Outwit game – a dynamical systems game for market dynamics

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Abstract

A dynamical systems game model of market, called Outwit Game, is proposed. In the game, individual behavior in a market is abstracted as orienting majority and orienting minority. We suppose that individuals' profit changes with a course of these two actions. Through computer simulations, we show that an index representing both micro and macro dynamics show a power law distribution.

Keywords: Outwit Game, Dynamical Systems Game, Market Dynamics, Micro-Macro Loop, Power Law Distribution

1 Introduction

In this paper, we propose a game model, called Outwit Game (OG), for capturing dynamics of market by considering individuals behavior in a market. In neo-classical economics, the assumptions of the perfect competitive market and the perfect rationality make the market model static. However, any market, in reality, dynamically changes. Therefore, in order to understand the real market, we need a model that includes dynamics of market and individual behavior.

The real market is composed of interactions among individuals in a game theoretic situation. Namely, others' action affects decision making and consequences of actions by an individual and vice versa. In addition to the interactions among individuals, there exist interactions between individuals' actions, that is, micro dynamics, and change of a market, that is, macro dynamics. This circular interacting causation is called "micro-macro loop" by Shiozawa[1]. We think that the essence of market dynamics consists of endogenous change induced by individuals' actions, interaction among individuals, and the micro-macro loop.

Constructing a game theoretic model of a market, we put importance on the following points:

- The perfect competition should not be assumed for a model of markets.
- The perfect rationality should not be assumed for a model of individuals.
- The model should include dynamic and strategic interaction among individuals.

In order to focus on the dynamical change of market, we adopt a framework of dynamical systems game[2]. It is an extension of the game theory for describing changes of game environment such as payoff matrix, options by players.

2 Individuals' Actions in Market

At first, we consider the properties of individual behavior in a market. Keynes[3] likens a market to a beauty contest, in which the prize goes to a parson who is voted at most and the voters for the winner earn. In this beauty contest, people try to vote for a parson who is thought of as the most beautiful by most of the people, not for a parson they themselves think as the most beautiful. In financial market, people try to invest a company that many investors invest. This is an action pursuing a trend and orienting a majority.

Investors do not always follow trends. They invest in a company that is not in a trend at present and may be in future, invest to a new business, or sell stocks they have before the upward trend ceases. Namely, people try to make a trend by themselves. This action is not to do the same action as others and favors minority.

In the market, people change over these two actions, orienting majority and orienting minority and gain or loss. Switching two actions may be a trigger of change of trends. The market change is often induced by the trend changes. Although the people favor majority or minority intentionally, the consequence of an action is determined by the market. This is an uncertainty of market.

3 Outwit Game

Based on the above discussion, we introduce a game abstracting the real market. In the game, individuals' actions have features of orienting majority and minority, that changes with the state of market. This type of game is named as Outwit Game (OG).

We define two versions of the OG, Simple Outwit Game (SOG) and Monetary Outwit Game (MOG). The MOG is relatively complex version than the SOG. In the MOG, so as to approach to the dynamics of real market, two forms of profit, capital gain and realized gain, are considered. These two games share the essential part. We analyze the SOG at first and report in this paper. The MOG is not treated in this paper.

3.1 Definition of Simple Outwit Game

The procedure of the SOG is the followings:

1. Each player selects one of two alternative moves at each time step t.
2. The majority and minority sides are decided from all players' moves.
3. Each player is grouped into the majority or the minority according to his/her move and given the payoff $p(t)$ defined by Table 1.

Table 1: The payoff matrix of Simple Outwit Game

move	payoff
Majority	$p(t) = \dfrac{N^{\mathrm{M}}(t) - N^{\mathrm{M}}(t-1)}{N}$
Minority	$p(t) = 0$

In Table 1, $N^{\mathrm{M}}(t)$ and $N^{\mathrm{M}}(t-1)$ are the number of majority players at the present (t) and the previous step $(t-1)$, respectively, and N is the number of all players. Note that $N^{\mathrm{M}}(t)$ and $N^{\mathrm{M}}(t-1)$ change with time, the players have the possibility both to gain and to loss when they keep in the majority side. If they are in the minority side, they are always risk-free.

3.2 Players' Action

In a game theoretic model, players pursue their own profit. In SOG, the player must predict the number of majority and decide the move at the next step according to the prediction in order to pursue his/her profit. Namely, they decide their move as

$$l(t+1) = \begin{cases} \text{Majority} & (\tilde{N}^{\mathrm{M}}(t+1) > N^{\mathrm{M}}(t)) \\ \text{Minority} & (\text{otherwise}) \end{cases} , \quad (1)$$

where $l(t+1)$ is the side of majority or minority and $\tilde{N}^{\mathrm{M}}(t+1)$ is the predicted number of majority at the next step. Note that the players cannot certainly be a majority/minority when they want to be so. They must predict which move is the majority/minority as well. Accordingly, SOG has double uncertainty in the prediction of the number of majority and its move.

