Abstract: This research aims to elucidate information systems in terms of embodied knowledge, and constructs a mathematical model for interaction between the teacher and learner from the viewpoint of information science, such as cognitive science and artificial intelligence.

As such, it is possible to build the information scientific stage model \((X, W, f, h)\) that will describe the coaching process. The mathematical model can potentially estimate the state of an expertise in embodied knowledge. Such approaches will not only benefit to elucidate research of embodied knowledge in cognitive science and artificial intelligence but also are applicable to coaching in sports science.

Key Words: Embodied Knowledge, Expertise, Mathematical Model, Interaction, Coaching

1. Introduction

1.1. Embodied knowledge as tacit knowledge

In recent years, there has been a growing interest in embodied knowledge because it is expected to elucidate an expertise in physical skills[1] from the viewpoints of information science, such as cognitive science and artificial intelligence. Several studies have been conducted on embodied knowledge, however, little is known about the information system through which physical skills are acquired. The reason is as follows. Embodied knowledge is tacit knowledge, which was conceptualized by Michael Polanyi[2] as bodily knowledge of how to act without any deliberation or verbalization[3]. Therefore, it is very difficult to explicate the information system of embodied knowledge.

1.2. The purpose of this study

As the first step in our study, we begin with a discussion on the coaching process. Typically, the true purpose of the teacher is not to transmit to the learner explicit knowledge such as that acquired from textbooks, but to teach and share somatic sensations which the teacher has mastered through experiences. Nevertheless, there are many cases in which it is difficult for the teacher to hand down physical skills to the learner during coaching because embodied knowledge is tacit knowledge. Therefore, the teacher gradually devises the optimal guidance for the learner, and will try to evoke somatic sensations like the teacher in the learner.

The purpose of this paper is to build a mathematical model of the interactions between the teacher and the learner in the abovementioned coaching process. It is hoped that the model will contribute to a better understanding of embodied knowledge, for example, the prediction about an expertise in physical skills and deeper elucidation related to the phenomenon of embodied knowledge according to the formula.

2. Mathematical model expressing an expertise in embodied knowledge

2.1. Process of coaching

The process by which teachers convey embodied knowledge does not require the learner to remember explicit knowledge written in textbooks or handouts. Instead, teachers pass their own physical sensations gained through their own experiences to their learners.
Although instruction is an ideal way to pass physical sensations from teacher to learner, this is often difficult in practice because the teacher has tacitly acquired that embodied knowledge. As the learner lacks this advantage, the teacher usually applies a step-by-step instruction method suited to the learner in which he copies and shares his own physical sensations.

Here, we denote the instruction of a teacher by \( x \). Assuming that \( x \) is a continuous function, the optimal \( x \) for a learner is an extremum of this function\(^1\). This argument can be reasoned as follows. The semantic space of the linguistics of a teacher’s instruction (hereafter referred to as the linguistic semantic space) is essentially a network of semantics constructed from the language. Within this linguistic network, teachers evaluate learners in the vicinity of expressions surrounding the meaning they want to convey to their learners. Therefore, by including distance or differences in the network and assuming that the network is continuous, the extreme value becomes a differential evaluation index.

Next, let us focus on the physical expression of learners who respond to \( x \). Here, by defining the learner’s physical expression as \( w \), we can exchange \( x \) and \( w \) in stages. At each stage, work is done in moving \( w \) to the teacher’s sensation (defined as \( x^k \), where \( k \) denotes the stage). The teacher’s instruction \( x^k \) is strongly correlated with the learner’s physical expression \( w^k \) but is expected to differ among individuals (diversity/identity).

Now, if we suppose that we can quantify the gap between \( x \) and \( w \), we can evaluate the level of an expertise in the learner’s embodied knowledge. In fact, these two processes are equivalent. First, we express the evaluation with respect to the \( k \)th teacher’s instruction as a function \( f^k \). Here, we set the teacher’s evaluation function because traditionally, the teachers grasp their coaching intersubjectively and impart their linguistic instruction to suit the learners. This function provides a theoretical description of this process. Such an evaluation function with respect to the teacher’s linguistic instruction is justified as follows: if the content of a teacher’s instruction at each stage is represented by \( x^k \), the teacher’s evaluation at that instruction stage should also be considered. A wise teacher often starts with simple instructions that are easily implemented by the learner and then gradually increases the difficulty level. The evaluation of the content instruction should increase accordingly.

