# Scalable and Practical Nonblocking Switching Networks

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# Switching Networks

- **Permutation Switching Networks.**
- **Multiple-multicasting Switching Networks.**

# Applications of Switching Networks

- **Communication subsystem (interconnection network) in a parallel computing system.**
- **Switching fabric in network switches and routers.**

# Permutation Switching Networks

**An $N \times N$ switching network has $N$ inputs and $N$ outputs. It usually comprises a number of** *switching elements* **(SEs) interconnected by a set of links.**

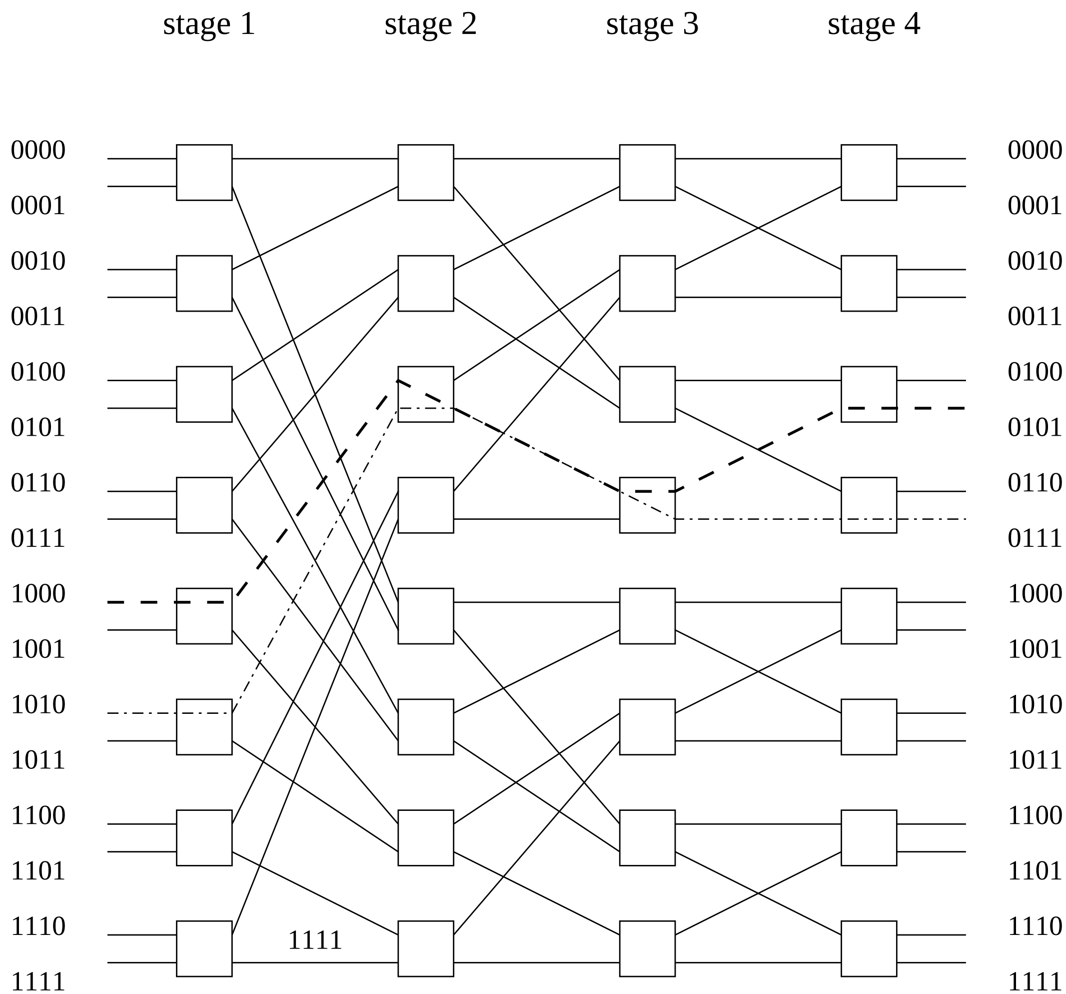
**Without loss of generality, we assume that an SE is of size $2 \times 2$; i.e. it has $2$ inputs and $2$ outputs.**

**The two inputs of an SE intending to be connected with the same output causes** *link conflict***.**

**If an I/O connection path does not have any link conflict with other connection paths, it is called a** *conflict-free* **path.**

**Nonblocking switching networks have been favored in switching systems because they can be used to set up conflict-free I/O connection paths for any (partial) I/O permutation.**

# Example




**A baseline network.**

# Three Basic Types of Nonblocking Switching Networks

- **Strictly Nonblocking (SNB) networks.**
  - **A connection from any idle input to any idle output can always be established regardless of how existing connections were established.**
  - **Useful for both circuit switching and packet switching.**
- **Wide-Sense Nonblocking (WSNB) networks**
  - **A connection from any idle input to any idle output can always be established by following a rule (algorithm).**
  - **Useful for both circuit switching and packet switching.**

- **Rearrangeable Nonblocking (RNB) networks.**
  - **A connection from any idle input to any idle output can be established if rearrangement of existing connections is allowed.**
  - **Useful for packet (cell) switching but not suitable for circuit switching.**

## Lower Bound for the Cost of Nonblocking Switching Networks

**Cost:** $\Omega(N \log N)$ **SEs.**

## Optimal SNB and WSNB Networks

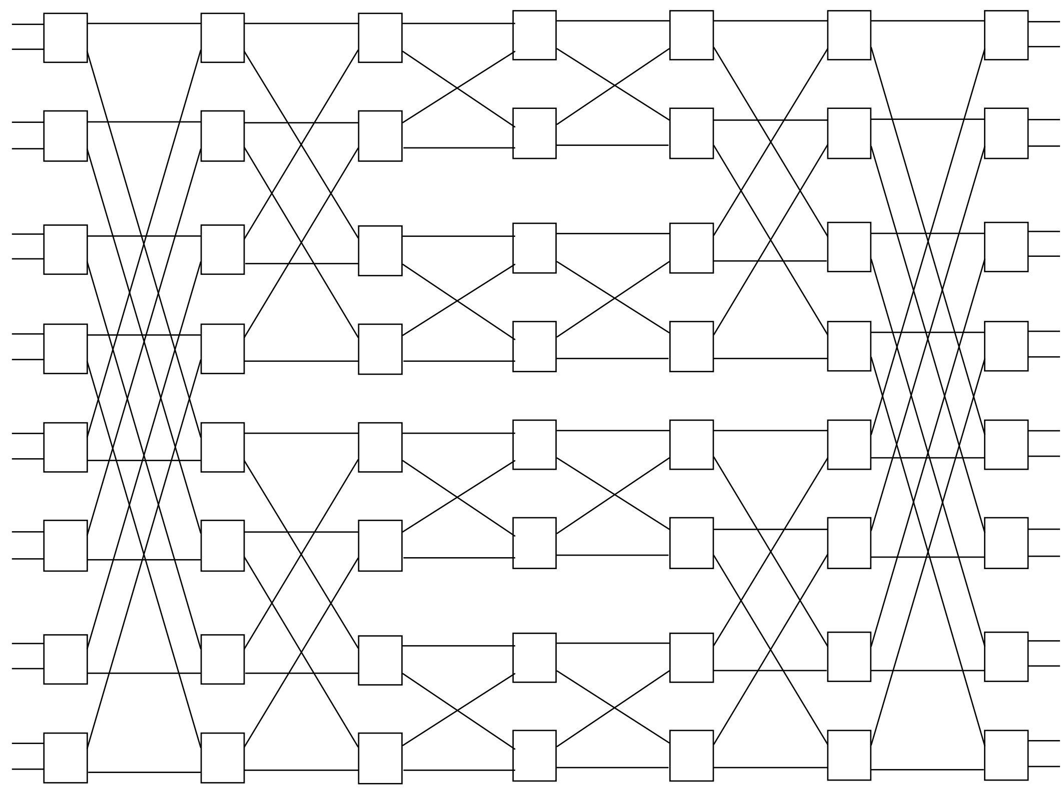
$O(N \log N)$**-cost theoretically possible, but not practical.**

**Suboptimal: Clos network of** $O(N^{1.5})$ **cost, and Cantor network of** $O(N \log^2 N)$ **cost. Routing I/O connections is time-consuming.**

