A DUALITY FOR QUASI ORDERED STRUCTURES (I)

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ABSTRACT

Recently, several authors extended Priestley duality for distributive lattices [9] to other classes of algebras, such as, e.g. distributive lattices with operators [7], MV-algebras [8], MTL and $IMTL$ algebras [2]. In [5] necessary and sufficient conditions for a normally presented variety to be naturally dualizable, in the sense of [6], i.e. with respect to a discrete topology, have been provided. Under this perspective, also bounded distributive quasi lattices (bdq-lattices), introduced in [4], are naturally dualizable.

Nonetheless, quasi lattices, constitute a generalization of lattice ordered structures to quasi ordered ones, i.e. structures in which the ordering relation is reflexive and transitive, but it may fail to be antisymmetric. Consequently, a sensible question arises: is there any "natural" candidate which stands to Priestley spaces as bounded distributive quasi lattices stand to bounded distributive lattices?

In this talk, following an idea from [1], where a representation of bdq-lattices has been proposed, we present an alternative form of dualization of bdq-lattices via the notion of *preordered Priestley spaces*. Preordered Priestley spaces are, in our opinion, of interest in that they naturally generalize Priestley spaces to a preordered setting, and, at the same time, they share with Priestley spaces desirable features. We will see that preordered Priestley spaces interpret, with respect to bdq-lattices, exactly the same rôle Priestley spaces play with respect to bounded distributive lattices.

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