

# Contents

<b>Abstract</b>	<b>i</b>
<b>Acknowledgments</b>	<b>iii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Chapter Introduction . . . . .	1
1.2 Background . . . . .	1
1.3 Statement of Research Question . . . . .	4
1.4 Structure of the Thesis . . . . .	4
<b>2 Literature Review</b>	<b>8</b>
2.1 Chapter Introduction . . . . .	8
2.2 Evolution of Puzzles . . . . .	8
2.3 Complexity of Solving Puzzles . . . . .	11
2.3.1 N-puzzle . . . . .	13
2.3.2 Minesweeper . . . . .	14
2.4 Game Refinement Theory . . . . .	15
2.4.1 Gamified experience for board games and sports games . . . . .	17
2.5 Motion in Mind Measure . . . . .	18
2.6 Chapter Summary . . . . .	22
<b>3 Entertaining Analysis of Solving Single-Agent Deterministic Puzzle</b>	<b>23</b>
3.1 Chapter Introduction . . . . .	23
3.2 Game Testbed: 8-puzzle . . . . .	24
3.3 Experimental Setup and Its Results . . . . .	26
3.3.1 A* Algorithm . . . . .	26
3.3.2 Analysis of the 8-puzzle for AI player . . . . .	27
3.3.3 Analysis of the 8-puzzle for human player . . . . .	31

3.3.4	Further analysis of the results . . . . .	33
3.4	Chapter Conclusion . . . . .	35
<b>4</b>	<b>Informational Dynamic of Single-Agent Stochastic Puzzle</b>	<b>37</b>
4.1	Chapter Introduction . . . . .	37
4.2	Game Testbed: Minesweeper . . . . .	39
4.3	Research Methodology . . . . .	41
4.3.1	Building Minesweeper AI . . . . .	41
4.4	Experiment Results and Analysis . . . . .	57
4.4.1	First action rule and safe first action rule . . . . .	57
4.4.2	Safe first action rule and safe neighborhood rule . . . . .	57
4.4.3	Guessing strategy and random guessing strategy . . . . .	58
4.4.4	Comparison of methods and strategy . . . . .	59
4.5	Discussion . . . . .	61
4.6	Chapter Conclusion . . . . .	62
<b>5</b>	<b>Finding the Border Between Games and Puzzles</b>	<b>64</b>
5.1	Chapter Introduction . . . . .	64
5.2	Games and Puzzles . . . . .	65
5.2.1	Current gaps in puzzle studies . . . . .	66
5.3	Informational Progress Model in Puzzle-solving . . . . .	68
5.4	Game Testbeds: N-puzzle and Minesweeper . . . . .	70
5.4.1	Informational progression of N-puzzle . . . . .	70
5.4.2	Informational progression of Minesweeper . . . . .	72
5.4.3	The solvability of single-agent game: evidence from Minesweeper . . . . .	74
5.5	Chapter Conclusion . . . . .	79
<b>6</b>	<b>Conclusion</b>	<b>81</b>
6.1	General Conclusion . . . . .	81
6.2	Answers to Research Questions . . . . .	83
	<b>Bibliography</b>	<b>84</b>
	<b>Publications</b>	<b>94</b>

# List of Figures

2.1	An illustration of three possible relationships among P, NP, NPC, and co-NP	12
2.2	Illustration of law of motion in mind over various mass ( $m$ ). The subjective motion( $p_2$ ) is derived from the objective ones( $p_1$ ), where subjective velocity ( $v_2$ ) was established. $\vec{p}_2$ is derived based on the conservation of $E_p$ .	21
3.1	An example of 8-puzzle	25
3.2	The value of $GR$ , $F$ , $p$ , and $E_p$ , with respect to $D$ in 8-puzzle. The gray area indicates the peak range of $F$ , $p$ , and $E_p$ and the $GR$ zone	30
3.3	The $GR$ value, $F$ , $p$ , and $E_p$ , with respect to the game depth in 8-puzzle based on human players	33
4.1	Two final states of Minesweeper $16 \times 30 99$ mines, the purpose is to find all hidden mines on the board without opening them. (a) is a losing state because the player opened on a mine in the game process, (b) is a winning state while the player revealed all mines on the board.	40
4.2	The basic information of the cells on the board for minesweeper ( $9 \times 9   10$ ), there are three unsolved blocks on the board: $U_1 = \{(2, 1), \dots, (2, 6), (3, 6), \dots, (3, 8), (4, 8), (5, 7), (5, 8)\}$ , $U_2 = \{(6, 3), \dots, (6, 6)\}$ , $U_3 = \{(7, 1), (7, 2)\}$ .	44
4.3	An illustration of primary reasoning strategy: The cells with “F” and numbers are flagged cell number cells, respectively.	44
4.4	Frontier division: an example of $9 \times 9$ Minesweeper configuration with the cells marked with blue, orange, and green colors to signify different and independent frontiers.	47
4.5	Boolean model of hidden cells: an example of a partial $9 \times 9$ Minesweeper configuration (top part) with each hidden cells were modelled with Boolean variable $x_i \in \{0, 1\}$ , where “0” means $c_i$ is a safe cell, and “1” means $c_i$ is a mine cell.	48



