Hyper Natural Deduction with Normalisation\(^1\)
WIP

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joint work with Arnold Beckmann

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Setting the stage
Setting the stage
Setting the stage

IL ⇐⇒ Curry Howard ⇔ LJ ⇔ ND ⇔ GL
Setting the stage

IL ⇐⇒ Curry Howard ⇔ GL

λ
Setting the stage

IL $\iff$ LJ

Curry Howard $\iff \lambda$

Gentzen ’34
Setting the stage

IL $\iff$ LJ

Curry Howard $\iff\lambda$

Sequent
$\Gamma \Rightarrow \Delta$

Axiom
$A \Rightarrow A$

Rules
$\frac{\Gamma \Rightarrow A}{\Gamma, \Delta \Rightarrow A \land B}$
Setting the stage

\[ \text{Sequent} \quad \Gamma \Rightarrow \Delta \]

\[ \text{Axiom} \quad A \Rightarrow A \]

\[ \text{Rules} \quad \frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{\Gamma, \Delta \Rightarrow A \land B} \]

\[ \text{(cut)} \quad \frac{\Gamma \Rightarrow A \quad \Pi, A \Rightarrow \Delta}{\Gamma, \Pi \Rightarrow \Delta} \]

IL ⇔ LJ

Curry Howard ⇔ \lambda
Setting the stage

IL ⇔ LJ

Curry Howard ⇔ \lambda

Sequent
\Gamma \Rightarrow \Delta

Axiom
A \Rightarrow A

Rules
\Gamma \Rightarrow A \quad \Delta \Rightarrow B
\Gamma, \Delta \Rightarrow A \land B

(cut)
\Gamma \Rightarrow A \quad \Pi, A \Rightarrow \Delta
\Gamma, \Pi \Rightarrow \Delta

cut elimination – consistency
Setting the stage

IL ⇔ LJ ⇔ ND  \iff  Curry Howard  \iff  \lambda

Gentzen ’34
Setting the stage

IL ⇔ LJ ⇔ ND ⇔ \lambda

Curry Howard

introduction rule

\[ \frac{A \quad B}{A \land B} \]

elimination rule

\[ \frac{A \land B}{A} \]
Setting the stage

IL $\iff$ LJ $\iff$ ND $\iff$ Curry Howard $\iff$ $\lambda$

introduction rule

$$
\frac{A \quad B}{A \land B}
$$

elimination rule

$$
\frac{A \land B}{A} \quad (\rightarrow\text{-}e \text{ or } mp) \quad \frac{A}{A \rightarrow B}
$$
**Setting the stage**

\[ \text{IL} \iff \text{LJ} \iff \text{ND} \iff \text{Curry Howard} \]

**Introduction rule**

\[
\frac{A \quad B}{A \land B}
\]

**Elimination rule**

\[
\frac{A \land B}{A}
\]

\[
(\rightarrow\text{-e or mp}) \quad \frac{A}{B}
\]

**Normalisation** — elimination/introduction
Setting the stage

\[\text{IL} \iff \text{LJ} \iff \text{ND} \iff \text{Curry Howard} \iff \lambda\]
Gödel Logics
History

Timeline

1933

Gödel

finitely valued logics
History

Timeline

1933

Gödel

1959

Dummett

infinitely valued propositional Gödel logics
Timeline

1933  1959  1969

Gödel  Dummett  Horn

linearly ordered Heyting algebras
intuitionistic fuzzy logic
Timeline


Gödel  Dummett  Horn  Takeuti-Titani  Avron

hypersequent calculus
Timeline

1933: Gödel
1959: Dummett
1969: Horn
1984: Takeuti-Titani
1991: Avron
1998: Hájek

t-norm based logics
Timeline

1933: Gödel
1959
1969: Horn
1984: Takeuti-Titani
1991
1998: Avron
since 90ies: Hájek