## Optimal RNB Network

$O(N \log N)$ **cost practically achievable. Example: Benes network.**

# Benes Network



# An Outstanding Open Problem

**Find practical $O(N \log N)$-cost SNB and WSNB Networks for circuit-switching applications.**

## Question: Do we really need SNB and WSNB networks?

**Practically, maybe not.**

# Virtual Nonblocking Networks

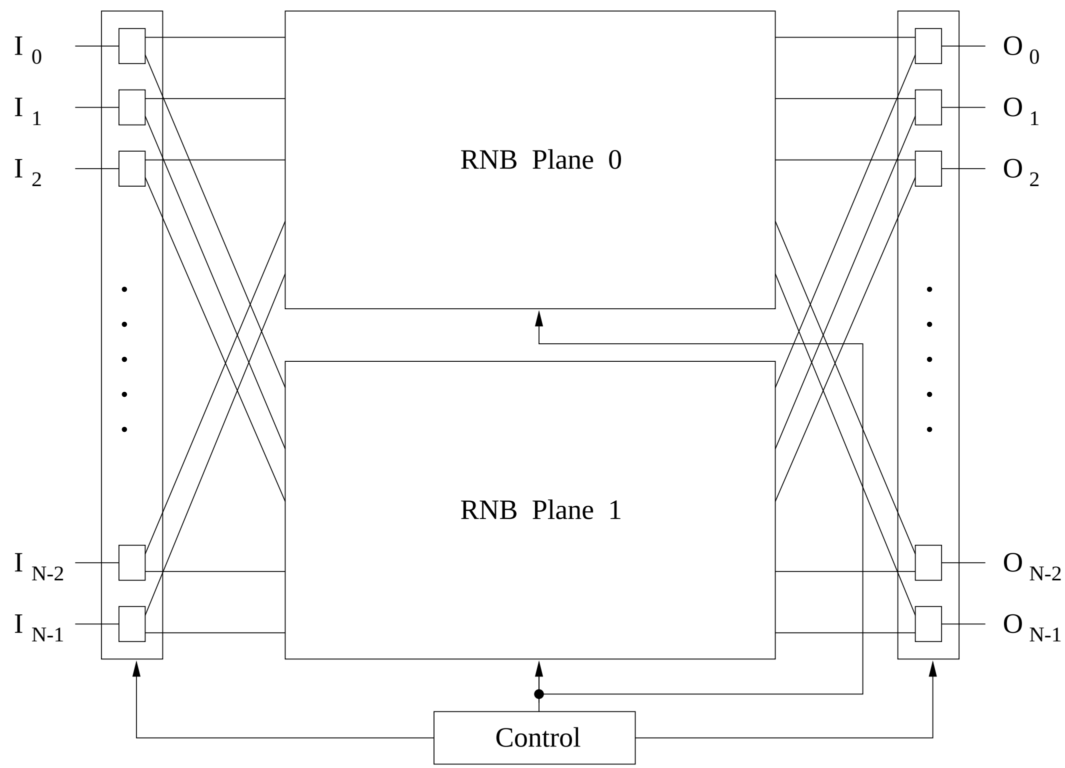
**A switching network is a virtual nonblocking (*VNB*) network if it satisfies the following conditions:**

**(i) it provides more than one path from any input (i.e. source) to any output (i.e. destination) such that different paths that connect the same source/destination pair can be momentarily set up simultaneously but any two paths connecting different input/output pairs are link-disjoint at any time; and**

**(ii) when properly controlled, the network can perfectly emulate an *WSNB* network.**

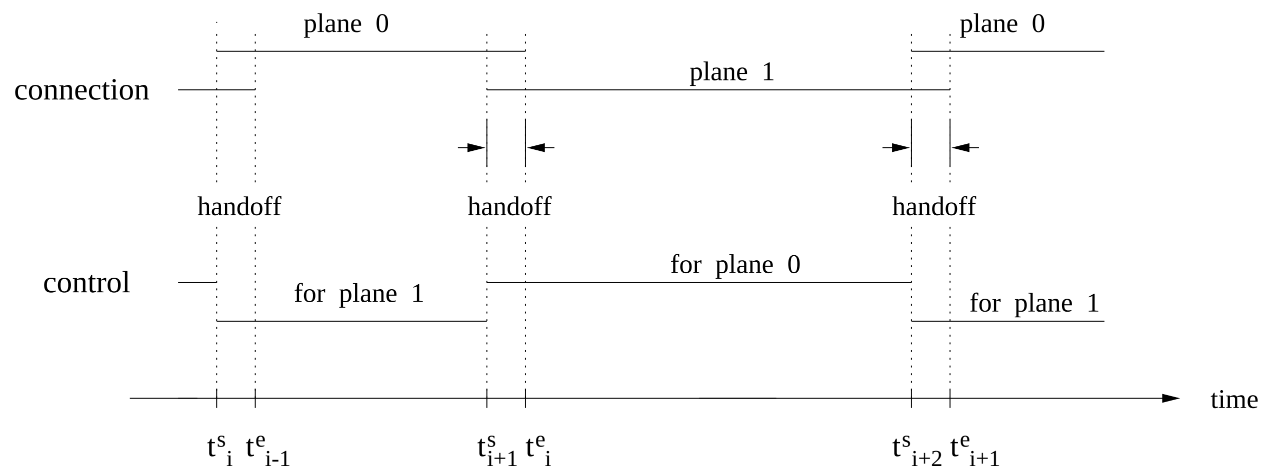
**This definition is broad enough to cover many embodiments of *VNB* network architectures.**

# Ping-Pang VNB Network




**Ping-pang *VNB* network architecture.**

# Handoff Process of Ping-Pang VNB Network




**Timing diagram of the operations of a ping-pang *VNB* network.**

# Handoff Schemes

*Lazy handoff* **scheme: handoff is only needed when finding conflict-free paths fails.**

*Eager handoff* **scheme: handoff is invoked when new connection requests arrive, regardless of whether conflict-free paths for the new connection requests exist or not.**

*VNB* **networks are useful in circuit switched networks. They are used as switching networks within network routers and switches to establish and maintain nondisruptive circuits.**

# Optimal VNB Networks

**The ping-pang VNB network constructed using two copies of Benes network has cost $O(N \log N)$, whitch is optimal.**

# Routing Performance Consideration

**Parallel Benes routing algorithm using an $N$-processor hypercube takes $O(\log^4)$ time, which is too slow.**

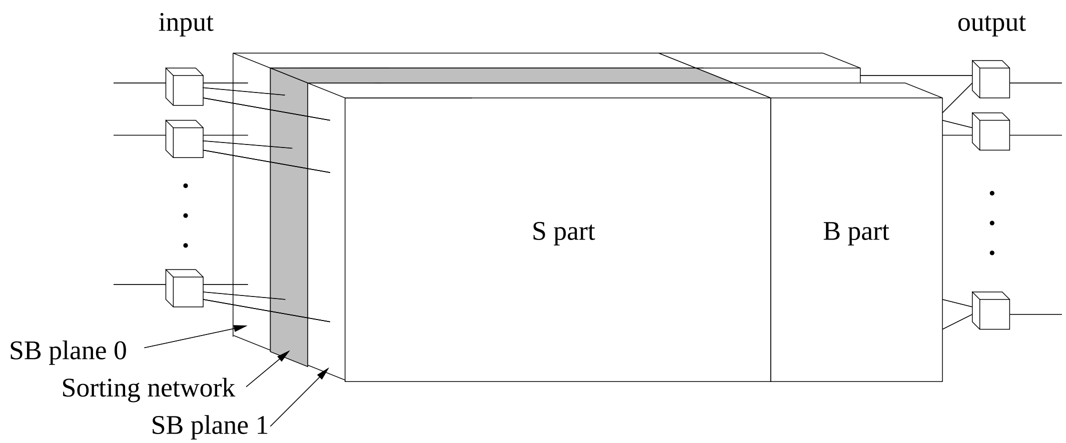
**Self-routing: the ability that any connection in a network can be established by inspecting addresses of its source and destination regardless of other connections.**

**Generalization: This property can be generalized to include the case that the setup of each SE only depends on the $O(\log N)$ information, which may not be the addresses of source and destination, available at the SE during routing.**

