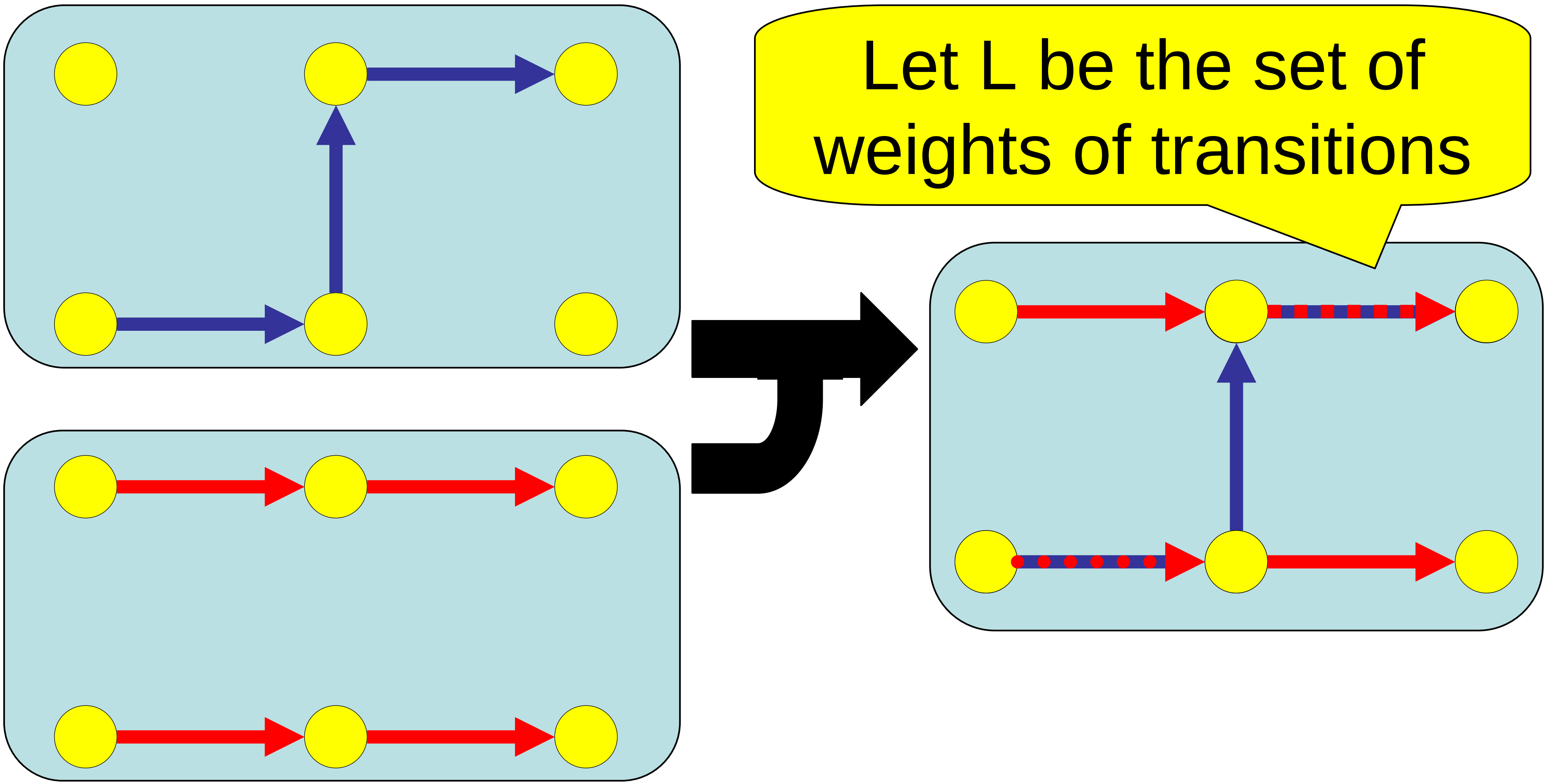
"Refinement of Models" in Model Checking

- Model Checking = Modeling + Checking
- Tatsumi and Kameyama tried to get minimal one among models checked successfully.
- They needed a number of model checking.



They wanted to perform a number of model checking all at once.

Superposition of Models



From 2={T,F} to general L

- Transition System, Kripke Model, Simulation
- State semantics of Modal μ -Calculus, Simulation Theorem
  - – De Morgan algebra [Tatsumi-Kameyama 2006]
  - – Complete Heyting algebra [This talk]
- Path semantics of Linear Modal μ -calculus, Simulation Theorem
  - – Complete Heyting algebra + condition [This talk]

Why complete Heyting algebra ?

- Sets and binary relations form a category.
- L must be a complete Heyting algebra for sets and binary L-valued relations to form a category [Johnstone 2002].

Complete Heyting algebra

is $(L, \leqq, \vee, \wedge, \Rightarrow)$ satisfying the following.

1. $(L, \leqq)$ is a partially ordered set.
2. An arbitrary subset of L has the join (so, also the meet).
3. $a \wedge b \leqq c \;\Leftrightarrow\; b \leqq a \Rightarrow c$

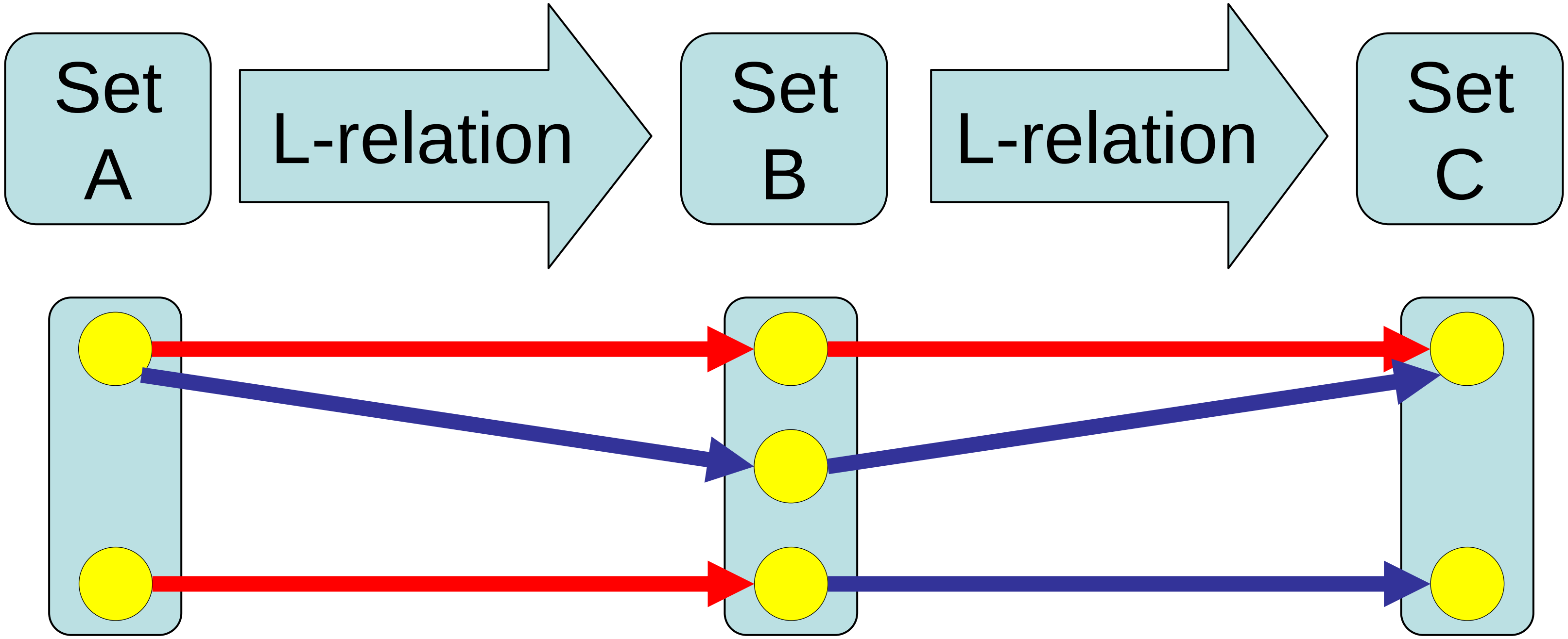
Example:

2, 2×2, …, 2^n, …

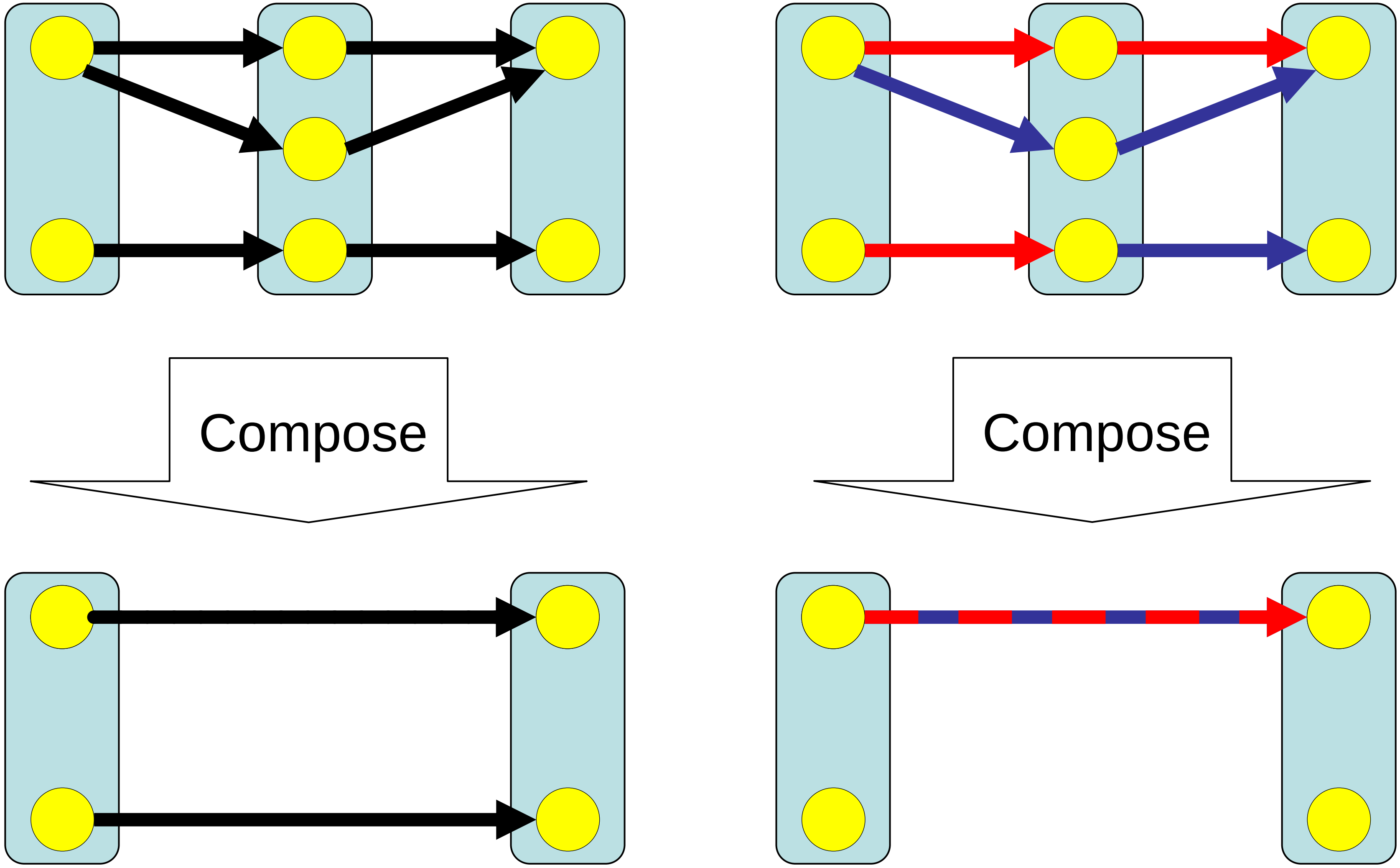
The open sets of a topological space

Category of L-valued relations

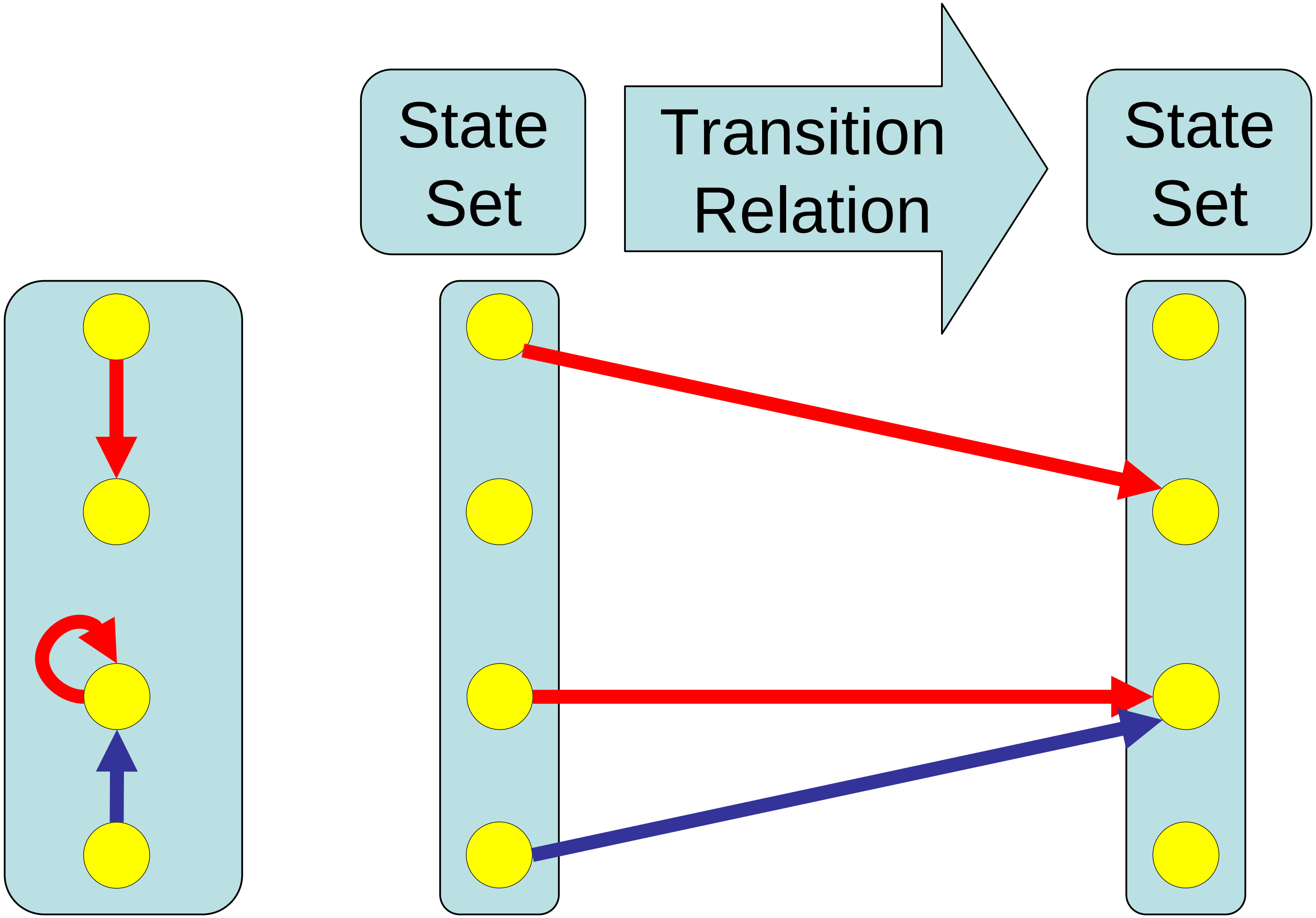
- Objects are sets
- Arrows from A to B are functions from $A \times B$ to L