Viennese group

proof theory, #, Kripke, qp, fragments, …
Standard propositional language

Semantics:
Fix $\varphi$ from set of propositional variables to $[0, 1]$

Extend valuations:

$$
\varphi(A \land B) = \min\{\varphi(A), \varphi(B)\}
$$

$$
\varphi(A \lor B) = \max\{\varphi(A), \varphi(B)\}
$$

$$
\varphi(A \rightarrow B) = \begin{cases} 
\varphi(B) & \text{if } \varphi(A) > \varphi(B) \\
1 & \text{if } \varphi(A) \leq \varphi(B) 
\end{cases}
$$
Using the definition of valuation we can collect all formulas that evaluate under all valuations based on $\varphi$ into the Gödel logic $G$:

$$G = \{ A : \text{for all } \varphi, \varphi(A) = 1 \}$$
Real valued based semantics

Using the definition of valuation we can collect all formulas that evaluate under all valuations based on $\varphi$ into the Gödel logic $G$:

$$G = \{ A : \text{for all } \varphi, \varphi(A) = 1 \}$$

Notes

- choice of $V$ is irrelevant as long as it is infinite
- often called Gödel-Dummett logic
- projecting logic, logic of order
**Equivalence result with linear Kripke frames**

### Gödel logic to Kripke frame
For each Gödel logic there is a countable linear Kripke frame such that the respective logics coincide.

### Kripke frames to Gödel logic
For each countable linear Kripke frame there is a Gödel truth value set such that the respective logics coincide.

Beckmann, P. ’07
Setting the stage

\[ IL \iff LJ \iff ND \iff \lambda \iff \text{Curry Howard} \iff \text{today's topic} \]

GL
Setting the stage

IL $\iff$ LJ $\iff$ ND $\iff$ Curry Howard $\iff$ $\lambda$

GL $\iff$ HLK

Avron ’91
Setting the stage

IL $\iff$ LJ $\iff$ ND $\iff$ Curry Howard

GL $\iff$ HLK

Hyper-Sequent

$\Gamma_1 \implies \Delta_1 \mid \ldots \mid \Gamma_n \implies \Delta_n$
Setting the stage

IL ⇔ LJ ⇔ ND

Curry Howard

GL ⇔ HLK

Hyper-sequent

\[ \Gamma_1 \Rightarrow \Delta_1 \mid \ldots \mid \Gamma_n \Rightarrow \Delta_n \]

deep inference
Setting the stage

\[
\begin{align*}
&\text{IL} \iff \text{LJ} \iff \text{ND} \iff \text{Curry Howard} \iff \lambda \\
&\text{GL} \iff \text{HLK} \\
\end{align*}
\]
Setting the stage

IL ⇔ LJ ⇔ ND \quad \text{Curry Howard} \quad \lambda

GL ⇔ HLK \quad \text{(com)} \quad \frac{\Gamma \Rightarrow A}{\Gamma \Rightarrow B} \mid \frac{\Delta \Rightarrow B}{\Delta \Rightarrow A}
Setting the stage

Curry Howard

IL ⇔ LJ ⇔ ND

GL ⇔ HLK

(com) \[
\frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{\Gamma \Rightarrow B \mid \Delta \Rightarrow A}
\]

Lorenzen style games, ...
Setting the stage

IL $\iff$ LJ $\iff$ ND $\iff$ Curry Howard $\iff$ $\lambda$

GL $\iff$ HLK $\iff$ ?
Setting the stage

IL ⇔ LJ ⇔ ND ⇔ Curry Howard ⇔ \( \lambda \)

GL ⇔ HLK ⇔ HND ⇔ ?
Setting the stage

IL $\Leftrightarrow$ LJ $\Leftrightarrow$ ND $\Leftrightarrow$ Curry Howard $\Leftrightarrow$ $\lambda$

GL $\Leftrightarrow$ HLK $\Leftrightarrow$ HND $\Leftrightarrow$ $\pi$?
Setting the stage

IL ⇔ LJ ⇔ ND \iff Curry Howard \iff \lambda

GL ⇔ HLK ⇔ HND \iff \pi?