**Self-routing VNB networks are desirable.**

# SB VNB Networks

**Each plane is a *concatenation* of a Sorting network and a Banyan (butterfly, Omega, baseline) network.**

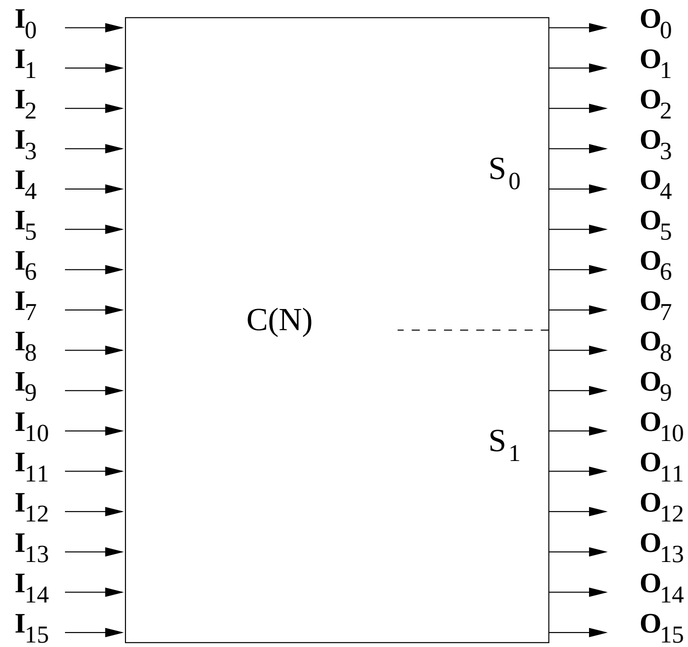



**An *SB* ping-pang *VNB* network.**

**Using AKS sorting network, this VNB network has cost $O(N \log N)$. But this is not practical because the coefficient hidden by big-$O$ is too large.**

# Q-sort: A New Sorting Network

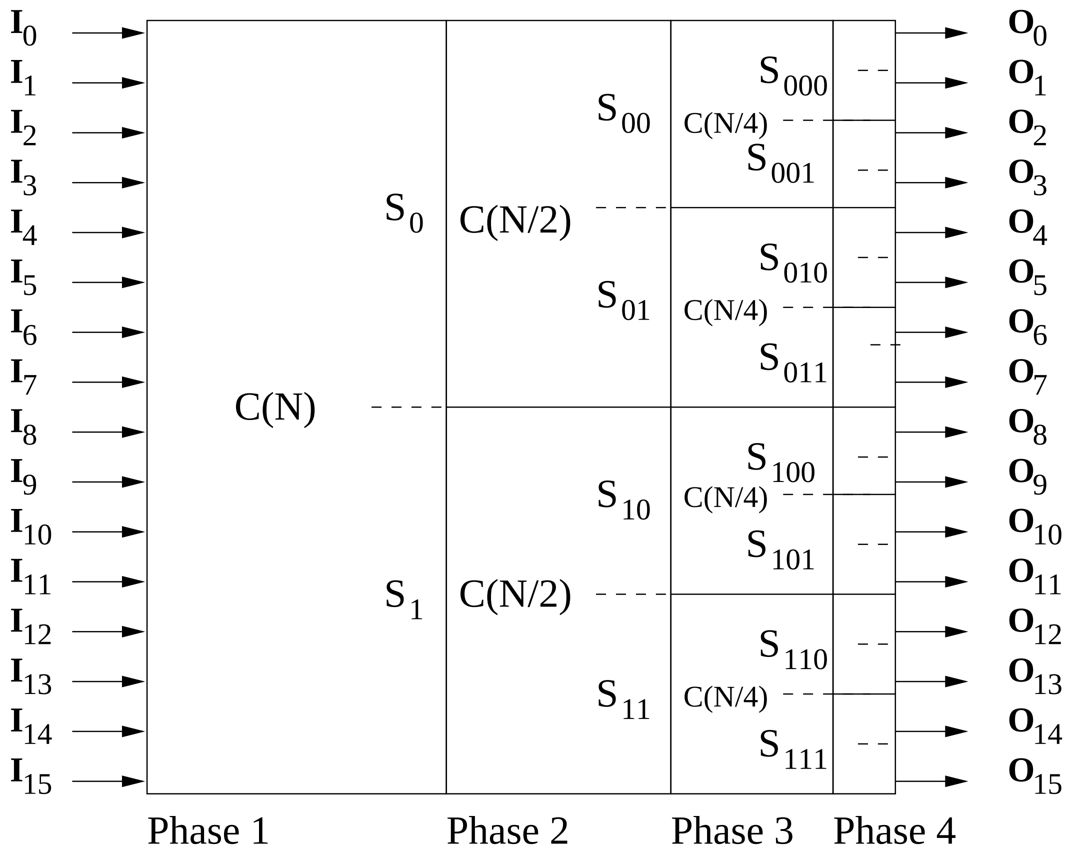
**Building block: $N$-classifier $C(N)$.**




**Classifier $C(16)$.**

**$C(N)$ is an $N \times N$ network that separates a set $S$ of $N$ input numbers into two subsets $S_0$ and $S_1$ such that $|S_0| = |S_1| = \frac{N}{2}$, no number in $S_0$ is larger than any number in $S_1$, and numbers in $S_0$ and $S_1$ appear at the first and second half of the outputs, respectively.**

# $Q$-Sort Network $Q(N)$: Constructed Using Classifiers Recursively

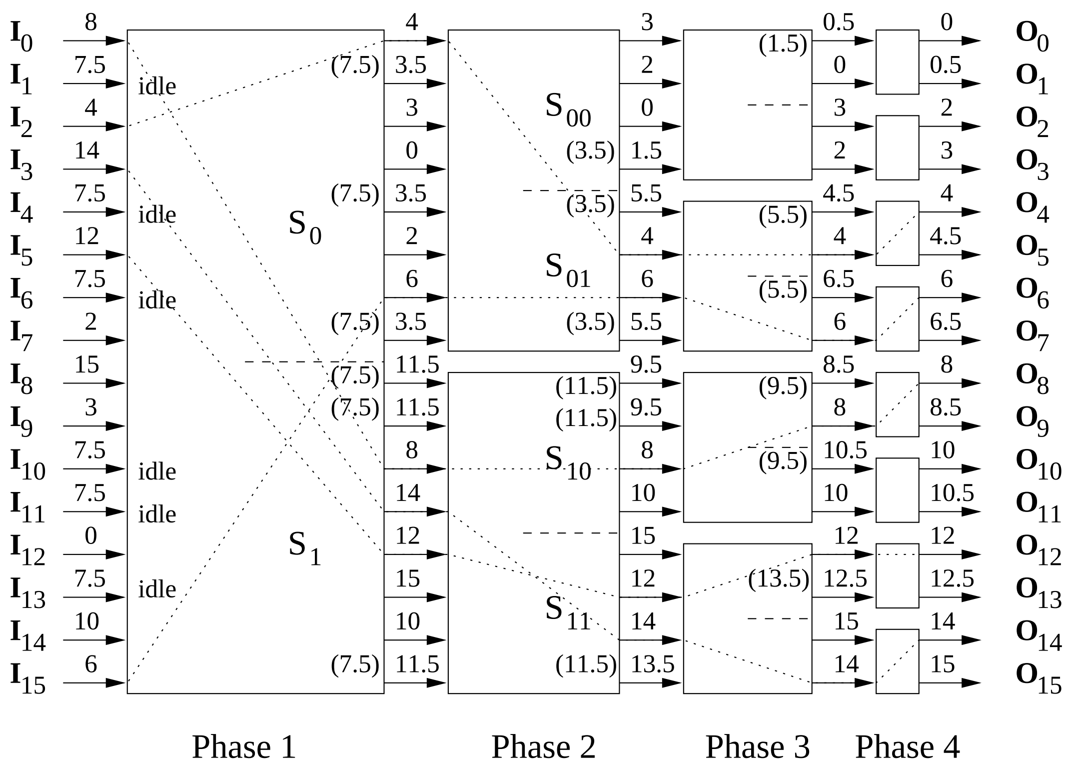



A $Q$-sort network $Q(N)$, $N = 16$. Cost: $O(N \log^2 N)$.

