

# Finite Embeddability Property of S4 modal residuated groupoids

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Abstract

We prove FEP of the class of residuated groupoids with an unary (modal) operator  $\diamond$ , and its residual  $\square$ , satisfying the S4-axioms. T:  $a \leq \diamond a$ , 4:  $\diamond \diamond a \leq a$  and a special axiom K:  $\diamond(a \cdot b) \leq \diamond a \cdot \diamond b$  introduced by Moortgat[10].  $NL_{S4}$  denotes Nonassociative Lambek Calculus with S4-modalities  $\diamond$  and its residual  $\square$ . The proof also yields FEP for the classes axiomatized by T and 4 or T only.

Blok and van Alten [5] prove FEP of the class of integral residuated groupoids, but they leave it as an open problem (Problem 4.2) whether the integrality assumption can be dropped. It was solved by Farulewski (2006, published RML 2008): the class of residuated groupoids has FEP. Buszkowski and Farulewski[3] and Buszkowski[4] show many other results of this kind, not assuming integrality. This paper continues line of research. Some techniques e.g. interpolation of formula-trees by formulae, are taken from [4]. However we directly prove the extended subformula property for systems with assumptions, using a cut-elimination theorem for these systems. (This theorem also holds for many other substructural logics, e.g Lambek calculus, Full Lambek Calculus.)

Further, we use nuclear completions of the form elaborated in Okada and Terui [9], Belardinelli, Jipsen and Ono [6], and adapted to nonassociative systems in Buszkowski and Farulewski [3].

In [8] we have shown that categorial grammars based on  $NL_{S4}$  with assumptions generate context-free languages.

The key argument of the papers is the extended subformula property for Gentzen style axiomatization and the interpolation lemma. By using an interpolation of members of the free groupoid, generated by a finite partial algebra by members of this algebra, we established FEP for the class of S4 modal residuated groupoids.

## References

1. Shun'ichi J.Amano, Finite Embeddability Property for Some Modal Algebra, Master thesis, School of information Science Japan Advance Insititue of Science and Techology.

2. W. Buszkowski, Lambek Calculus with Nonlogical Axioms. In C. Casadio, P. J. Scott, and R. A. G. Seely (eds.), *Language and Grammar. Studies in Mathematical Linguistics and Natural Language*, CSLI, Lecture Notes 168:77-93, Stanford 2005
3. W. Buszkowski, and M. Farulewski, Nonassociative Lambek Calculus with Additives and Context-Free Languages. In: O. Grumberg et al. (eds.), *Languages: From Formal to Natural*, LNCS 5533:45-58, 2009.
4. W. Buszkowski, Interpolation and FEP for Logic of Residuated Algebras, *Logic Journal of the IGPL*, To appear (available in an electronic version via Advanced Access).
5. W. J. Blok, C. J. Van Alten On the finite embeddability property for residuated ordered groupoids, *Transactions of the AMS* 357 (10) (2005).
6. F. Belardinelli, P. Jipsen, and H. Ono, Algebraic Aspects of Cut Elimination, *Studia Logica* 77:209-240, 2004.
7. J. Lambek, The mathematics of sentence structure. *American Mathematical Monthly* 65:154-170, 1958.
8. Z. Lin, Modal Nonassociative Lambek Calculus with Assumptions: Complexity and Context-freeness, In: A.H. Dediu, H. Fernau, C. Martin-Vide (eds.), *Language and Automata Theory and Applications*, Fourth International Conference, LATA 2010, LNCS 6031, 414-426.
9. M. Okada, and K. Terui, The Finite Model Property for Various Fragments of Intuitionistic Linear Logic *Journal of Symbolic Logic* 64:790-802, 1999.
10. M. Moortgat, Multimodal linguistic inference. *Journal of Logic, Language and Information* 5:349-385, 1996.
11. A.R. Plummer, S4 enriched multimodal categorial grammars are context-free. *Theoretical Computer Science* 388:173-180, 2007. Corrigendum. *Theoretical Computer Science* 403:406-408, 2008.
12. N. Galatos, P. Jipsen, T. Kowalski and H. Ono, *Residuated Lattices: An Algebraic Glimpse at Substructural Logic*. Elsevier, 2007.