today’s topic
Wishlist items

Properties we want to have

- local, or at least quasi-local
Wishlist items

Properties we want to have

- local, or at least quasi-local
- normalisation
Wishlist items

Properties we want to have

► local, or at least quasi-local
► normalisation

Previous work: Baaz, Ciabattoni, Fermüller ’00
Insufficient in both respects:
► local, but adding hyper-structure to natural deduction, just copying
► no normalisation, only via translation to HLK
Natural Deduction rules

$\land$-i \[ \frac{A \quad B}{A \land B} \]

$\land$-e \[ \frac{A \land B}{A} \quad \frac{A \land B}{B} \]

$\lor$-i \[ \frac{A}{A \lor B} \quad \frac{B}{A \lor B} \]

$\lor$-e \[ \frac{A \lor B}{C} \quad \frac{[A]}{C} \quad \frac{[B]}{C} \]

$\to$-i \[ \frac{B}{A \to B} \]

$\to$-e \[ \frac{A}{A \to B} \quad \frac{A}{B} \]

$\bot$-I \[ \frac{\bot}{A} \]
Additional rules

Ideas:

- split rules
- require additional properties on proof figure to guarantee correctness
**Additional rules**

Idea:
- split rules
- require additional properties on proof figure to guarantee correctness

Let $X$ countable set of names, assume duality of names

\[ \Gamma \]

\[ A \]

\[ B \]

\[ \text{where } x \text{ is a name in } X \]
Linearity in LJ

\[ \Rightarrow (A \rightarrow B) \lor (B \rightarrow A) \]
Linearity in LJ

\[ \Rightarrow A \rightarrow B \]

\[ \Rightarrow (A \rightarrow B) \lor (B \rightarrow A) \]
Linearity in LJ

\[ ??? \]
\[ A \Rightarrow B \]
\[ \Rightarrow A \rightarrow B \]
\[ \Rightarrow (A \rightarrow B) \lor (B \rightarrow A) \]
Linearity in HLK

\[ \Rightarrow (A \rightarrow B) \lor (B \rightarrow A) \]
Linearity in HLK

\[ \Rightarrow (A \rightarrow B) \lor (B \rightarrow A) \mid \Rightarrow (A \rightarrow B) \lor (B \rightarrow A) \]

\[ \Rightarrow (A \rightarrow B) \lor (B \rightarrow A) \quad \text{e-contr} \]
\[ \Rightarrow (A \rightarrow B) \vee (B \rightarrow A) \mid \Rightarrow B \rightarrow A \]
\[ \Rightarrow (A \rightarrow B) \vee (B \rightarrow A) \mid \Rightarrow (A \rightarrow B) \vee (B \rightarrow A) \]
\[ \Rightarrow (A \rightarrow B) \vee (B \rightarrow A) \]
\[ \vee -r \quad \text{e-contr} \]
Linearity in HLK

\[
\begin{align*}
\Rightarrow A \rightarrow B & \mid \Rightarrow B \rightarrow A \\
\Rightarrow (A \rightarrow B) \lor (B \rightarrow A) & \mid \Rightarrow B \rightarrow A \\
\Rightarrow (A \rightarrow B) \lor (B \rightarrow A) & \Rightarrow (A \rightarrow B) \lor (B \rightarrow A) \\
\Rightarrow (A \rightarrow B) \lor (B \rightarrow A) & \Rightarrow (A \rightarrow B) \lor (B \rightarrow A) \\
\end{align*}
\]
**Linearity in HLK**

\[
\Rightarrow \frac{A \rightarrow B \mid B \Rightarrow \bar{A}}{\Rightarrow A \rightarrow B \mid \Rightarrow B \rightarrow \bar{A} \quad \rightarrow -r}
\]

\[
\Rightarrow \frac{A \rightarrow B \mid \Rightarrow B \rightarrow \bar{A}}{\Rightarrow(A \rightarrow B) \lor (B \rightarrow A) \mid \Rightarrow B \rightarrow \bar{A} \quad \lor -r}
\]

\[
\Rightarrow \frac{(A \rightarrow B) \lor (B \rightarrow A) \mid \Rightarrow (A \rightarrow B) \lor (B \rightarrow A)}{(A \rightarrow B) \lor (B \rightarrow A) \quad \lor -r \quad \text{e-contr}}
\]