# $Q$-Sort Network $Q(N)$ as an RNB Network

## Routing a partial permutation

$$\pi = (h_0, h_1, \cdots, h_{N-1}) = (8, \Lambda, 4, 14, \Lambda, 12, \Lambda, 2, 15, 3, \Lambda, \Lambda, 0, \Lambda, 10, 6)$$

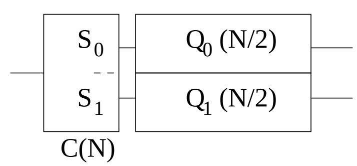



**Self-routing process in $Q(16)$ for partial permutation $\pi$, where $h_i = \Lambda$ indicates that input $i$ is idle (not to be connected to output).**

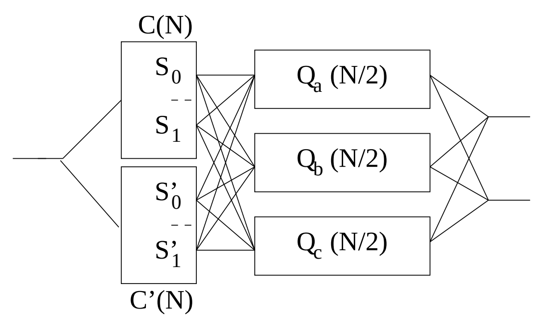
## Self-Routing Ping-Pang VNB Network Based in $Q(N)$

**Using two copies of $Q$-sort network, we can construct a self-routing ping-pang *VNB* network.**

## $Q^*(N)$ Network: Cost Saving VNB Network



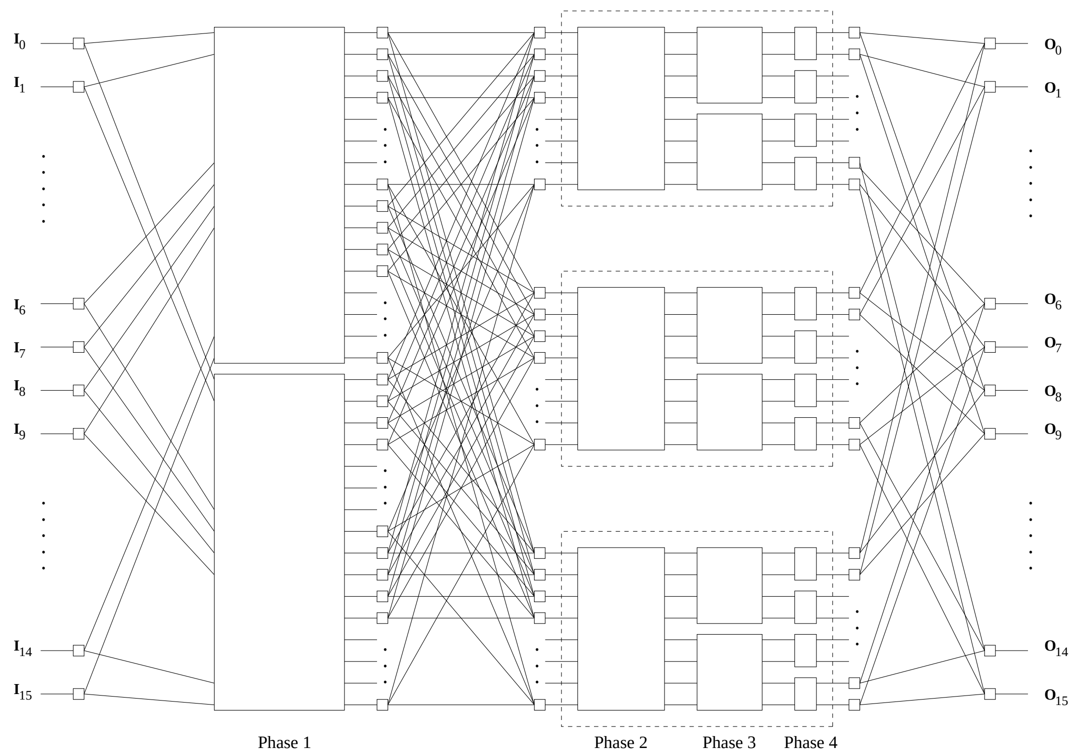

(a)




(b)

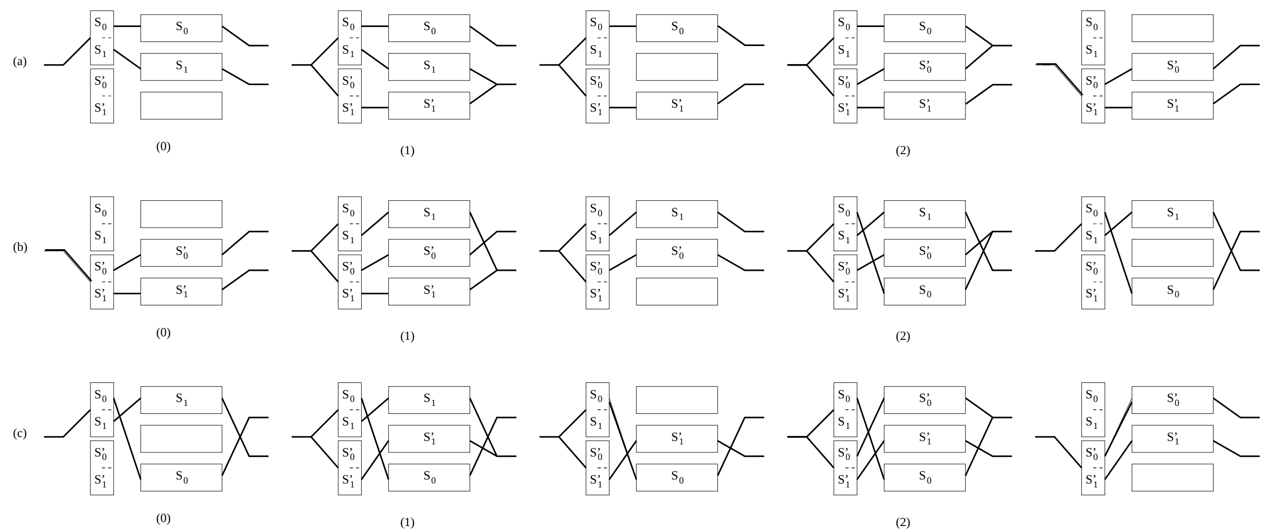
**(a) The structures of a $Q$-sort network $Q(N)$. (b) The cost-saving *VNB* network $Q^*(N)$ based on $Q(N)$.**

# More Details of $Q^*(N)$ Network




The interconnection details of $Q^*(16)$ The part in a dashed block is a $Q(8)$.

# Cyclic Handoff Scheme for $Q^*(N)$ Network



Handoff operations of $Q^*(N)$.

# Summary for Permutation VNB Switching Networks

- **The concept of virtual nonblocking (*VNB*) networks, which are operationally equivalent to SNB or WSNB networks but as simple as *RNB* networks.**

- **The the ping-pang structure, for designing a class of *VNB* networks.**

- **It is simple to design an optimal *VNB* network that contains $O(N \log N)$ SEs using two Benes networks. In contrast, no explicitly constructed *SNB* network of cost in the order of $O(N \log N)$ cost with a small coefficient has been reported in the literature.**

- **Theoretically it is possible to construct self-routing *VNB* networks with $O(N \log N)$ SEs. Practically it is easy to construct self-routing *VNB* networks with $O(N \log^2 N)$ SEs by adopting sorting networks. To the best of our knowledge, no self-routing *SNB* network of $O(N \log^2 N)$ SEs has been reported in the literature.**

- **A new sorting network, called the $Q$-sort network, based on classifiers, and show how to construct self-routing *RNB* networks with $O(N \log^2 N)$ cost.**

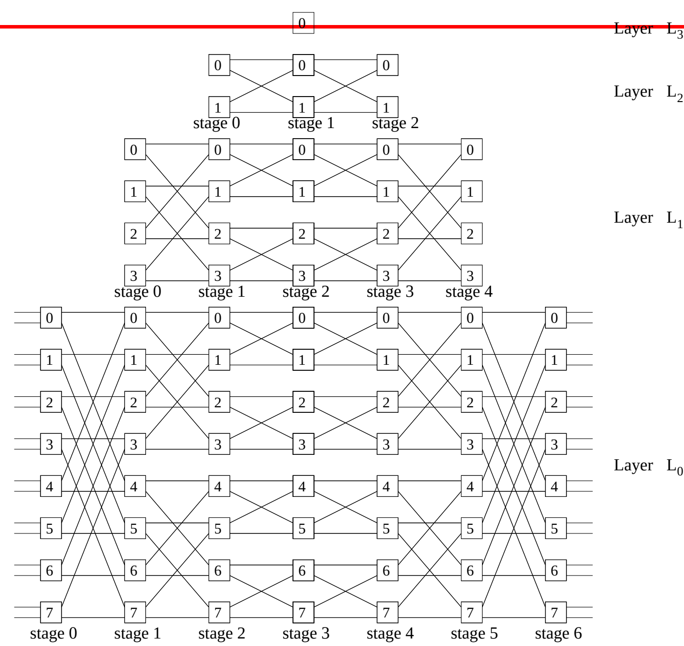
- **A cyclic handoff scheme for constructing *VNB* network $Q^*(N)$ with reduced cost.**

