SMT for polynomial constraints over Real numbers

To Van Khanh

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Japan Advanced Institute of Science and Technology

Outline

Aim Combination of SAT solver and two theories

- Interval Arithmetic: aiming to decide unsatisfiability
- Testing: aiming to decide satisfiability

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- Interval arithmetic and Testing — CAI1 and CAI2
- Framework of SMT solver
- Preliminary experiments — QF_NRA of SMT-LIB

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Introduction

- Presburger arithmetic is decidable. linear constraints
- Tarski proved that polynomial constraints over real numbers is decidable. 1930
- Collins proposed Cylindrical Algebraic Decomposition (CAD). DEXPTIME, 1975

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Related works Not many SMT for polynomial constraints

- iSAT applies classical interval arithmetic.
- MiniSmt performs on rational (possibly irrational) domains.
- Barcelogic focuses on integer numbers.
- CVC3 is also a popular SMT.

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Applications

- checking roundoff/overflow error
- measures for proving termination

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Interval Arithmetic (IA)

Why IA?

- Example 1

  $x = [0, 2]$ and $y = [-1, 3]$

  (check-sat $(x^2 - 2x^2y + y > 16)$)

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  (unsatisfiable)

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- Example 2

  $x = [-2, 2]$ and $y = [-1, 3]$

  (check-sat $(2xy - 2x^3y + x > -2)$)

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- Example 2

  $x = [-2, 2]$ and $y = [-1, 3]$

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  With $x = [0, 1]$ and $y = [0, 1]$

  $2xy - 2x^3y + x =$ [-1.9375, 3.03125]

  (satisfiable)

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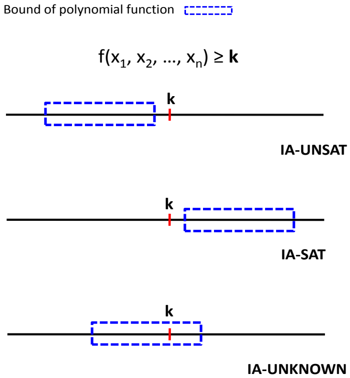

Classical Interval and Affine Interval

Classical Interval - CI

Let $x = [0, 2]$, $x - x = [0, 2] - [0, 2] = [-2, 2]$.

Classical Interval and Affine Interval

Classical Interval - CI

Let $x = [0, 2]$, $x - x = [0, 2] - [0, 2] = [-2, 2]$.

Affine Interval

- Introducing noise symbols ϵ which is interpreted as a value in $[-1, 1]$.
- Noise symbols are used for symbolic manipulation (to get better precision of substraction).
  $x = [0, 2] = 1 + \epsilon$ then $x - x = (1 + \epsilon) - (1 + \epsilon) = 0$.
- The problem is how to treat multiplication like ϵ^2, $\epsilon\epsilon'$

Affine Interval

Ideas Choices for multiplication are:

- $\epsilon\epsilon'$ is replaced by a fresh noise symbol (AF) Stolfi, 93
- $\epsilon\epsilon'$ is pushed into the fixed noise symbol $\epsilon_\pm$ (AF1, AF2) Messine, 02
- ϵ^2 is replaced by ϵ_+ ($-\epsilon^2$ by ϵ_-) (AF2) Messine, 02
- $\epsilon\epsilon'$ is replaced with $[-1,1]\epsilon$ or $[-1,1]\epsilon'$ (EAI) Ngoc, 09

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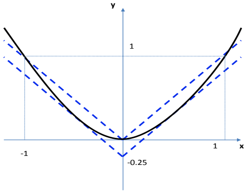
Example 3 $x = [0,2]$ and $y = 2 - x$. Compute $x * y$?

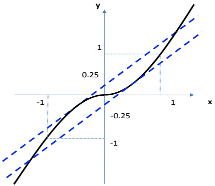
$x = 1 + \epsilon$, $y = 2 - x = 1 - \epsilon$ where ϵ is interpreted as a value in $[-1,1]$.

- AF: $x * y = 1 - \epsilon^2 = 1 + \epsilon'$, ϵ' is interpreted as a value in $[-1,1]$.
- AF1: $x * y = 1 - \epsilon^2 = 1 + \epsilon_\pm$, $\epsilon_\pm$ is interpreted as a value in $[-1,1]$.
- AF2: $x * y = 1 - \epsilon^2 = 1 + \epsilon_-$, ϵ_- is interpreted as a value in $[-1,0]$
- EAI: $x * y = 1 - \epsilon^2 = 1 + [-1,1]\epsilon$