4.6	Binary tree search for free variables: $x_1$ and $x_2$ are free variables, blue nodes are leaf nodes, which represents four variable states: $x_1 = 0, x_2 = 0$ ; $x_1 = 0, x_2 = 1$ ; $x_1 = 1, x_2 = 0$ ; and $x_1 = 1, x_2 = 1$ .	55
4.7	Safe first action rule. The winning rate of different first opened cells (game configuration: $8 \times 10$   12 mines with “PAR” AI agent strategy on safe first action rule).	58
4.8	Safe neighborhood rule. The winning rate of different first opened cells (game configuration: $8 \times 10$   12 mines with “PAR” AI agent strategy on safe neighborhood rule).	59
5.1	An illustration of move selection model based on skill and chance (adopted from [1])	68
5.2	Dynamic process of solving rate of 8-puzzle	71
5.3	The distribution of the solving rate in the search process when solving 8-puzzle.	72
5.4	The distribution of solving rates for the three standard Minesweepers board sizes	73
5.5	Game length and the number of guesses for solving Minesweeper $9 \times 9$ board size various on the number of mines, 2000 runs each mine. (a) is the scatter diagram of the average game length, and (b) is the scatter diagram of the guess times for solving Minesweeper $9 \times 9$ board size.	75
5.6	The winning rate ( $p$ ) of the $9 \times 9$ board size Minesweeper is based on the number of mines $M \in [1, 72]$ , with 2000 runs per mine. The winning rate $p = 1$ for $M \leq 3$ indicates a deterministic puzzle; $0 < p \leq 1$ for $M \in [4, 39]$ and $M \in [67, 71]$ indicates a stochastic puzzle; otherwise, $p = 0$ for $M \in [40, 66]$ indicates a game.	77
5.7	The winning rate ( $p$ ) of the $16 \times 16$ board size Minesweeper is based on the number of mines $M \in [1, 247]$ , with 2000 runs per mine. The winning rate $p = 1$ for $M \leq 13$ indicates a deterministic puzzle; $0 < p \leq 1$ for $M \in [14, 84]$ and $M \in [245, 246]$ indicates a stochastic puzzle; otherwise, $p = 0$ for $M \in [86, 244]$ indicates a game.	78

# List of Tables

2.1	Analogical link between physics and game (adopted from [1]) . . . . .	18
3.1	Game refinement value of 10000 simulated times of 8-puzzle, where $n$ and $D$ stands for the average number of plausible options and steps to solve respectively . . . . .	28
3.2	Motion in mind measures and $GR$ value over different total steps to solve ( $D$ ) for the AI player ( $A^*$ algorithm). . . . .	29
3.3	Total steps to solve 8-puzzle games for human players, as well as various motion in mind measures . . . . .	32
3.4	Analysis of 8-puzzle game as a board game based on $GR$ and motion in mind measure . . . . .	34
3.5	Analysis of 8-puzzle game as a scoring sports based on $GR$ and motion in mind measure . . . . .	34
4.1	Comparison for first action rule and safe first action rule based on different Minesweeper configuration. . . . .	57
4.2	Comparison of various methods and strategies from the previous works against the proposed PAFG strategy performance (winning rate in percentage) in solving the Minesweeper. . . . .	60
5.1	Analysis of $n$ -puzzle game on motion in mind . . . . .	70