- ***VNB* networks can achieve lower cost and faster routing, which are not easy to achieve simultaneously for *SNB* networks. Thus, for all circuit switching applications, all we need are *VNB* networks.**

# Almost-Nonblocking Permutation Switching Networks

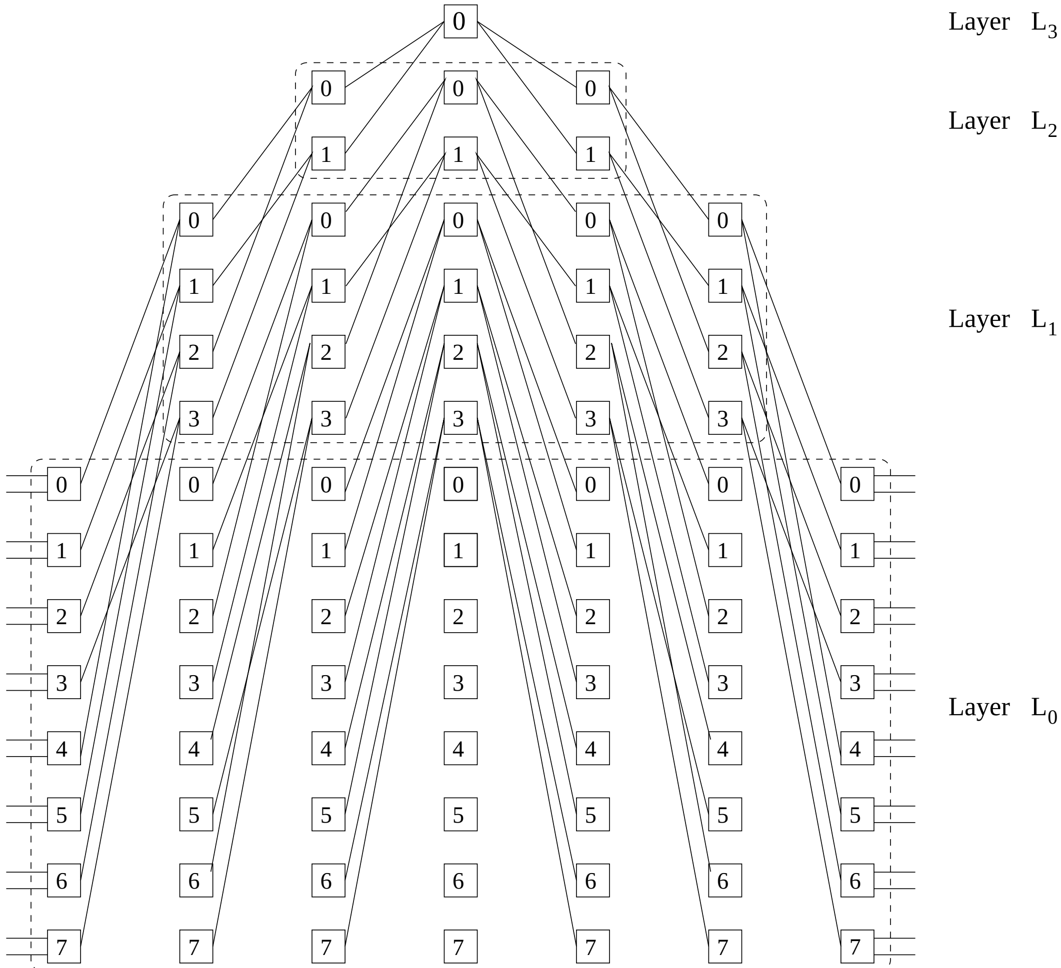
*Pyramid Benes network* $PB(2^n)$ **has** $n-1$ **layers** $L_i$**,** $0 \leq i \leq n-2$**, such that** $L_i$ **is a** $2^{n-i} \times 2^{n-i}$ **Benes network.**

## Intra-layer Connections




**Topology of** $PB(16)$**: Intra-layer connections.**

# Inter-layer Connections




**Topology of $PB(16)$: Inter-layer connections.**

# Nonblokingness of $PB(N)$ Networks

**$PB(N)$ has $O(N \log N)$ cost.**

**By extensive simulations, we have not found any blocking case for $PB(N)$.**

**We cannot claim that $PB(N)$ is strictly nonblocking.**

**However, we can claim that $PB(N)$ is** *almost strictly nonblocking*.

# Multiple-Multicasting Switching Networks

**Broadcasting is a communication pattern which sends a message from a given input port $i$ to all output ports.**

**Multicast is a communication pattern that sends a message from a given input port $i$ to a subset $O_i$ of output ports.**

**A multicast I/O connection pattern is represented by a pair $(i, O_i)$.**

**A** *legal multiple-multicast* **is a communication pattern that is represented by a set of pairs $\{(i_1, O_{i_1}), (i_1, O_{i_1}), \cdots, (i_m, O_{i_m})\}$ such that $i_j \neq i_k$ and $O_{i_j} \cap O_{i_k} = \emptyset$ if $j \neq k$.**

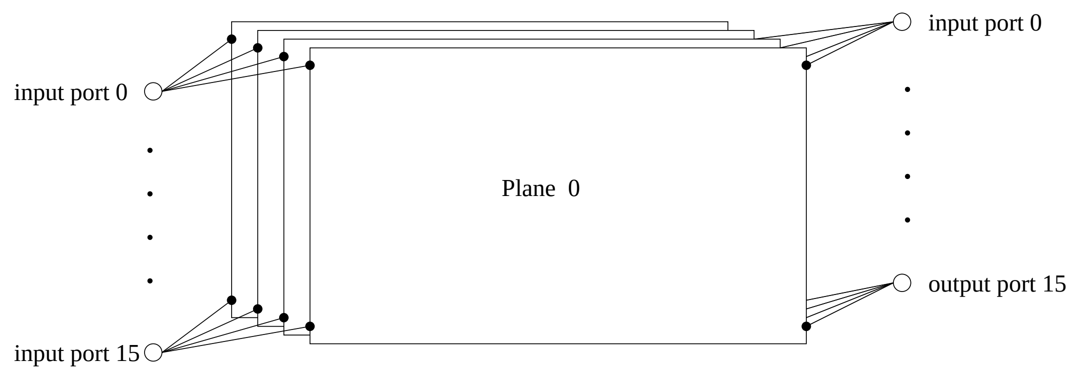
**Permutation (i.e one-to-one) and broadcast (i.e. one-to-all) are special cases of legal multiple-multicast.**

# VNB Networks for Multiple-Multicast

**In order to obtain a VNB network with multiple-multicast capability, we have to design a RNB network with multiple-multicast capability.**

# Multi-log$_2N$ Networks

**An $N \times N$ network of $p$ identical planes, each being a multistage network of $O(\log_2 N)$ stages.**

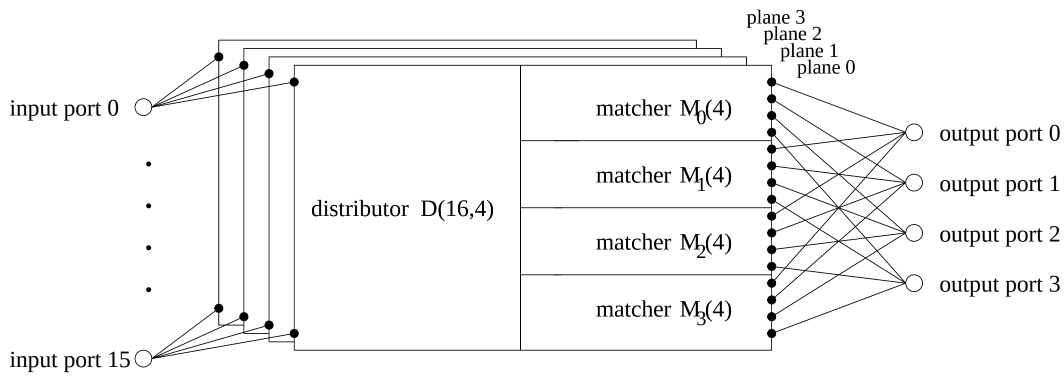



**Multi-$\log_2 N$ networks.**

**If each plane is a Banyan network, then $p \geq 2^{\lfloor \frac{n}{2} \rfloor}$ planes are required for the multi-log$_2N$ network to be an RNB permutation network. Cost: $O(N^{1.5} \log N)$.**