Composition: L=2 and L=2 × 2

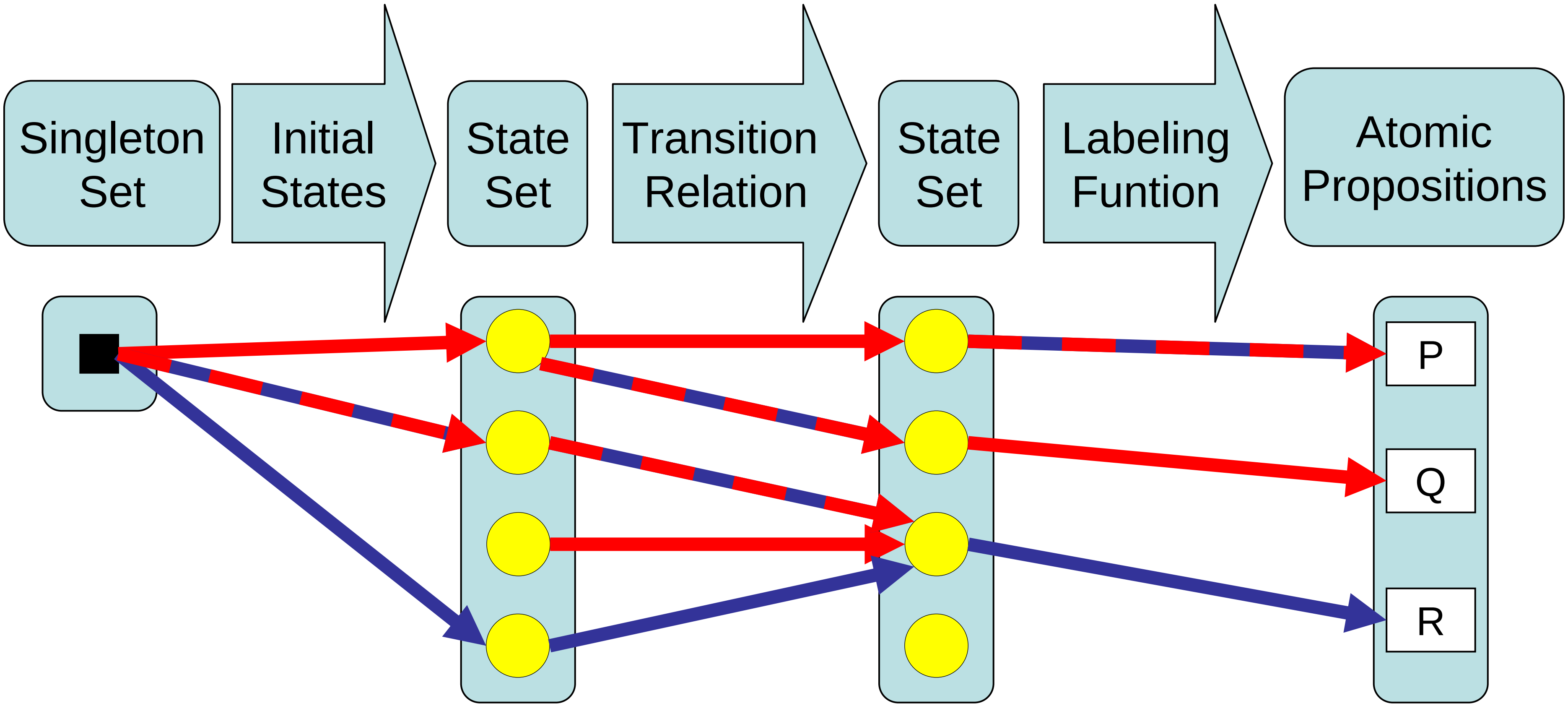


L-valued Transition System

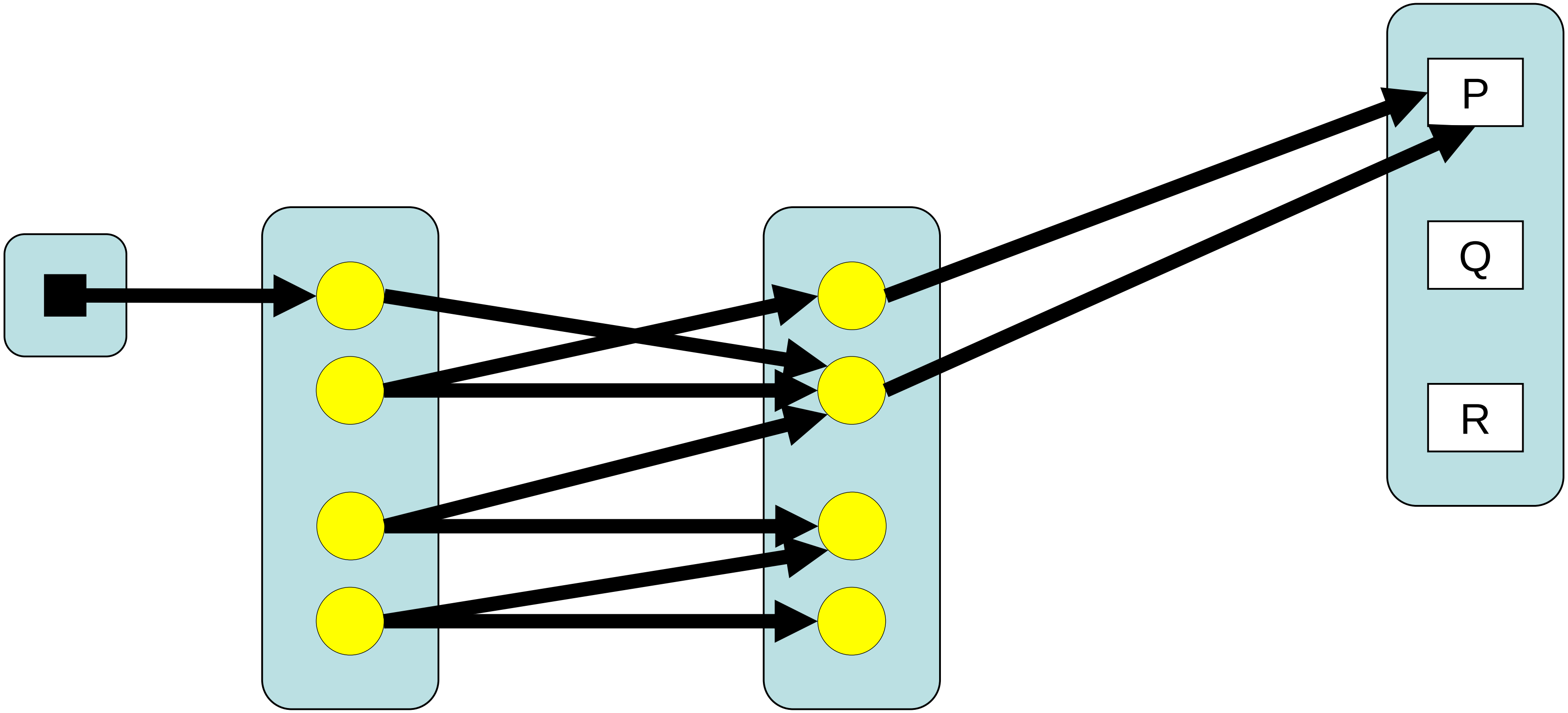


L-valued Kripke model

consists of the following L-relations.



2-valued Simulation



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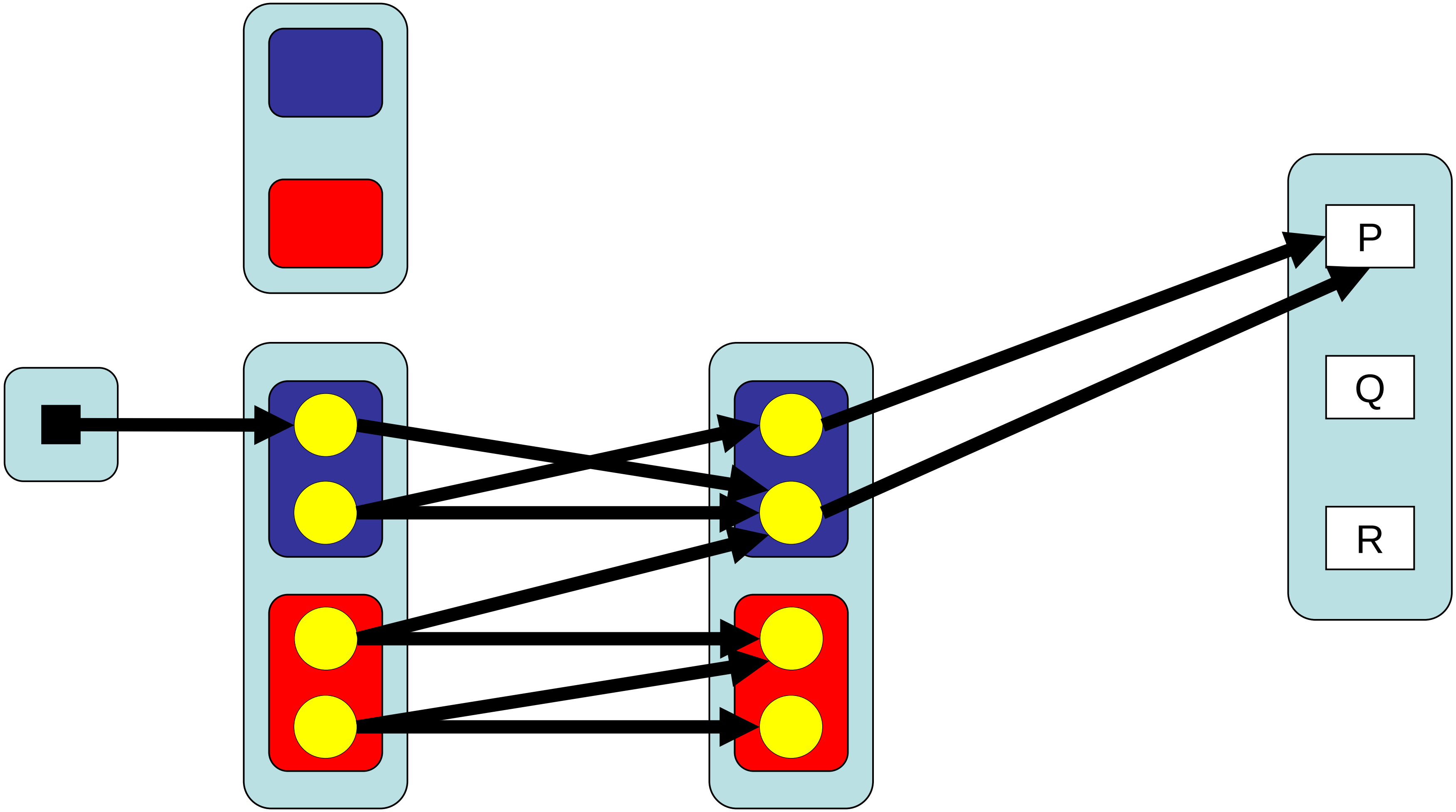

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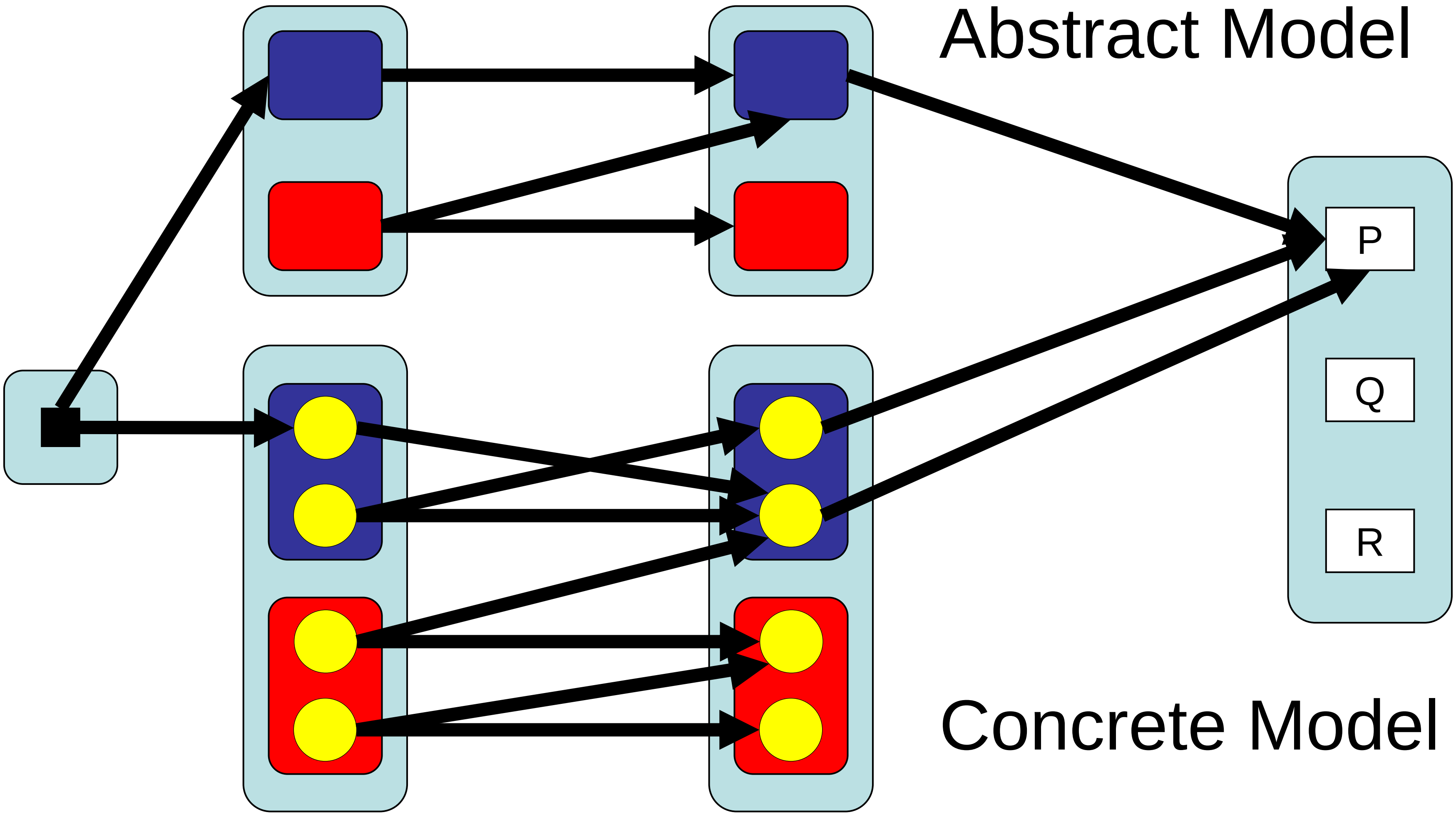