Chebyshev Approximation Interval - CAI1, CAI2



(a) Chebyshev approximation for y = x.x



(b) Chebyshev approximation for y = x.|x|

- Symbolic manipulation: CAI1, CAI2

$$\epsilon \times \epsilon = |\epsilon| \times |\epsilon| = |\epsilon| + [-\frac{1}{4}, 0] \text{ and } \epsilon \times |\epsilon| = \epsilon + [-\frac{1}{4}, \frac{1}{4}]$$

- Keeping products $\epsilon_i \epsilon_j$ of noise symbols in their forms CAI2
- Example 3: $x = [0, 2]$ and $y = 2 - x$. Compute $x * y$?
  $x = 1 + \epsilon$, $y = 2 - x = 1 - \epsilon$.
  $x * y = (1 + \epsilon)(1 - \epsilon) = 1 - \epsilon^2 = 1 - (|\epsilon| + [-\frac{1}{4}, 0]) = [1, \frac{5}{4}] - |\epsilon|$.

Example

Example 4 Given $f = x^3y - 2xy + x^2y^2 - x^2$ with $x \in [-1, 1]$ and $y \in [-2, 0]$, the bounds of f are as follows:

- AF1 : $[-15, 15]$
- AF2 : $[-15, 14]$
- CAI1: $[-13.75, 12]$
- CAI2: $[-12, 10.25]$

Example

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- AF1 : $[-15, 15]$
- AF2 : $[-15, 14]$
- CAI1: $[-13.75, 12]$
- CAI2: $[-12, 10.25]$

Example 5 Taylor expansion of $sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$ with $x \in [0, 0.523598]$ (x from 0 to $\frac{\pi}{6}$), the bounds of $sin(x)$ are:

- AF1 : $10^{-6}[-6290.49099241,\ 523927.832027]$
- AF2 : $10^{-6}[-6188.00580507,\ 514955.797111]$
- CAI1: $10^{-6}[-1591.61467700,\ 503782.471931]$
- CAI2: $10^{-6}[-1591.61467700,\ 503782.471931]$

Comparision

Dependent operands Affine interval may give better results than CI.

$x = [0, 2]$ and $y = 2 - x$. Compute $x * y$?

$x = 1 + \epsilon$ and $y = 2 - x = 1 - \epsilon$ where $\epsilon \in [-1, 1]$

- $CI \quad : x * y = [0, 2] * ([2, 2] - [0, 2]) = [0, 2] * [0, 2] = [0, 4]$
- $AF \quad : x * y = 1 - \epsilon^2 = 1 + \epsilon' \ = 1 + [-1, 1] = [0, 2]$
- $AF2 : x * y = 1 - \epsilon^2 = 1 + \epsilon_- = 1 + [-1, 0] = [0, 1]$

Comparision

Dependent operands Affine interval may give better results than CI.

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- CI :$x * y = [0,2] * ([2,2] - [0,2]) = [0,2] * [0,2] = [0,4]$
- AF :$x * y = 1 - \epsilon^2 = 1 + \epsilon' = 1 + [-1,1] = [0,2]$
- $AF2$:$x * y = 1 - \epsilon^2 = 1 + \epsilon_- = 1 + [-1,0] = [0,1]$

Independent operands CI often gives better results than affine interval.

$x \in [0,2]$ and $y \in [-1,5]$. Compute $x * y$?

- CI : $x * y = [0,2] * [-1,5] = [-2,10]$
- AF: $x = 1 - \epsilon_1$ and $y = 2 + 3\epsilon_2$ where $\epsilon_1, \epsilon_2, \epsilon'$ are noise symbols and they are interpreted as values in $[-1,1]$.
  $x * y = (1 - \epsilon_1)(2 + 3\epsilon_2) = 2 - 2\epsilon_1 + 3\epsilon_2 + 3\epsilon' = [-6,10]$

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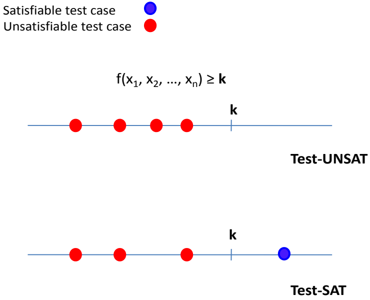
QF_NRA of SMT-LIB

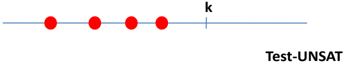
Testing

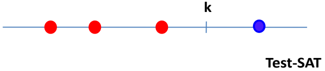

Satisfiable test case
Unsatisfiable test case
$f(x_1, x_2, \ldots, x_n) \geq \mathbf{k}$
k
Test-UNSAT
k
Test-SAT

Testing

Satisfiable test case (blue dot)
Unsatisfiable test case (red dot)

$f(x_1, x_2, \ldots, x_n) \geq \mathbf{k}$

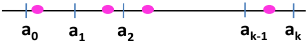




A random test case (magenta dot)
A periodic test case (cyan dot)

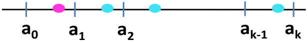
k-random ticks = {$c_1, c_2, \ldots, c_k$}

$c_i \in [a_0+(i-1)\theta, a_0+i\theta]$ with $\theta = (a_k-a_0)/k$



k-periodic ticks = {$c, c+\theta, \ldots, c+(k-1)\theta$}

$c \in [a_0, a_0+\theta]$ is randomly generated with $\theta = (a_k-a_0)/k$



- k-random ticks
- k-periodic ticks
- Sensitive variables, which has large coefficient, are considered for generating more test cases.