**No known multi-log$_2N$ of cost $O(N^{1.5} \log N)$ can realize conflict-free multiple-multicast.**

# $DM(N)$: A Self-Routing Multi-$\log_2 N$ Network for Multiple-Multicast

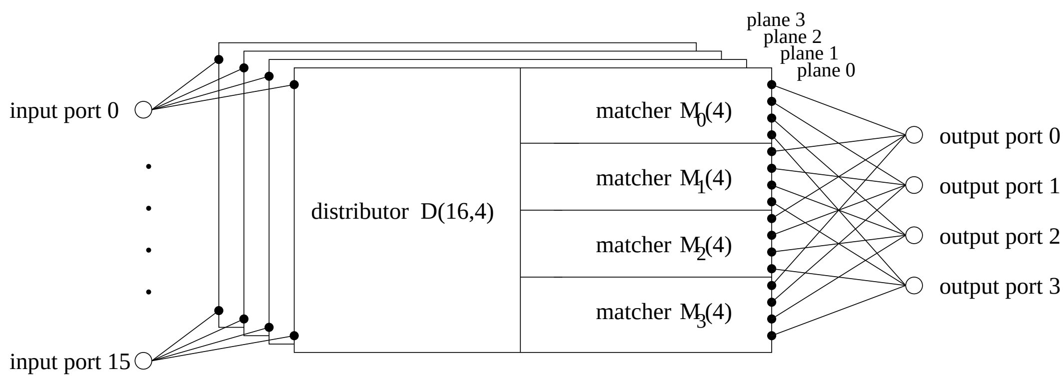



**Structure of $DM(N)$ networks.**

# Structure of $DM(N)$ Network

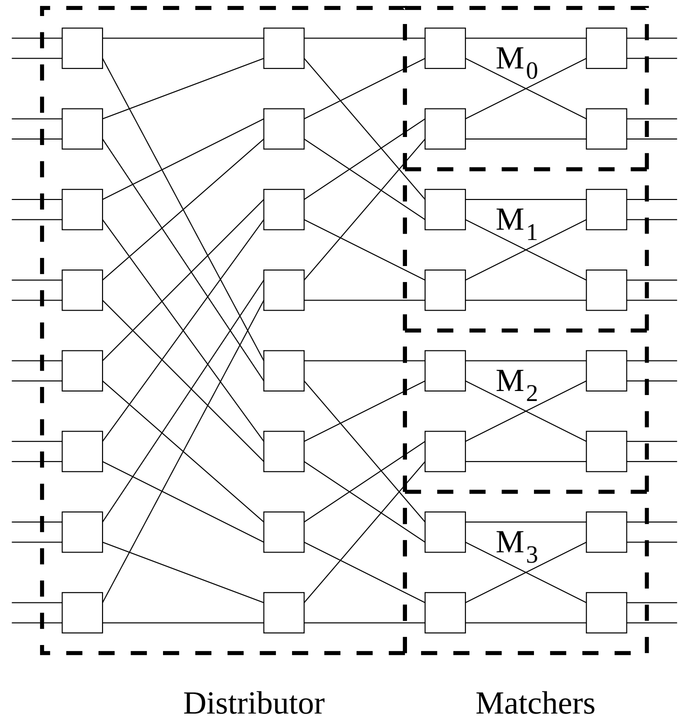
**Assume that $N = 2^n$, where $n$ is even.**

- **$\sqrt{N}$ planes, $P_0, P_1, \cdots, P_{\sqrt{N}-1}$, each being a baseline network.**
- **Input port $i$ is connected to the $i$-th inputs of all planes through a tree $T_i$.**
- **Plane $P_j$ is dedicated to connecting any input port to any output ports $j \cdot \sqrt{N} + k$, $0 \leq k < \sqrt{N}$.**
- **Plane $P_j$ has a distributor, and $\sqrt{N}$ matchers , each having $\sqrt{N}$ outputs, with the $k$th outputs of the matchers connected to output port $j \cdot \sqrt{N} + k$.**




**Structure of $DM(N)$ networks.**

# Structure of a Plane




**A plane of $DM(16)$ is a Baseline network.**

# 3-Phase Routing Algorithm

**An input is *active* if it is to be connected to an output.**

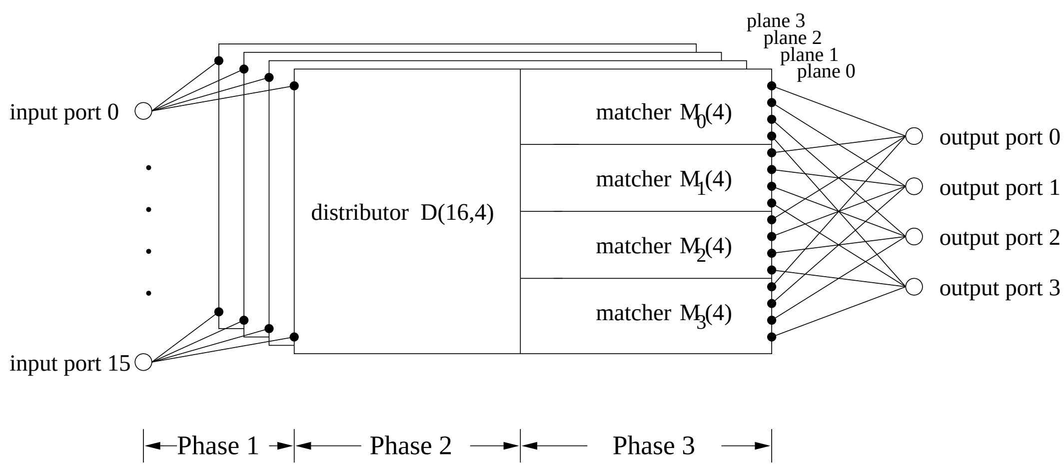
**Multiple-multicast communication pattern:** $\{(i_1, O_{i_1}), (i_1, O_{i_1}), \cdots, (i_m, O_{i_m})$ **such that** $i_j \neq i_k$ **and** $O_{i_j} \cap O_{i_k} = \emptyset$ **if** $j \neq k$**.**

**Let** $O_{i_j,p} = \{q|$ **output** $q$ **belongs to the** $k$**-th plane** $\}$**.** $O_{i_j,p}$ **is the set of output ports to be connected to input port** $i_j$ **in plane** $P_p$**.**

# Phase 1: Route inputs to the inputs of planes.

**For all $(i_j, O_{i_j})$ do the following in parallel:**

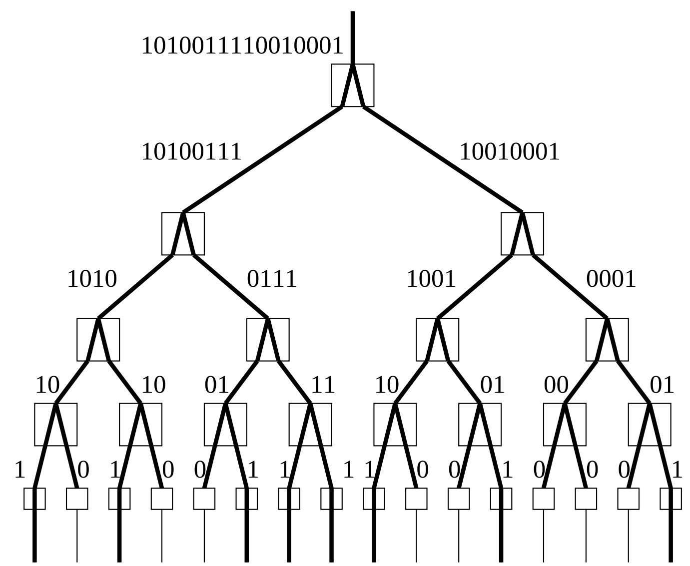
**Connect input port $i_j$ to the $k$-th plane if $O(i_j, k) \neq \emptyset$ using input tree $T_{i_j}$.**




**3-phase routing in $DM(N)$ networks.**

# Implementatin of Phase 1

Each input port $i_j$ computes a bit string for planes it has to be connected. Then, this bit string is used to route through the tree $T_{i,j}$ using broadcast-and-select technique.