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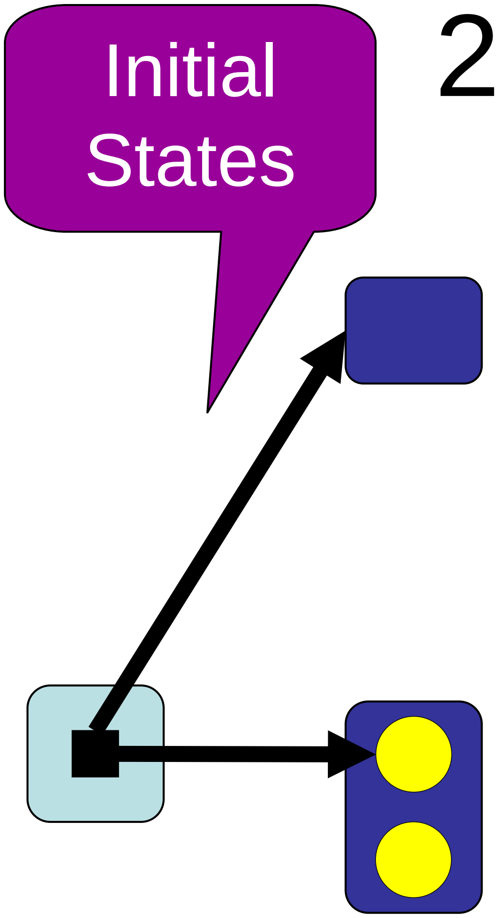

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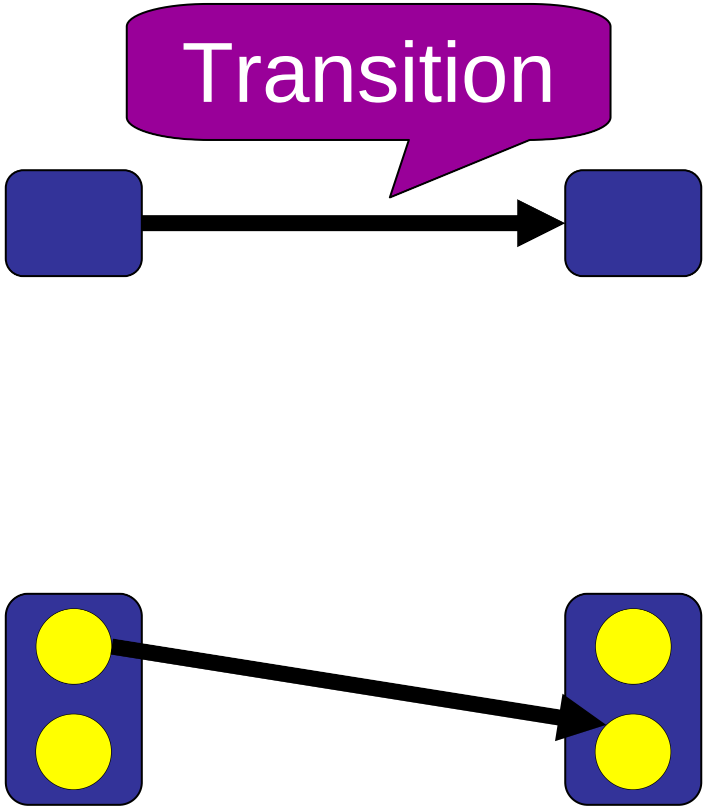

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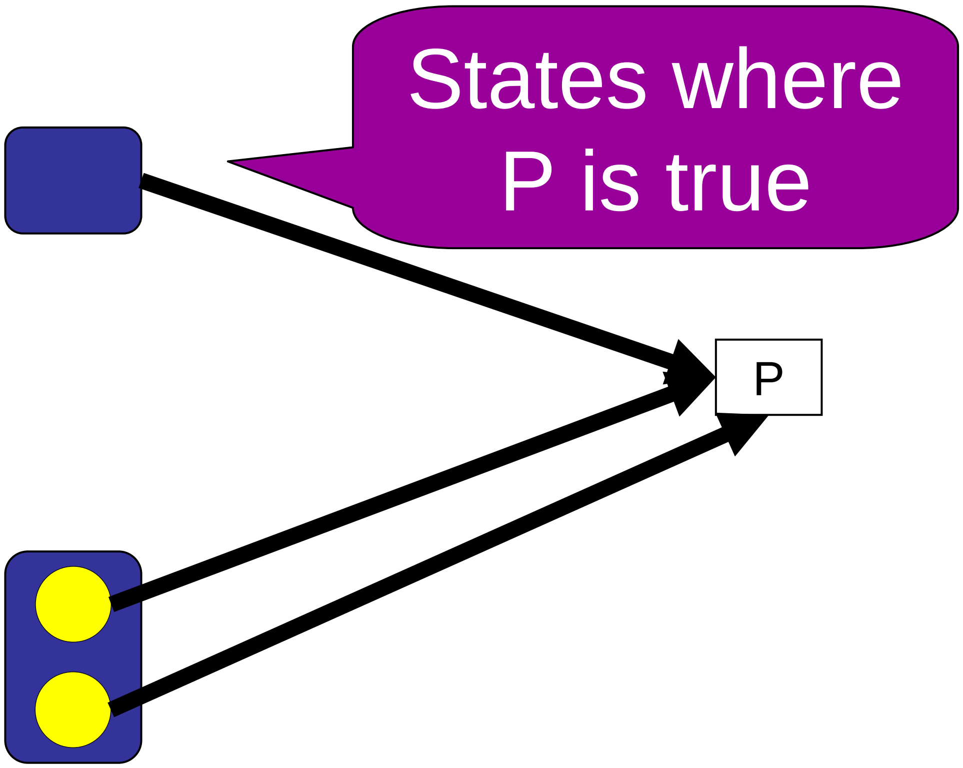

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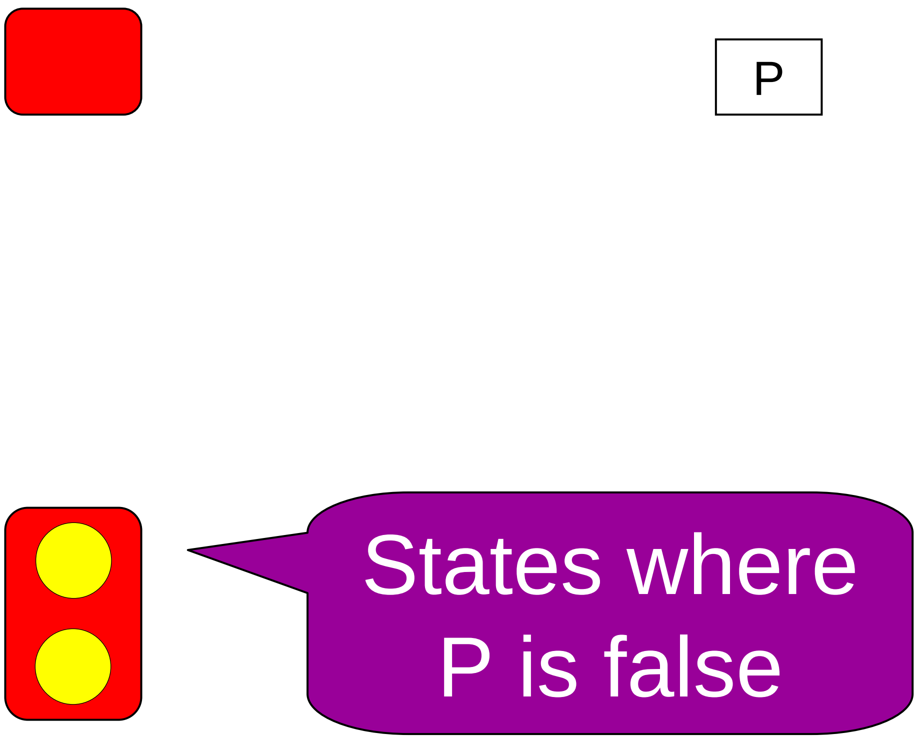