**Linearity in HLK**

\[
\frac{A \Rightarrow B \mid B \Rightarrow A}{\Rightarrow A \rightarrow B \mid B \Rightarrow A} \rightarrow -r
\]

\[
\frac{\Rightarrow A \rightarrow B \mid B \Rightarrow A}{\Rightarrow A \rightarrow B \mid \Rightarrow B \rightarrow A} \rightarrow -r
\]

\[
\frac{\Rightarrow (A \rightarrow B) \lor (B \rightarrow A) \mid \Rightarrow B \rightarrow A}{\Rightarrow (A \rightarrow B) \lor (B \rightarrow A)} \lor -r
\]

\[
\frac{\Rightarrow (A \rightarrow B) \lor (B \rightarrow A) \mid \Rightarrow (A \rightarrow B) \lor (B \rightarrow A)}{\Rightarrow (A \rightarrow B) \lor (B \rightarrow A)} \lor -r
\]

\[
\text{e-contr}
\]
Linearity in HLK

\[
\begin{align*}
\frac{A \Rightarrow A}{A \Rightarrow B} & \quad \frac{B \Rightarrow B}{B \Rightarrow A} \quad \text{com} \\
\Rightarrow A \Rightarrow B & \quad \Rightarrow B \Rightarrow A \\
\Rightarrow A \Rightarrow B & \quad \Rightarrow B \Rightarrow A \\
\Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) & \quad \Rightarrow B \Rightarrow A \\
\Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) & \quad \Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) \\
\Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) & \quad \Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) \\
\Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) & \quad \Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) \\
\Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) & \quad \Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) \\
\Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) & \quad \Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) \\
\Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) & \quad \Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) \\
\Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A) & \quad \Rightarrow (A \Rightarrow B) \lor (B \Rightarrow A)
\end{align*}
\]
Linearity in HND

\[(A \rightarrow B) \lor (B \rightarrow A)\]
Linearity in HND

\[
\begin{align*}
(A \rightarrow B) \lor (B \rightarrow A) & \quad (A \rightarrow B) \lor (B \rightarrow A) \\
\hline
(A \rightarrow B) \lor (B \rightarrow A) & \text{contr}
\end{align*}
\]
Linearity in HND

\[
\frac{A \rightarrow B}{(A \rightarrow B) \lor (B \rightarrow A)} \lor \neg i \quad \frac{(A \rightarrow B) \lor (B \rightarrow A)}{(A \rightarrow B) \lor (B \rightarrow A)} \text{ contr}
\]
Linearity in HND

\[
\begin{align*}
&\frac{\overline{B}}{A \rightarrow B} \quad \rightarrow \quad -i \\
&\quad \frac{(A \rightarrow B) \lor (B \rightarrow A)}{\lor -i}
\end{align*}
\]

\[
\frac{(A \rightarrow B) \lor (B \rightarrow A)}{(A \rightarrow B) \lor (B \rightarrow A)} \quad \text{contr}
\]
**Linearity in HND**

\[
\frac{\overline{B}}{A \rightarrow B} \quad \rightarrow -i
\]

\[
\frac{(A \rightarrow B) \lor (B \rightarrow A)}{(A \rightarrow B) \lor (B \rightarrow A)} \quad \lor -i
\]

\[
\frac{\overline{B} \rightarrow A}{B \rightarrow A} \quad \lor -i
\]

\[
\frac{(A \rightarrow B) \lor (B \rightarrow A)}{(A \rightarrow B) \lor (B \rightarrow A)} \quad \lor -i
\]

\[
\text{contr}
\]
**Linearity in HND**

\[
\begin{align*}
\frac{\overline{B}}{A \rightarrow B} \quad \rightarrow & \quad \neg i \\
\frac{\overline{A}}{B \rightarrow A} \quad \rightarrow & \quad \neg i \\
\frac{(A \rightarrow B) \lor (B \rightarrow A)}{(A \rightarrow B) \lor (B \rightarrow A) \lor (A \rightarrow B) \lor (B \rightarrow A)} & \lor \quad \neg i \\
\end{align*}
\]