4 Players' Model for Simulation of SOG

We analyze the characteristics of SOG using computer simulations. In the simulation, we use a model of players with prediction and learning. Because of the space limit, we briefly explain the players' model.

Each player has two kind of prediction functions for the number and the move of majority from the present information. At first a player predict the number of majority in the next step,

$$\tilde{N}^{\mathrm{M}}(t+1) = P^{\mathrm{N}}(N^{\mathrm{M}}(t), m^{\mathrm{M}}(t), m(t)) \ . \quad (2)$$

This expression represents that a player produce a predicted number of majority $\tilde{N}^{\mathrm{M}}$ at the next step $t+1$, based on the number of majority, $N^{\mathrm{M}}(t)$, the move of the majority side, $m^{\mathrm{M}}(t)$, and the move of the player itself, $m(t)$, at the present step t.

Further, using the same information at the present and the output from the prediction function, P^{N}, the player try to predict the move of the majority $\tilde{m}^{\mathrm{M}}(t+1)$ at the next step using the other prediction function P^{m},

$$\tilde{m}^{\mathrm{M}}(t+1) = P^{\mathrm{m}}(N^{\mathrm{M}}(t), m^{\mathrm{M}}(t), m_i(t), \tilde{N}^{\mathrm{M}}(t+1)) \ . \quad (3)$$

Every time step, the players adjust the prediction function according to success and unsuccess of the predictions.

5 Simulation Results

We analyze the dynamic behavior and statistical properties of the game. The total number of players is 51, two moves are -1 and 1.

5.1 Dynamics of Game

To see the dynamics of SOG, we observe the transition of the players' payoffs. The most of players can gain the payoffs averagely. Three examples of the transitions of accumulated payoffs,

$$S_i(t) = \sum_{t'=0}^{t} p_i(t') \ , \tag{4}$$

are depicted in Fig. 1, where $p_i(t)$ is the payoff of the i-th agent at the time step t. We find three types of the transition: rapid increasing in a long range, slow increasing, and decreasing.

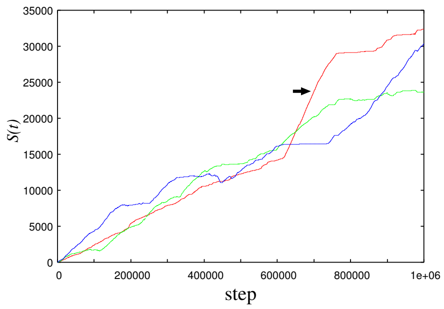

Figure 1: The transition of the accumulated payoff $S_i(t)$ of tree typical players. The x axis is time step, the y axis is the accumulated payoff. The arrow indicates a player depicted in Fig. 2.

The increasing of the accumulated payoff means that the players are able to predict the transition of the majority and it's move to some extent. We observe more closely the dynamics of the game in order to know how they obtain the payoff (Fig. 2). Figure 2(a) depicts the accumulated payoff of a player whose accumulated payoff grows rapidly in the period from 630000 to 730000 steps in Fig. 1 (indicated by an arrow). The player gains some payoff every three steps. The time series of the player's moves is period three (-1, -1, 1) as shown in Fig. 2(b). Figure 2(c) is the time series of the majority's move. This shows period three dynamics (-1, 1, -1). Namely, the player's belonging group changes as (majority, minority, minority). Finally, we draw the change of the number of minority from last step, that is, $N^{\mathrm{M}}(t) - N^{\mathrm{M}}(t-1)$, in Fig. 2(d). This value also shows period three dynamics as (positive, negative, negative). Accordingly, the player belongs to the majority when he/she can gain and to the minority when he/she may loss. In other words, he/she can appropriately outwit.