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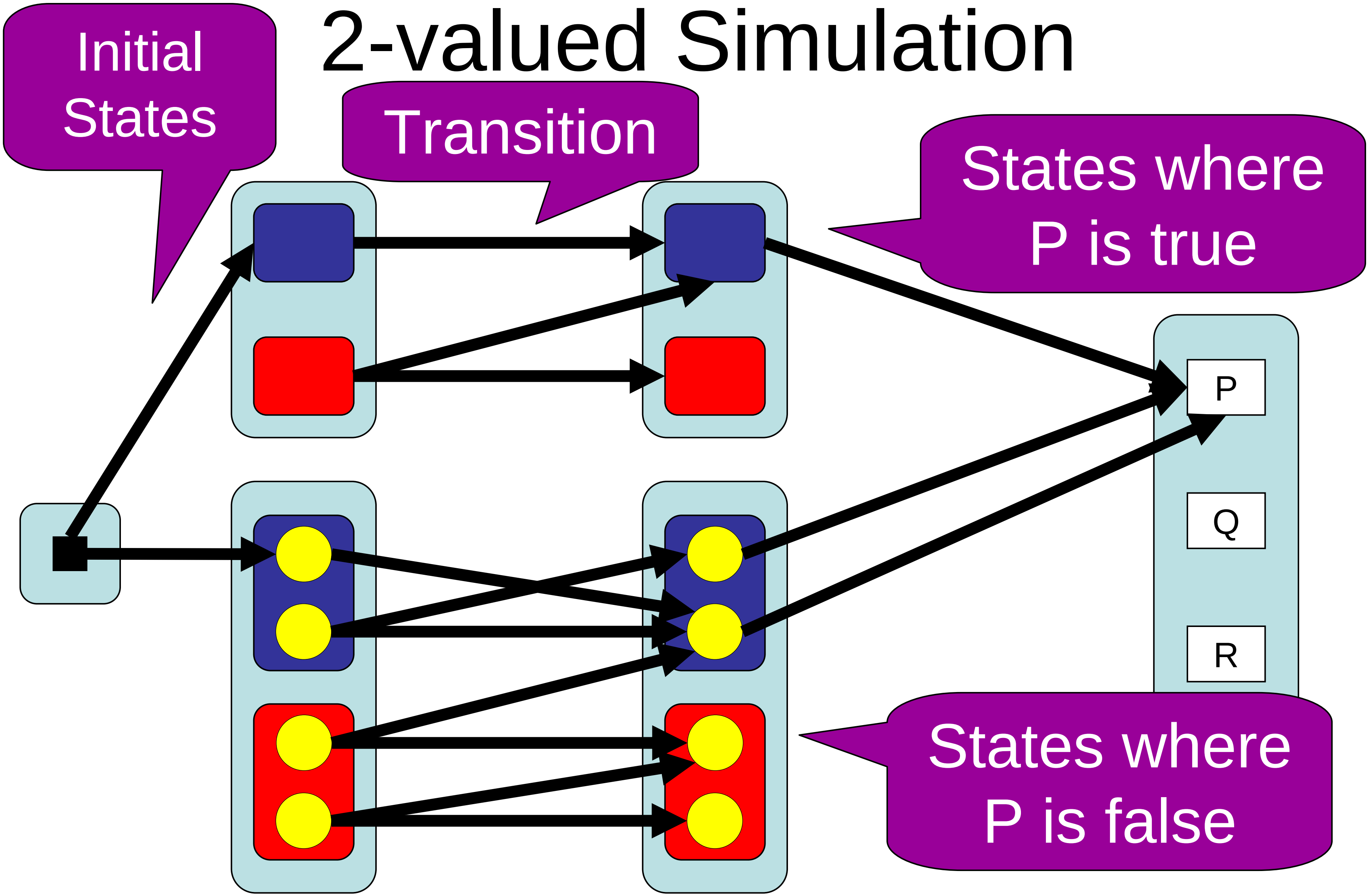
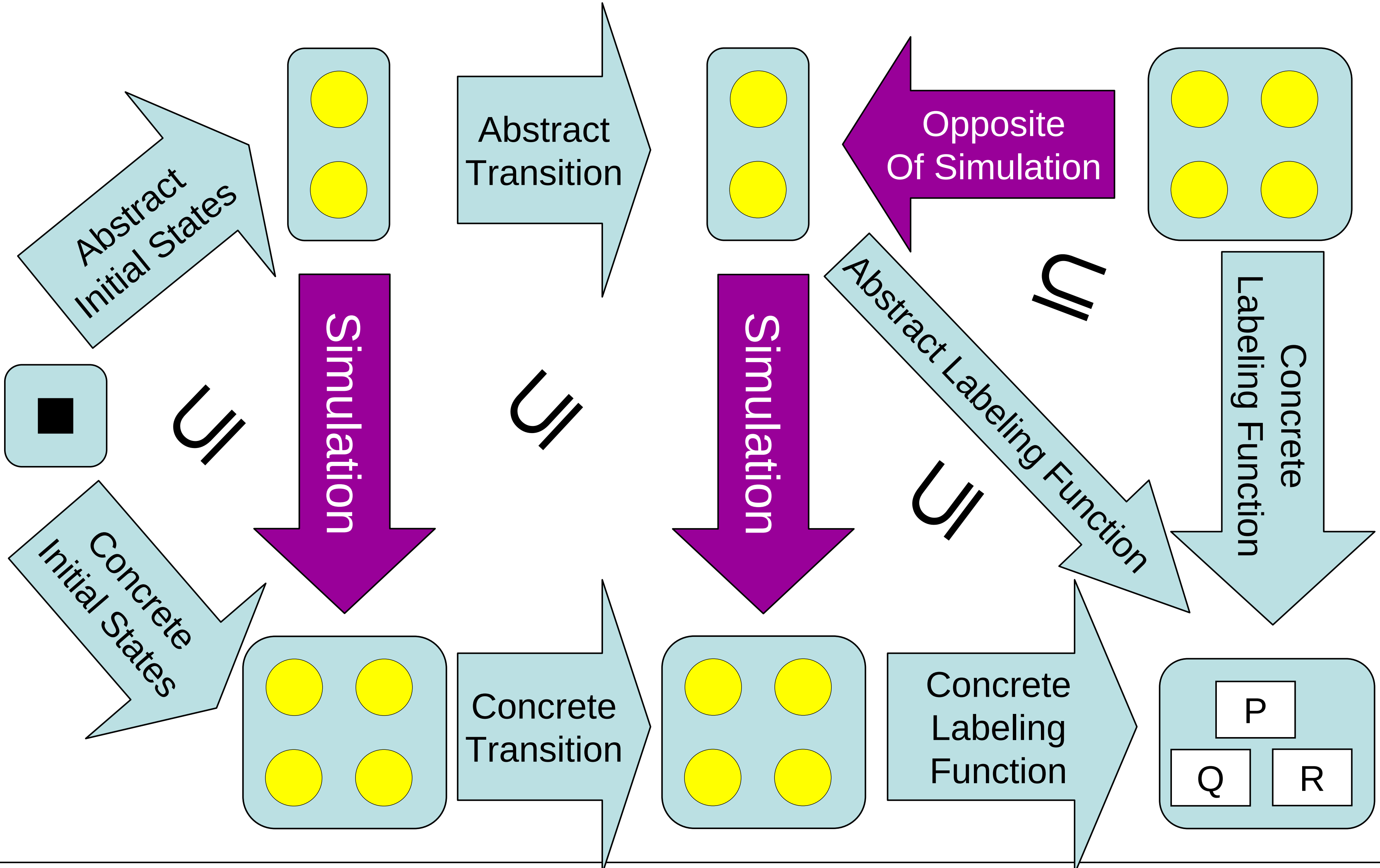


L-valued Simulation



L-valued State Semantics

Modal μ -Calculus

$$\psi ::= p \mid \bot \mid \mathrm{T} \mid \psi \vee \psi \mid \psi \wedge \psi \mid \psi \Rightarrow \psi$$
$$\mid x \mid \mu x.\psi \mid \nu x.\psi \mid \Diamond \psi \mid \Box \psi$$

K,s,V $\vDash \psi$ is an element of L

- Natural definition (no details in this talk)
- Intuitionistic version

$$K,s,V \vDash \psi \quad \neq \quad K,s,V \vDash (\psi \Rightarrow \bot) \Rightarrow \bot$$

Simulation Theorem

For any simulation,

if the abstract model satisfies ψ,

then the concrete model satisfies ψ.

- When ψ has no $\Box$ in the negative positions and no $\Diamond$ in the positive positions
- Example: $\nu X.P \wedge \Box X$

  “P always globally holds”.

This theorem holds in L-valued context.

L-valued Path Semantics

Linear Modal μ -Calculus (generalization of LTL)

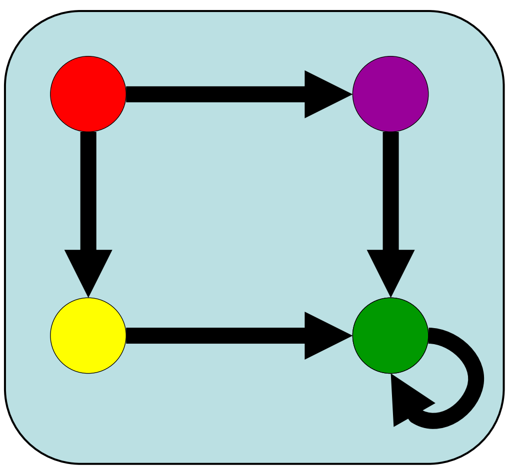
ψ ::= p | $\perp$ | T | $\psi \vee \psi$ | $\psi \wedge \psi$ | $\psi \Rightarrow \psi$

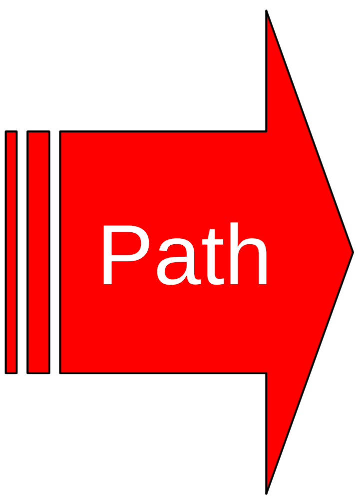
| x | μ x. ψ | ν x. ψ | Next ψ

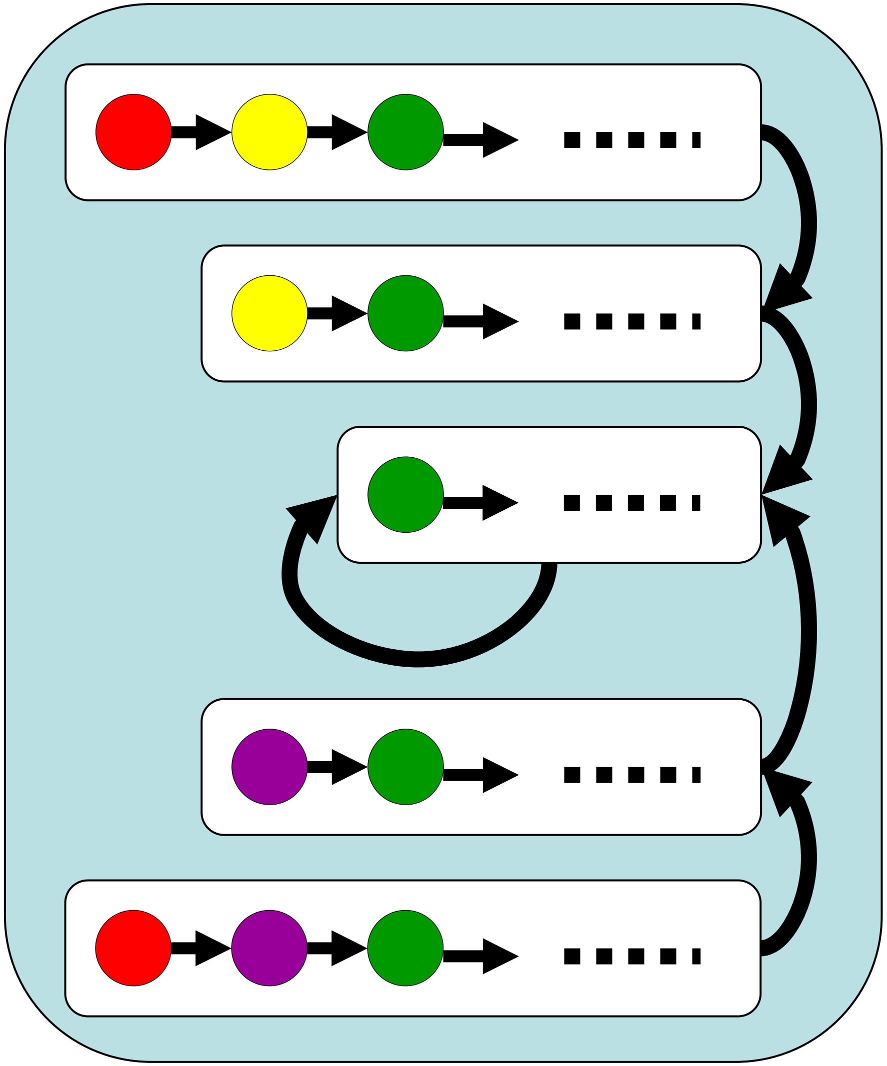
K, π ,V $\vDash \psi$ is defined for a path π .

