Product of Hybrid Logics

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Hybrid logic is an extended modal logic with nominals \( i \) (a syntactic name of a state) and satisfaction operators \( @i\varphi \) (\( \varphi \) is true at the named state by \( i \)). In this talk, I will propose how to combine two hybrid logics, i.e., a way of dealing with two dimensions (e.g., any two domains from time, space, possible worlds, and individuals) at the same time in one setting. My way of combining hybrid logics can be regarded as an expansion of product of modal logics [1]. In the first part, I will explain how to ‘product’ two hybrid logics over Kripke frames and establish a general completeness result (called pure completeness) for Kripke semantics [2]. A key idea is that two kinds of nominals and satisfaction operators enable us to capture the interaction between two dimensions in terms of the following five axioms:

\[
\begin{align*}
\text{Com} & : @1 @2 \quad @1 @a \leftrightarrow @a @1p \\
\text{Com} & : @2 @1 \quad @2 @i \leftrightarrow @i @2p \\
\text{Red} & : @i a \leftrightarrow a \\
\text{Red} & : @i i \leftrightarrow i \\
\text{Com} & : @a @1p \leftrightarrow @i @a p
\end{align*}
\]

In the second part, I will extend this product method to hybrid logics over topological spaces and show that we can still keep a general completeness result [3] (remark that the above axioms are still valid on all product of topological spaces). This can be regarded as a two-dimensional hybrid expansion of [5]. If time permits, I will also explain how to capture the dependence of one dimension (space) over the other (time) and touch on how to generalize the method of this talk into the coalgebraic level [4].

References