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Polynomial Constraints

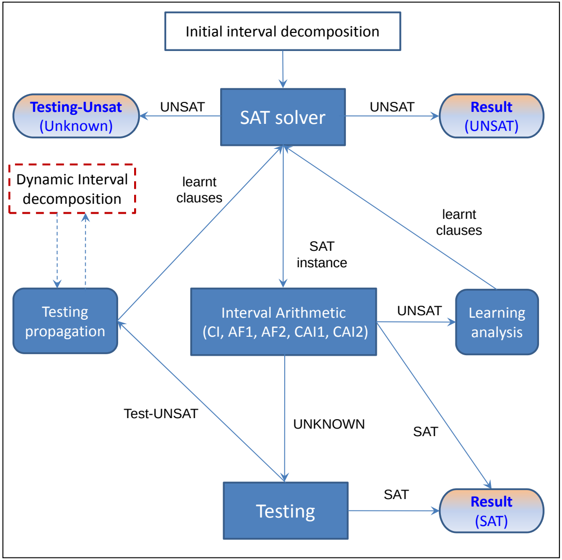
Definition

A polynomial inequality constraint is in the form of

$$(\exists x_1 \in [l_1, h_1] \cdots x_n \in [l_n, h_n]. \bigwedge_{j}^{m} f_j(x_1, \cdots, x_n) > 0)$$

where $l_i, h_i \in \mathbb{R}$ and $f_j(x_1, \cdots, x_n)$ is a polynomial over variables $x_1, \cdots, x_n$.

Framework of SMT solver: refinement loop



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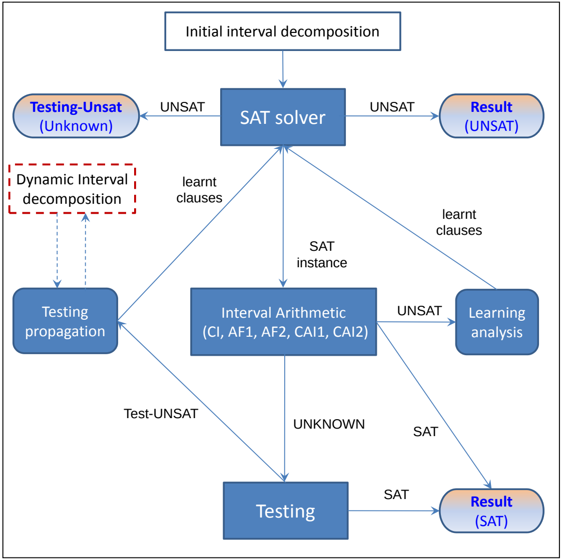


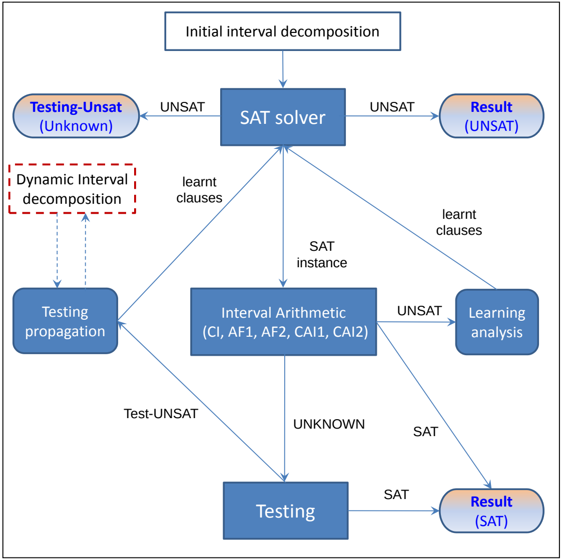
Initial interval decomposition

$x_1 \in [a_0, a_1] \vee x_1 \in [a_2, a_3]$
$\vee \ldots \vee x_1 \in [a_{n-1}, a_n]$

$x_2 \in [b_0, b_1] \vee x_2 \in [b_2, b_3]$
$\vee \ldots \vee x_2 \in [b_{m-1}, b_m]$

...