2-valued Path Semantics

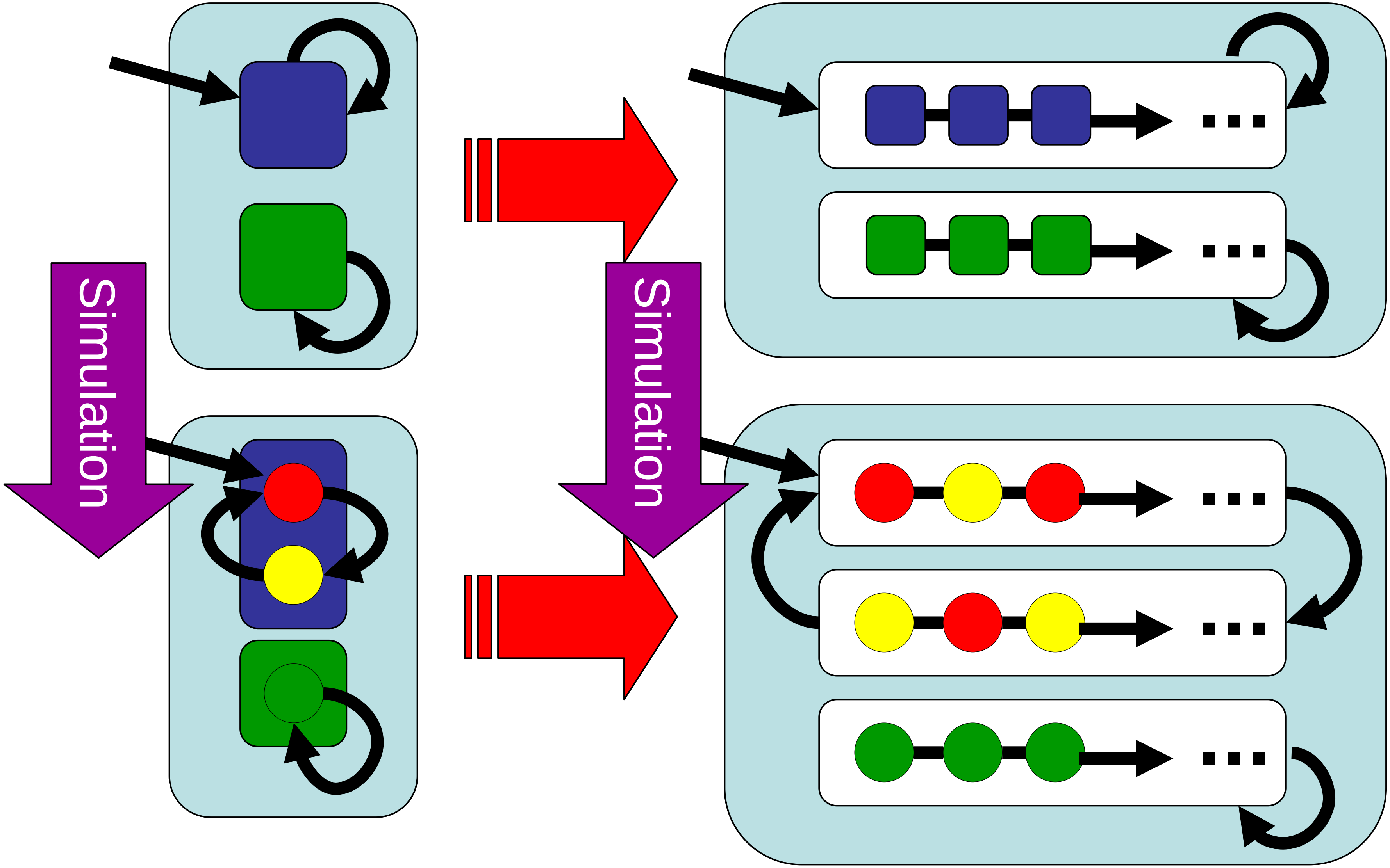
Path Semantics =
Path Construction
\+ State Semantics







Simulation is lifted



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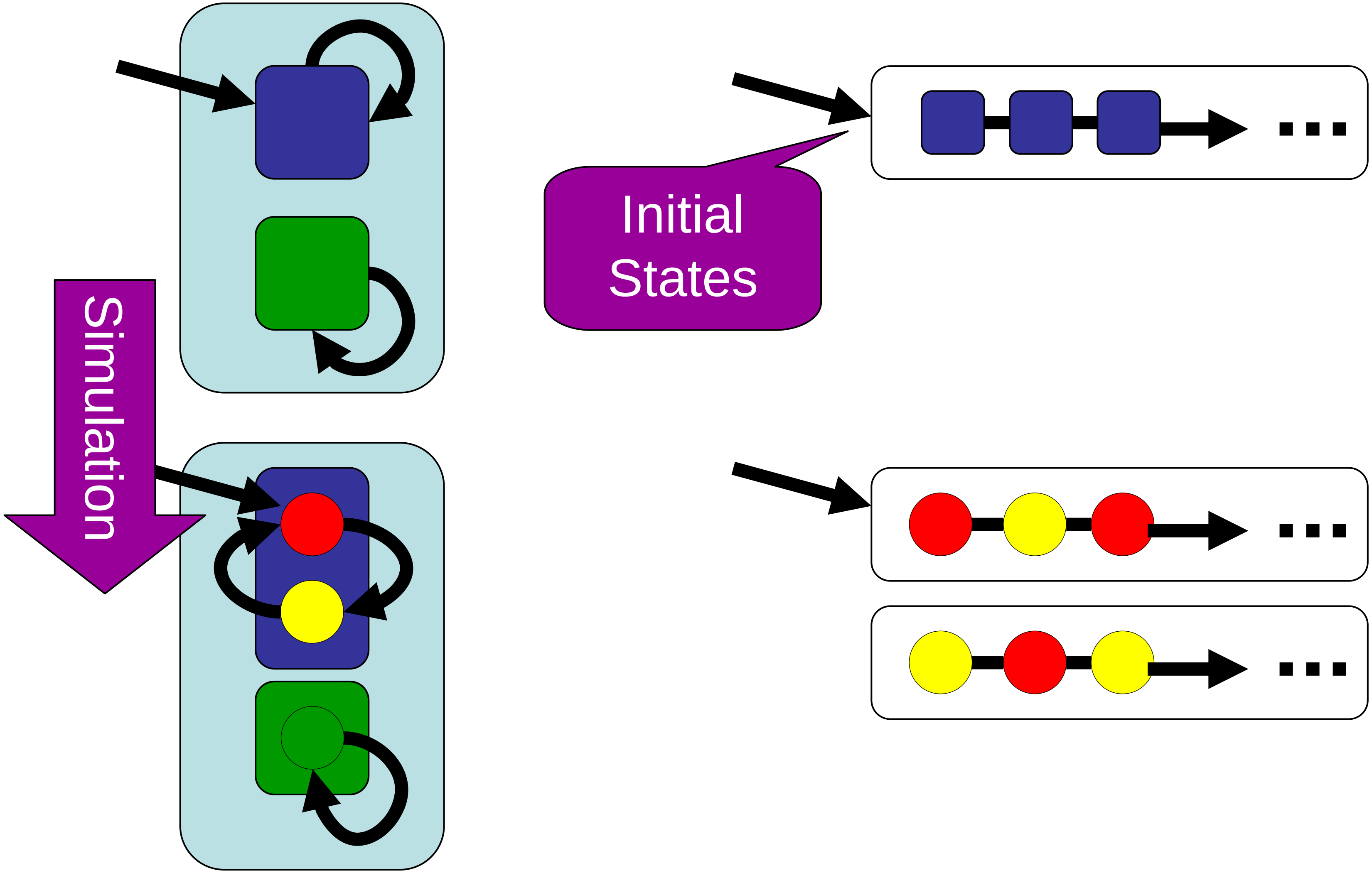

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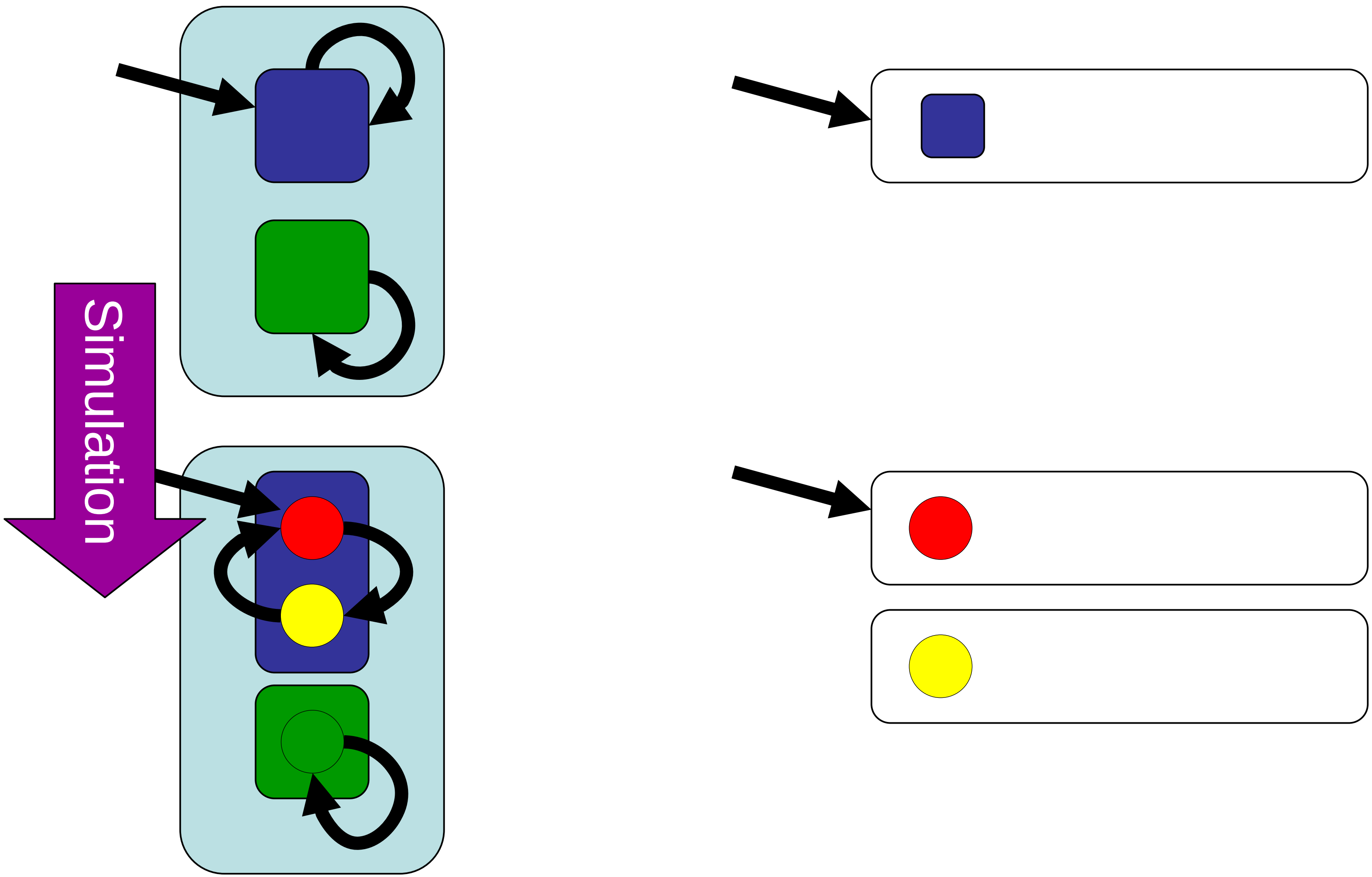