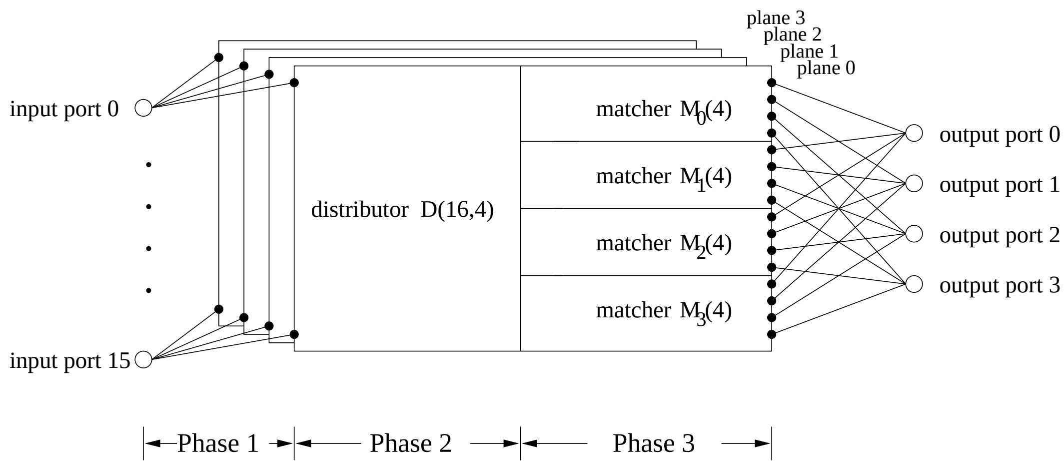



Broadcast-and-select: with a bit-string 101001111001000, the input of the tree is connected to the outputs specified by the bit-string.

# Phase 2: Routing through distributors.

**For all planes do the following in parallel:**

**Connect active inputs of each plane to the inputs of the plane's matchers such that each matcher has at most 2 active inputs.**




**3-phase routing in $DM(N)$ networks.**

# Implementation of Phase 2

**Each plane is equipped with a circuit for generating prefix sums of $N$ Boolean values, which are used for self-routing in the plane. We call this circuit *intraplane routing preprocessing* (IRP) circuit.**

**It is sufficient to consider routing operations in a single plane.**

**If input $i$ is active, set $R_i = 1$; else set $R_i = 0$.**

**The IRP circuit is used to compute binary prefix sums of $R_i$s, where the binary prefix sum $S_i$ is defined as**

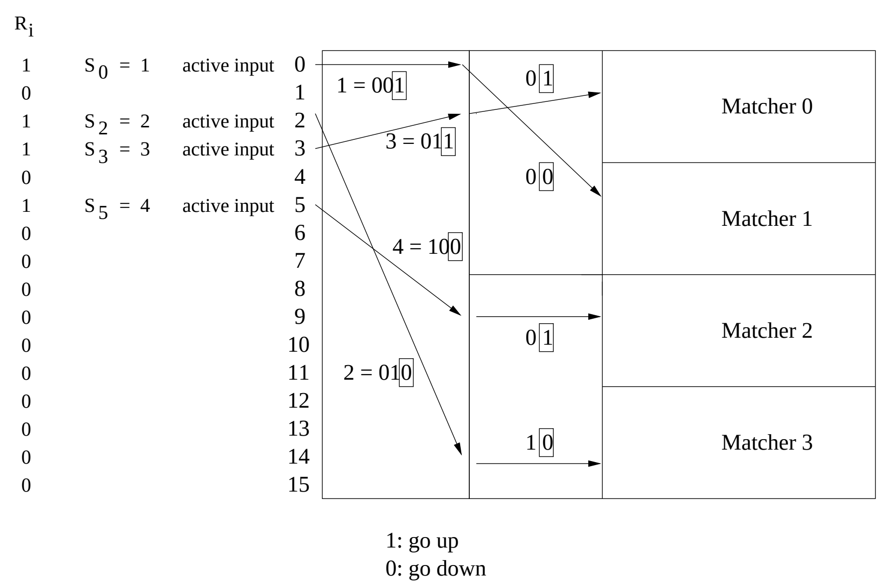
$$S_i = R_0 + R_1 + \cdots + R_i \tag{1}$$

**$S_i$s can be computed in $O(\log N)$ time by an IRP circuit of tree structure.**

**Use the bits of $S_i$ to route active inputs up or down recursively.**

# Example:

**For** $R_0 = 1, R_1 = 0, R_2 = 1, R_3 = 1, R_4 = 0, R_5 = 1, R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_15 = 0$, **we have** $S_0 = 1, S_1 = 1, S_2 = 2, S_3 = 3, S_4 = 3, S_5 = 4, S_6 = S_7 = S_8 = S_9 = S_{10} = S_{11} = S_{12} = S_{13} = S_{14} = S_{15} = 4$.

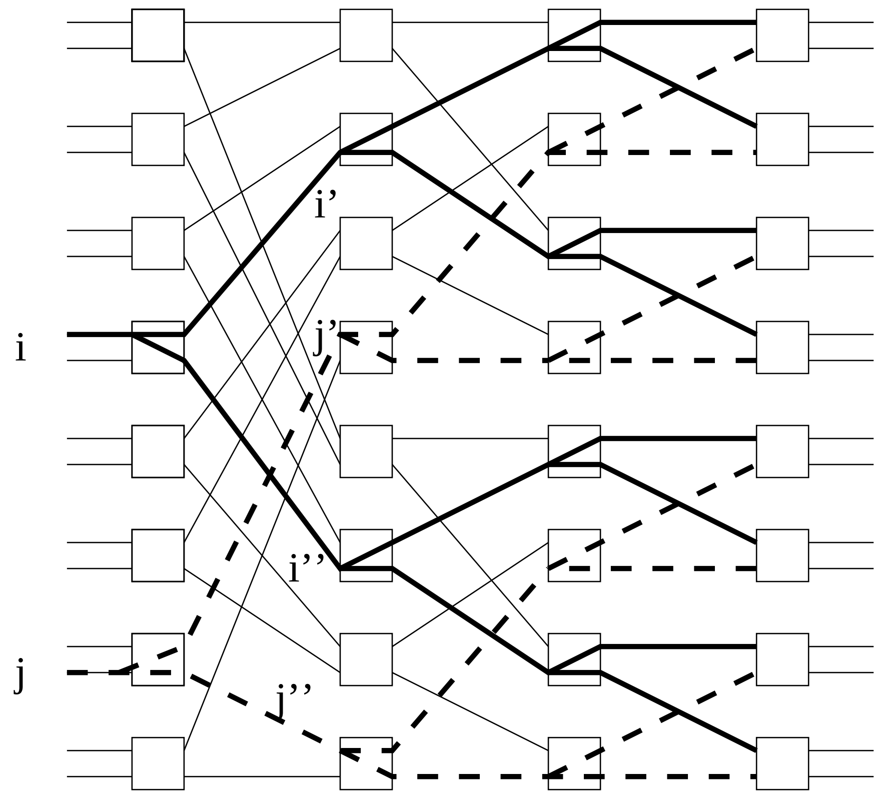



**Self-routing in Phase 2.**

# Active Inputs of Matchers

**Phase 2 guarantees that each matcher has at most 2 active inputs.**

**Phase 2 guarantees the two active inputs have link-disjoint broadcast trees in the matcher, which is a Baseline subnetwork.**