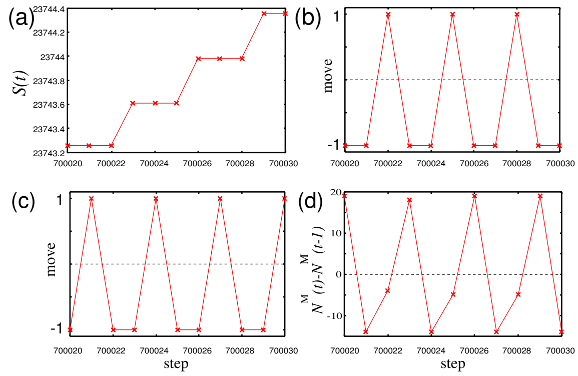

Figure 2: Transitions of (a) the accumulated payoff, $S(t)$ of a player, (b) the moves of the player, (c) the majority's move, and (d) the number of change of majority per one step, $N^{\mathrm{M}}(t) - N^{\mathrm{M}}(t-1)$, at the period of time step (the x axis) between 700020 and 700030.

5.2 Power Law Distribution of Players' and Market Dynamics

We observe some statistical characteristics of SOG. Figure 3(a) shows a distribution of the length that the players continue to take the same move. The straight line is a fitting taken at the range longer than 30. This observation value obeys a power law distribution in the range of longer length, while in the range of shorter length the value is much larger than the extrapolation of the power law fitting in the longer range. This means that the players mostly change their move shortly, but some players often do not change their moves very long periods.

Figure 3(b) is a distribution of the length that the majority continues to take the same move, namely a version of global value or market dynamics of Fig. 3(a). This value also obeys a power law.

6 Discussion

6.1 Power Law Distribution in Market

We found the power law distributions in the length of consecutive time steps that individuals take the same move and that the majority is in the same move. The former can be thought of as a characteristic of individual action, that is, a micro dynamics, and the latter as that of market, that is, a macro dynamics. If we interpret the latter index as the length of trends, this result means that infinitely long trends can occur in a market. Namely, there is possibilities of large

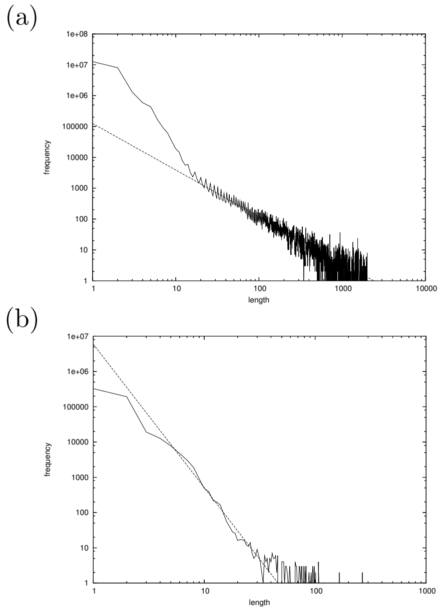

Figure 3: The distribution of the consecutive length of the same move taken by (a) the players and (b) the majority side. The x and y axes are the length and the frequency, respectively. The straight lines are fitting by pow functions at ranges that length is (a) longer than 30, and (b) 1 to 100.

stock bubbles, long term inflations and deflations in financial markets.

Power laws have been found in several indices in real financial markets. We have not, however, made correspondence between such power laws and the power law distribution in our game model. The reason why such power law is observed in our game model is also an open problem.

6.2 Comparison with Minority Game

A game that can be considered as a model of market is the Minority Game (MG)[4]. The Outwit Game (OG) is thought of as a modification of MG. The differences between the OG and the MG are the followings:

1) In the OG, two kind of actions, orienting majority and minority are taken into consideration, while the MG takes only the action orienting minority.

2) The OG has a payoff matrix that changes with time (explicitly includes a time variable t), while the payoff matrix of MG is fixed[1].

3) In the OG, to win the game, players should select the majority/minority appropriately responding to changes of game situations, while in the MG to be the minority is always to win.

[1] As a simple modification of the MG, the number of majority can be given to the minority. Although the relative value of the payoff changes with time in this modification, the essential payoff structure that the minority is always win does not change.

6.3 SOG as Tragedy of a Common

The SOG has a characteristic of tragedy of a common. If a player in the SOG change from the majority to the minority in order to avoid a loss, the other players in the majority suffer a loss. For example, when n players are in the majority, suppose a player changes to the minority. Since the number of the majority decreases to $n-1$, the players remaining in the majority lose their payoff. But the player changed to the minority eludes a loss. Namely, an action pursuing individual's own profit conflicts with profit of the whole.

7 Conclusion

We propose a new dynamical systems game theoretic model of financial market. This game is named the Outwit Game. The simplest version of the game, the Simple Outwit Game, is analyzed using computer simulations. We show that indices corresponding to a micro and macro dynamics obey the power law. Thus, it can be said that the Outwit Game reflect some nature of market dynamics. The analysis of the Outwit Game should be progressed for showing the utility of this game model for understanding market dynamics.

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