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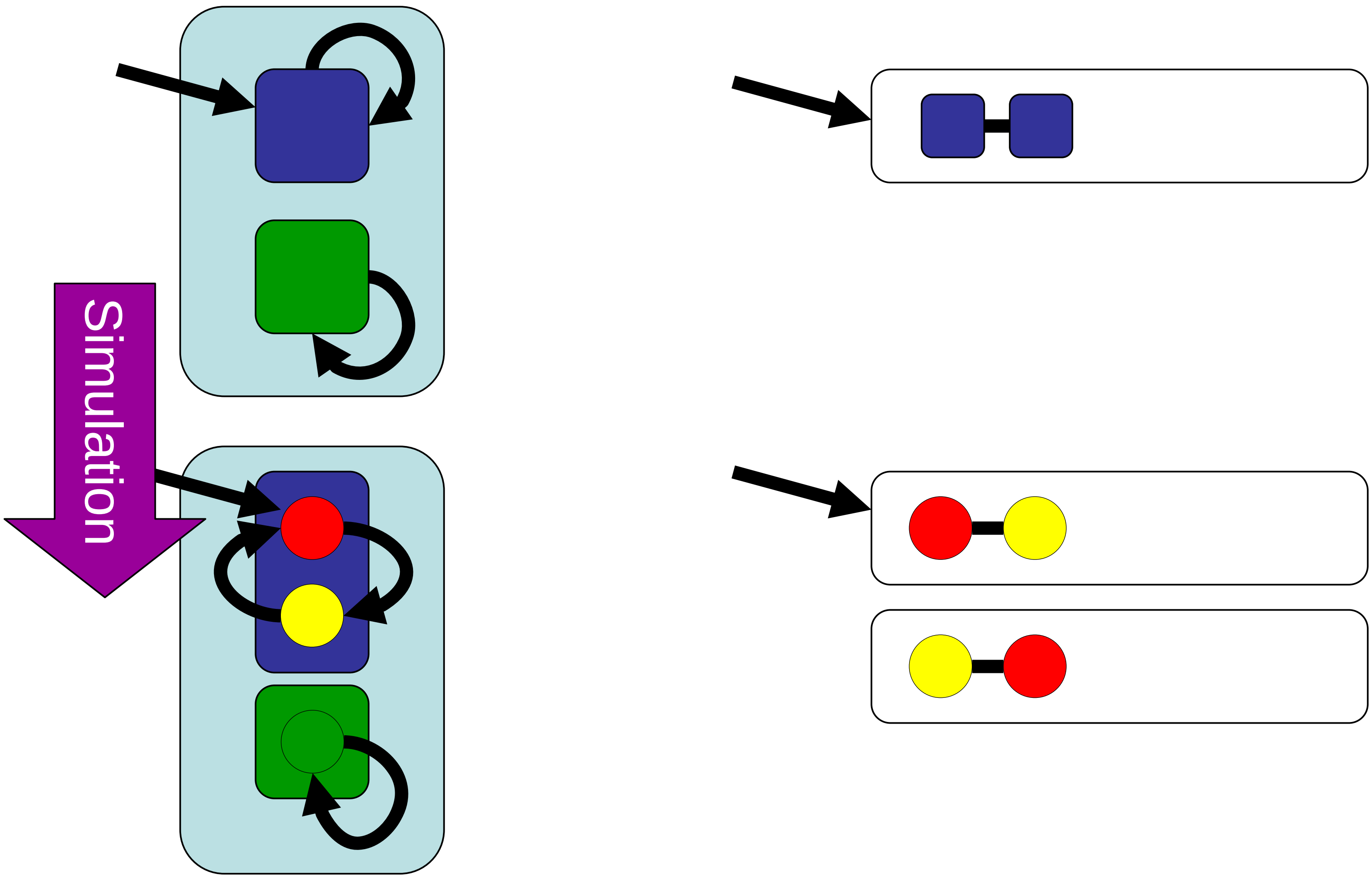