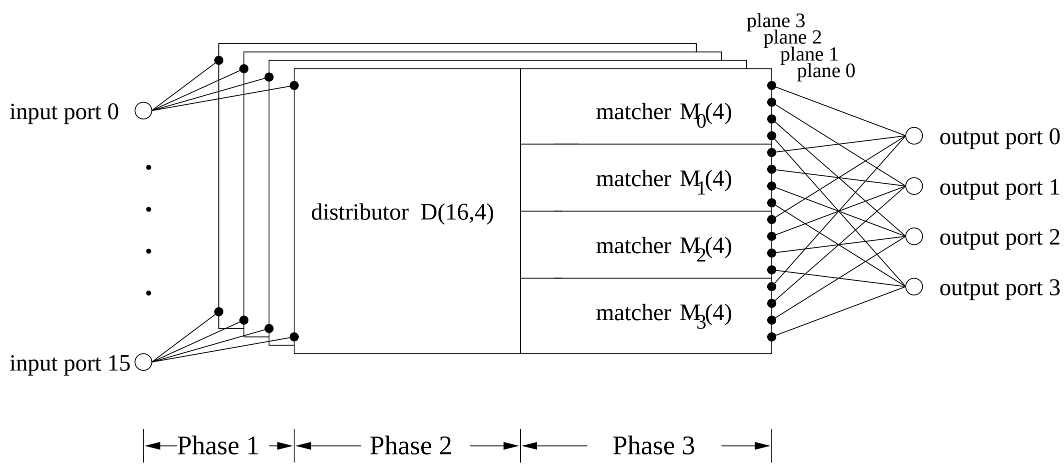



**Two broadcasting trees in $M(16)$.**

# Phase 3: Routing through matchers.

For all matchers in all planes $P_k$, $0 \le k < \sqrt{N}$, do the following in parallel:

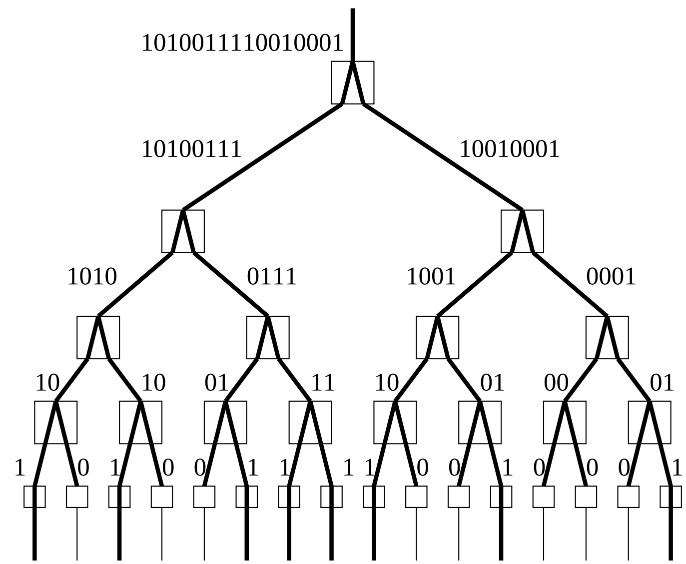
If an active input is originated from input port $i_j$, then connect the active input to all outputs of $O_{i_j,k}$ within the matcher.




3-phase routing in $DM(N)$ networks.

# Implementation of Phase 3

## Using broadcast-and-select technique based on bit strings..




## Broadcast-and-select.

# Summary on Routing $DM(N)$ Network

**Routing in $DM(N)$ is performed in 3 phases.**

**Bit strings for Phase 1 and Phase 3 are computed by input ports.**

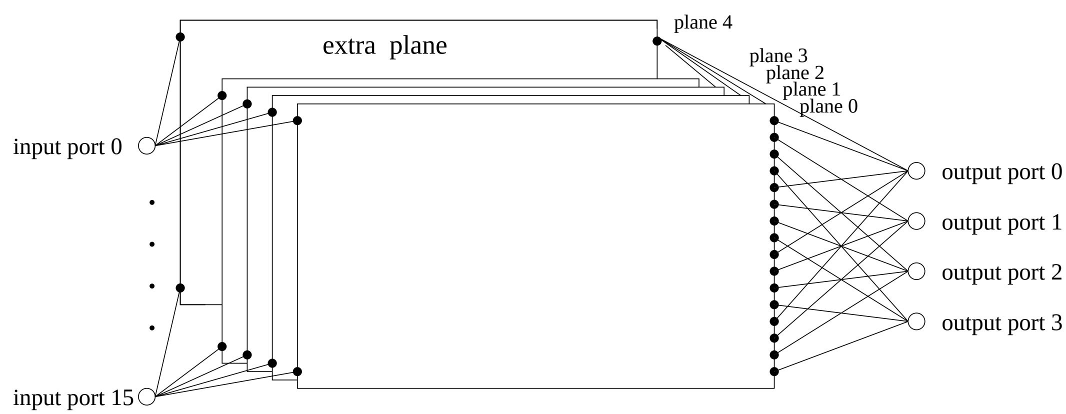
**Binary profix sums for Phase 2 are performed by a circuit at the input of each plane.**

**Then, self-routing is conducted in all phases.**

# Making $DM(N)$ Virtually Nonblocking

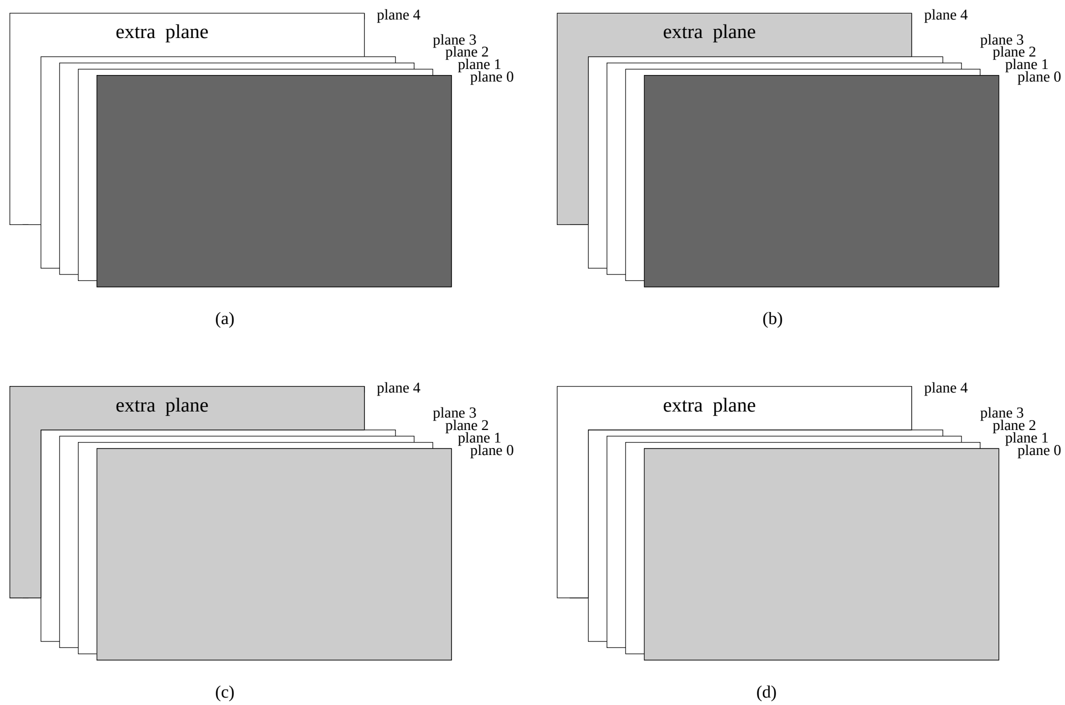
**Using ping-pang scheme, we need two copies of $DM(N)$.**

**But we can reduce the cost by only introduce 1 extra plane.**




**Extended $DM(16)$ network.**

# Hand-off in Virtually Nonblocking Multiple-Multicast $DM(N)$ Network




**Using 1 extra plane to do hand-off.**

# Concluding Remarks

**The high degree of connection capability in** *SNB* **and** *WSNB* **networks is achieved at much higher cost, which implies their lower scalability.**

**We introduce the concept of virtual nonblockingness. We show that a virtual nonblocking network functions like a strictly or wide-sense nonblocking network, but it is constructed with the cost of a rearrangeable nonblocking network.**

**While finding low cost** *SNB* **networks remains to be a challenging theoretical and scientific pursuit, VNB networks can satisfy all the requirements of an** *SNB* **or** *WSNB* **in practice.**

**Our results indicate that for large-scale circuit switching applications, we only need to build virtual nonblocking networks.**