**contr**
Linearity in HND

\[
\frac{\overline{A}}{B} \quad \text{com}^\chi_{A,B} \quad \frac{\overline{B}}{A} \quad \text{com}^\chi_{B,A} \\
\frac{A \rightarrow B}{(A \rightarrow B) \lor (B \rightarrow A)} \quad \lor \quad \frac{B \rightarrow A}{(A \rightarrow B) \lor (B \rightarrow A)} \\
\frac{(A \rightarrow B) \lor (B \rightarrow A)}{\lor \quad \text{contr}}
\]
Linearity in HND

\[
\begin{array}{cccc}
[A] & \frac{A}{A} & \frac{B}{B} & \text{com}^\chi_{A,B} \\
\frac{A \rightarrow B}{A \rightarrow B} & \rightarrow & -i & \frac{B}{B} \\
\frac{A \rightarrow B}{(A \rightarrow B) \lor (B \rightarrow A)} & \lor & -i & \frac{B}{B} \\
\frac{B \rightarrow A}{(A \rightarrow B) \lor (B \rightarrow A)} & \lor & -i & \frac{B}{B} \\
\frac{A \rightarrow B}{(A \rightarrow B) \lor (B \rightarrow A)} & \lor & -i & \frac{B}{B} \\
\frac{B \rightarrow A}{(A \rightarrow B) \lor (B \rightarrow A)} & \lor & -i & \frac{B}{B} \\
\end{array}
\]

\[\text{contr}\]
Basic conceptualisation of HND

- set of pre-derivations (tree of rules)
- dual communication and splitting labels connect different pre-derivations
- connected components of an HND prove one hyper-sequent
- conditions on the independence of connected components when applying non-unary rules
**Independence condition by construction**

\[ \Gamma_1 A_1 R_1 = \{ \rho_1^1, \rho_2^1, \ldots, \rho_{n_i}^1 \} \]

\[ \Gamma_2 A_2 R_2 = \{ \rho_1^2, \rho_2^2, \ldots, \rho_{n_i}^2 \} \]

Combined to:

\[ R = \{ \rho, \rho_2^i, \ldots, \rho_{n_i}^i : i = 1, 2 \} \]
**Independence condition**

Assume in a proof figure with

\[
\begin{align*}
\Gamma_1 & \quad \Gamma_2 \\
\sigma_1 & \quad \sigma_2 \\
\Lambda_1 & \quad \Lambda_2 \\
\land-i & \quad B \\
\cdot & \quad \\
\cdot & \quad \\
\cdot & \quad \\
\rho_1 & 
\end{align*}
\]
**Independence condition**

Assume in a proof figure with

\[
\begin{array}{cccc}
\Gamma_1 & & \Gamma_2 \\
\vdots & & \vdots \\
\sigma_1 & & \sigma_2 \\
\vdots & & \vdots \\
\Lambda_1 & \land-i & \Lambda_2 \\
\end{array}
\]

\[\land-i \quad \bar{B} \quad \rho_1\]

Condition: \([\sigma_1]_{R_1^*} \cap [\sigma_2]_{R_2^*} = \emptyset\]
Explicit definition
Set of conditions on a set of pre-derivations that guarantee correctness.
Advantage: good for proof analysis and translation
Two definitions of HND

Explicit definition
Set of conditions on a set of pre-derivations that guarantee correctness.
Advantage: good for proof analysis and translation

Inductive/Operational definition
Set of rules how to combine simple HNDs to more complex ones.
Advantage: good for actual deriving, but bad for proof analysis
Explicit definition of HND

Set of pre-derivations (tree of rules)

Additional conditions:
- duality of labels
- sub-derivations of dual labels don’t overlap
- sub-derivations above splitting labels are the same
- total order on the set of labels conforming with the order they appear on any branch
- independence of non-unary rule
Inductive definition of iHND

Set of pre-derivations (tree of rules), constructed from axioms by applying rules of HND:

- unary rules can be applied without conditions
- for non-unary rules we have to have two independent iHNDs where names of labels are disjoint
- contraction rule requires two pre-derivations of the same iHND
- rules are applied per pre-derivation
Properties

Theorem
Every iHND is also a HND. HNDs with additional conditions on contractions are also iHNDs.
Properties

Theorem
Every iHND is also a HND. HNDs with additional conditions on contractions are also iHNDs.