Therefore, \( w^k \) must be evaluated at each stage. To this end, we define an evaluation function \( h^k(w^k) \) of \( w^k \) imposed by the teacher. Note that \( h(w) \) assesses the proximity to the teacher’s instruction and hence determines the highest state of evaluation \( w \). When the \( h(w) \) of \( w^k \) is time-independent, it is represented by the extremum of the evaluation function.

\[
\frac{dh(w)}{dx} = 0 \tag{1}
\]

Moreover, if the evaluation function \( u^k \) of physical expression depends on a parameter such as time, the problem becomes that of finding a stationary curve.

Next, it is hoped that a learner will faithfully implement and notice a word-based instruction \( x^k \). Insufficient attention paid by the learner to the instruction will be reflected in \( w^k \), which dictates the next instruction \( x^{k+1} \) imparted by the teacher. This yields a stage-by-stage interaction in which the learner implements \( x^k \) and outputs a physical expression \( w^k \), which then guides the teacher’s next instruction \( x^{k+1} \), eliciting a response in the learner’s physical expression \( w^{k+1} \). This interaction is iterated until the teacher observes the learner’s physical expression and that “nature as nature is as good as it can be” in a more sophisticated form.

\(^1\) Expressing a natural phenomenon as a minimum or maximum value of a fixed physical quantity echoes Aristotle’s sentiment.

concludes that the learning has been accomplished.

This mathematical model comprises the teacher’s instruction and its evaluation function ($x^k$ and $f^k$, respectively) and the learner’s physical expression and its evaluation function ($w^k$ and $h(w)$, respectively). Thus, the constructed mathematical model is denoted $(X, W, f, h)$, where $X = x^k$, $f = f^k(x^k(t), dx^k(t)/dt)$, $W = w^k$, and $h = h^k(w^k)$ for $k = 1, 2, ..., n$. As shown in Fig 1, this model converges the learner’s linguistic instruction and teacher’s physical expressions as the stages progress.

2.2. Devising the function

As shown in the constructed mathematical model, the evaluation function is a point in the interaction between the teacher and learner. The teacher’s evaluation function provides the physical sensation that the teacher wishes to explicitly impart (specifies the function parameters explicitly). At the practically verified stage, we require a method that effectively decides the evaluation functions. For this purpose, we consider the following two methods.

The first method is a sensational representation that focuses strictly on the quantifiable expressions and withholds the analysis on the physical operations. In this approach, the variable determinations are based on a complex of two standards. More specifically, the composite function of standards $A_i$ and $B_j$ is a tensor function $C_{ij}$ on which the evaluation function can be based. A theoretical explanation of this approach is possible but would involve complex calculations such as those of gravity and acceleration at each stage.

The second method determines the topologies of the learner’s physical expressions that are expected and definitely not expected by the teacher. In this approach, the learner’s physical expression is regarded as a physical meaning space. For example, if the learner’s sensation becomes slightly closer to the teacher’s physical representation near the teacher’s linguistic instruction $x^1$, it can be assigned a continuous variable $w^1$, which becomes the physical meaning space. In other words, by distributing the physical expressions that are close to and far from the physical sensation of the teacher along a straight line, we can evaluate the learner’s physical expression.
2.3. **Stationary curve and Euler equation**

When the evaluation function depends on parameters such as time, any differences between the teachers’ and learners’ physical sensations can be expressed by the variation principle, which seeks the stationary curve\(^2\) of a functional as shown in Fig 2. The least-action principle is formulated by an equation that relates force and work, leading to important concepts such as potential and kinetic energy. For example, if a learner’s physical representation is the evaluation function \(h^k(w^k(t), dw^k(t)/dt)\), we can specify its action integral \(H^k[w^k]\).

\[
H^k[w^k] = \int_{t_0}^{t_1} h^k(w^k(t), dw^k(t)/dt) dt
\]  \hspace{1cm} (2)

The stationary curve of this action integral can be derived from the following Euler equation and is calculable for a given evaluation function.

\[
\frac{dh^k(w^k(t), dw^k(t)/dt)}{dt} - \frac{dw^k(w^k(t), dw^k(t)/dt)}{d(dw^k(t)/dt)} = 0
\]  \hspace{1cm} (3)

However, although this method is theoretically sound, it involves complicated calculations such as those of gravity and acceleration in practical implementation.

3. **Conclusion**

For the purpose of our work, as such, it is possible to build the information scientific stage model \((X, W, f, h)\) that will describe the coaching process between the teacher and the learner. The mathematical model can potentially estimate the state of an expertise in embodied knowledge. Such approaches will not only benefit to elucidate research of embodied knowledge in cognitive science and artificial intelligence but also are applicable to coaching in sports science.

A further study of the stage model of embodied knowledge should be conducted to verify the validity empirically.

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**References**


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\(^2\) The stationary curve follows the principle of least action, which states that “nature’s work always takes the easiest and the shortest path,” i.e., any event occurs with the least effort.[4].