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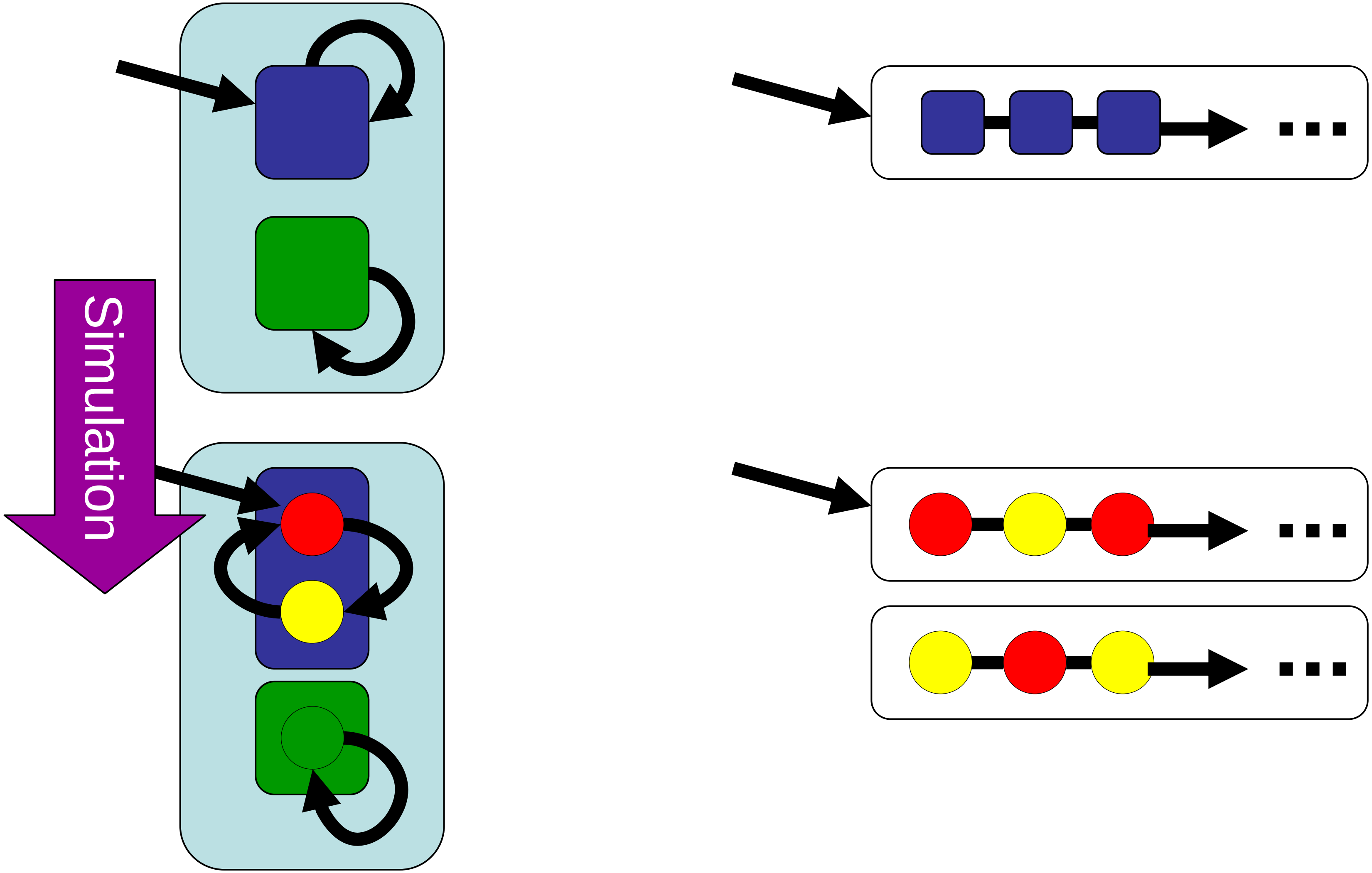
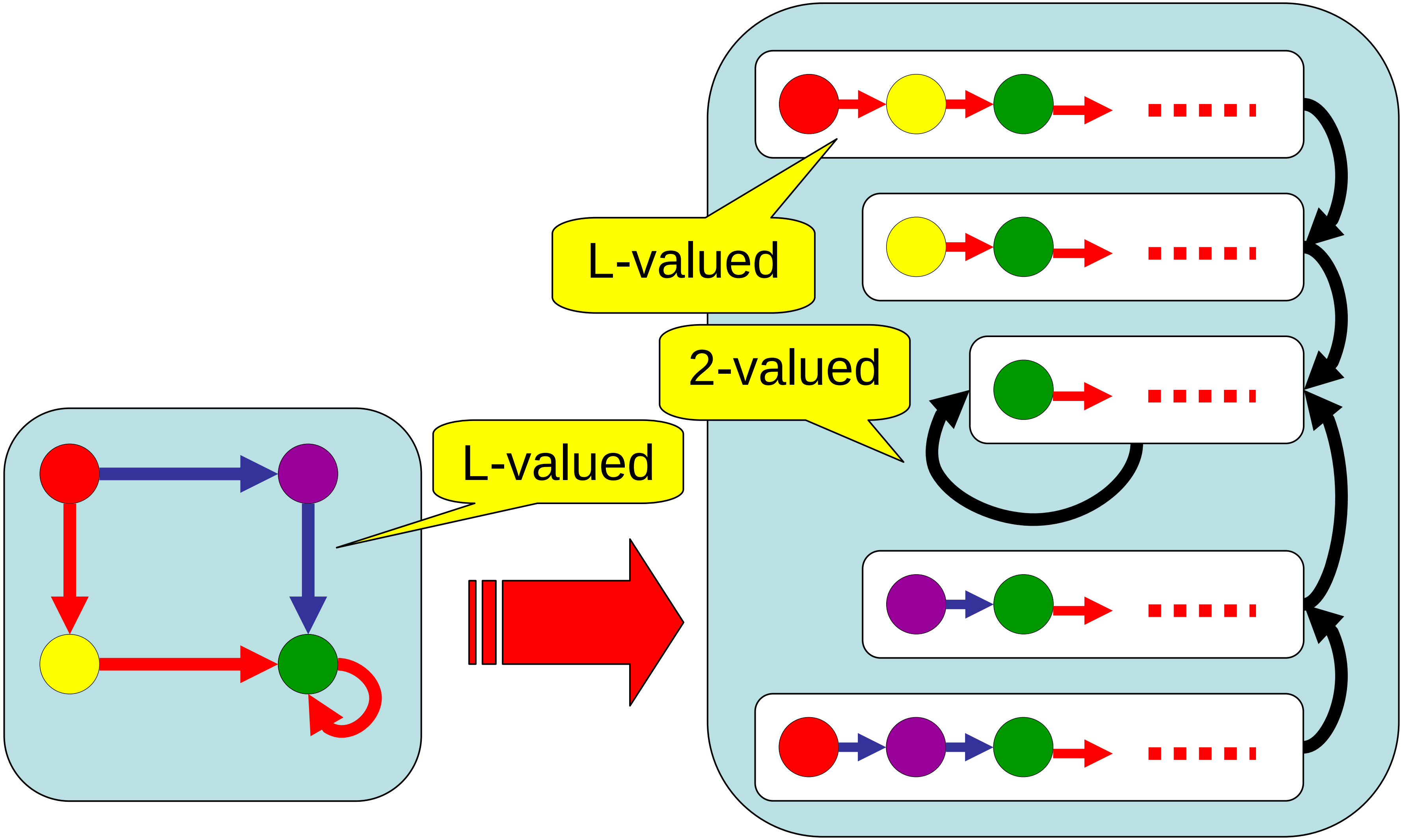
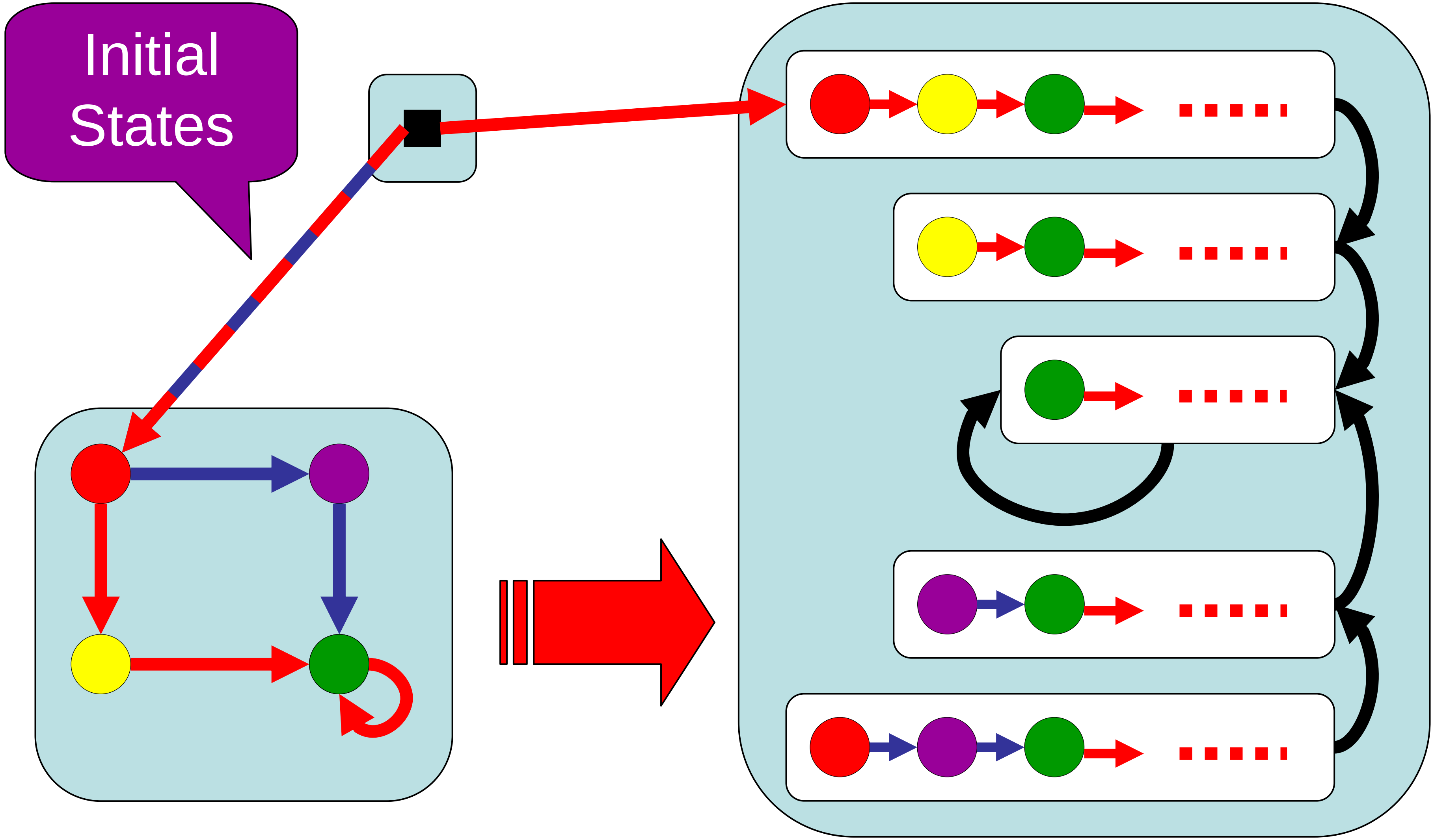


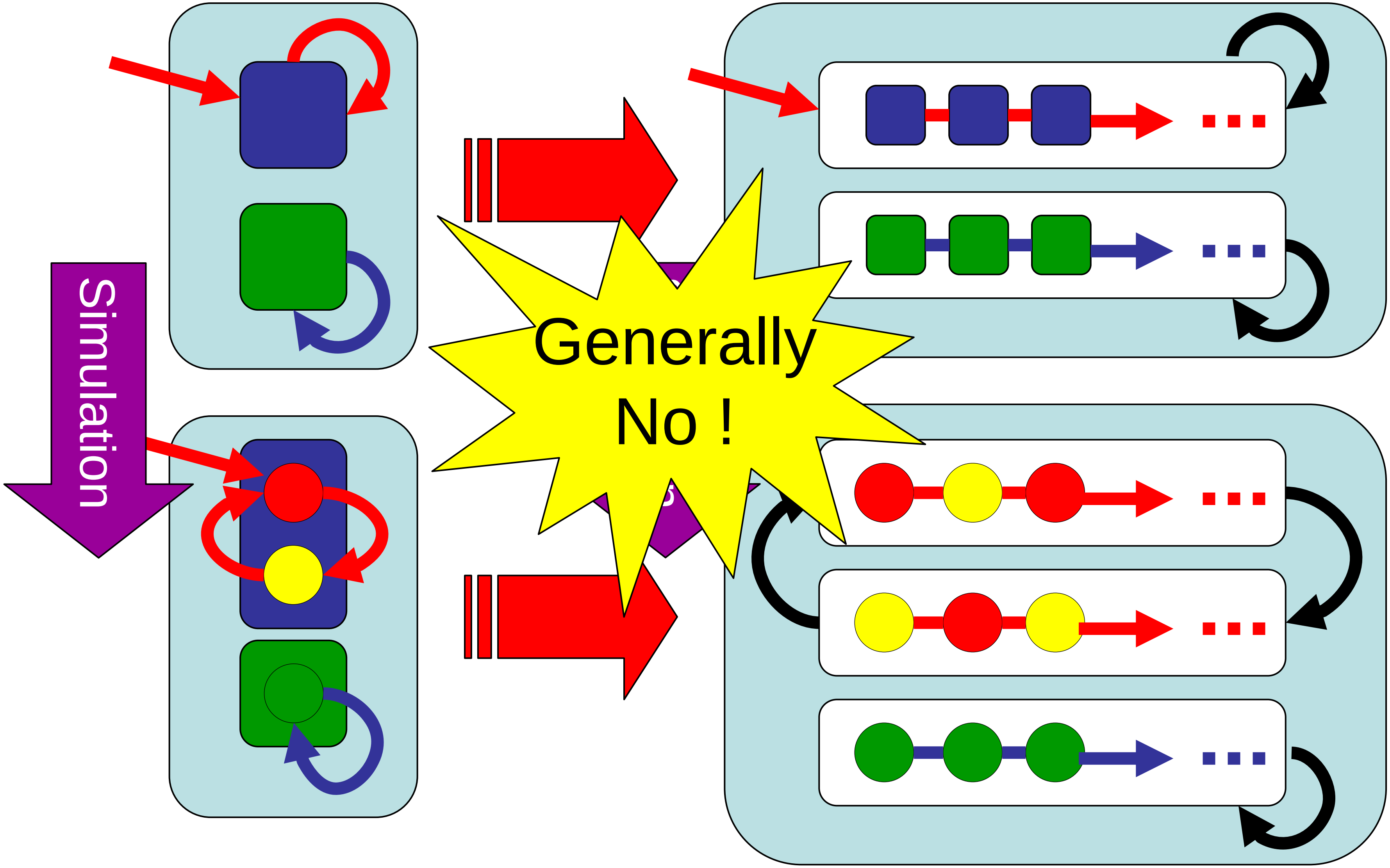
L-valued Path Semantics



L-valued Path Semantics



Is simulation lifted ?



Is simulation lifted ?

- No. We found a counterexample.
- We gave a sufficient condition:
  - A simulation is lifted if L is the open sets of a topological space and closed for countable intersections.
  - Examples: power sets, $\mathrm{Nat} \cup \{\omega\}$
- Under the condition, the simulation theorem for path semantics holds.

Conclusion

- Complete Heyting algebra valued
  - Transition System, Kripke Model, Simulation
  - State Semantics for Modal μ -Calculus, Simulation Theorem
- Under our new condition
  - Path Semantics for Linear Modal μ -Calculus, Simulation Theorem

Future Work

- To relate this work to fuzzy relations or probabilistic relations