Theorem
Every HLK proof can be translated into an HND derivation. Every HND derivation can be translated into an HLK proof.
Properties

Theorem
Every iHND is also a HND.
HNDs with additional conditions on contractions are also iHNDs.

Theorem
Every HLK proof can be translated into an HND derivation.
Every HND derivation can be translated into an HLK proof.
Consequence: Sound and complete for $G_{[0,1]}$
The intended translation into Hyper Natural Deduction (HND) would be:
**Normalisation**

Idea: Get rid of wrongly ordered rules, that is elimination after introduction:
NORMALISATION

Idea: Get rid of wrongly ordered rules, that is elimination after introduction:

\[
\begin{array}{c}
\vdash [A] \\
\vdash \vdots \\
\vdash \vdots \\
\vdash \vdots \\
\vdash \Gamma \\
\end{array}
\]

\[\rightarrow_i \quad \frac{B}{A \rightarrow B} \quad A \]

\[\rightarrow_e \quad \frac{A \rightarrow B}{B} \]

\[
\text{convert to} \\
\Gamma \\
A \\
B
\]
NORMALISATION

Idea: Get rid of wrongly ordered rules, that is elimination after introduction:

\[
\begin{array}{c}
[A] \\
\vdots \\
\vdots \\
\vdots \\
\vdash i \frac{B}{\Gamma} \\
\vdash e \frac{A \rightarrow B \quad A}{B} \\
\end{array}
\]

convert to

\[
\begin{array}{c}
\Gamma \\
\vdots \\
A \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
B
\end{array}
\]
NORMALISATION

Idea: Get rid of wrongly ordered rules, that is elimination after introduction:

\[
\begin{array}{cc}
\hline
[A] & \Gamma \\
\vdots & \Gamma \\
\vdots & \vdots \\
\vdots & \vdots \\
\hline
\rightarrow-i & B \\
\rightarrow-e & A \rightarrow B \\
\hline
\end{array}
\]

convert to

\[
\begin{array}{cc}
\Gamma & A \\
\vdots & \vdots \\
\vdots & \vdots \\
B & \vdots \\
B & B \\
\hline
\end{array}
\]

In \(\lambda\): \((\lambda x.t)s\) reduces to the \(t[x := s]\).
Normalisation

Idea: Get rid of wrongly ordered rules, that is elimination after introduction:

\[
\begin{array}{c}
[A] \\
\vdots \\
\vdots \\
\vdots \\
\rightarrow^\text{-i} B \\
\rightarrow^\text{-e} A \rightarrow B \\
\hline
B \\
\end{array}
\]

\[
\begin{array}{c}
\Gamma \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{array}
\]

convert to

\[
\begin{array}{c}
\Gamma \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{array}
\]

\[
\begin{array}{c}
A \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
B \\
\end{array}
\]

In $\lambda$: $(\lambda x. t)s$ reduces to the $t[x := s]$.

Effect of normalisation: hourglass form of ND proofs
Normalisation in HND

Theorem

HND admits normalisation, that is that all elimination rules can be moved above introduction rules.
Current status and next steps

Already done:

- explicit, implicit HND defined
- relation between HND and iHND
- translation between HLK and HND
- normalisation (needs write-up)
- discussion about possible connections with $\pi$-calculus

Next steps:

- finish normalisation
- create term system for HND
- explore connections to $\pi$-calculus or similar
**Current status and next steps**

Already done:

- explicit, implicit HND defined
- relation between HND and iHND
- translation between HLK and HND
- normalisation (needs write-up)
- discussion about possible connections with π-calculus

Next steps:

- finish normalisation
- create term system for HND
- explore connections to π calculus or similar