Framework of SMT solver: refinement loop



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...

SAT solver MiniSAT2.2

Theory propagation

- Very lazy approach

Sat example by Testing

Problem P1

$(x \in [-1,0]) \vee (x \in [0,1]) \vee (x \in [1,2]) \vee (x \in [2,3]) \vee (x \in [3,4])$
$(y \in [-1,0]) \vee (y \in [0,1]) \vee (y \in [1,2]) \vee (y \in [2,3]) \vee (y \in [3,4])$
$(assert\ (f = 4x + 3y - xy > 12))$

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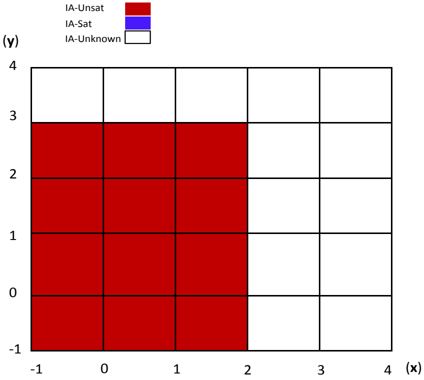
Unsat areas (red) are marked by IA.



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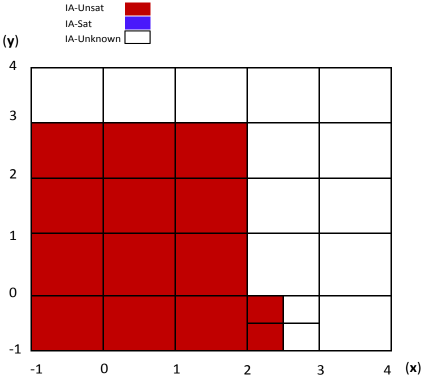
Dynamic interval decomposition



Sat example by Testing

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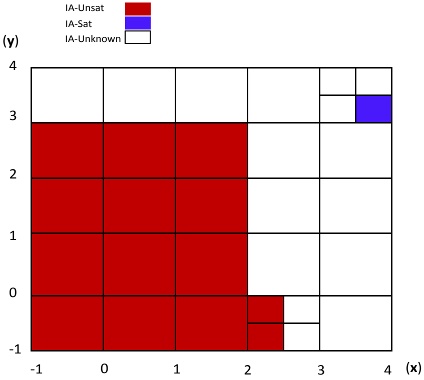

Sat example by Testing

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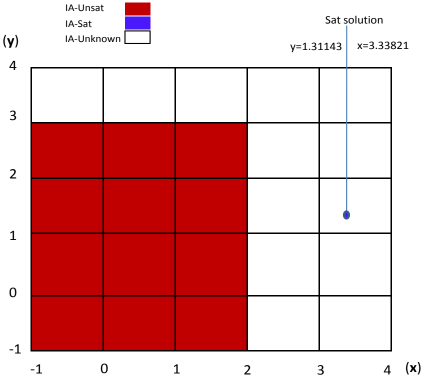
$(assert\ (f = 4x + 3y - xy > 12))$

Sat solution by testing:

$x = 3.33821$ and $y = 1.31143$



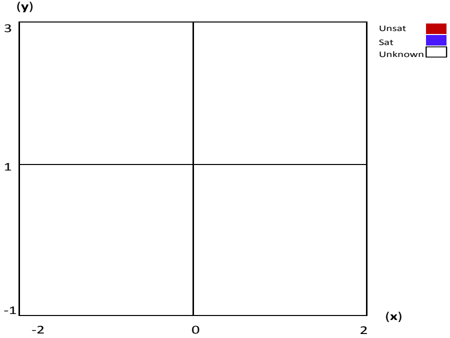
Unsat example by IA

Problem P2

$(x \in [-2, 0]) \vee (x \in [0, 2])$

$(y \in [-1, 1]) \vee (y \in [1, 3])$

$(assert\ (f = x^3 - 2x^2(1 + y^2) - 2y(x + y) + y > 6.5))$



Unsat example by IA

Problem P2

$$(x \in [-2,-1]) \vee (x \in [-1,0]) \vee (x \in [0,2])$$

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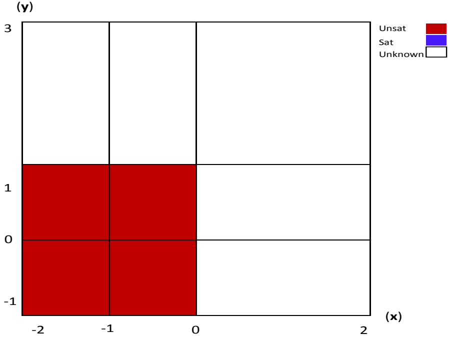

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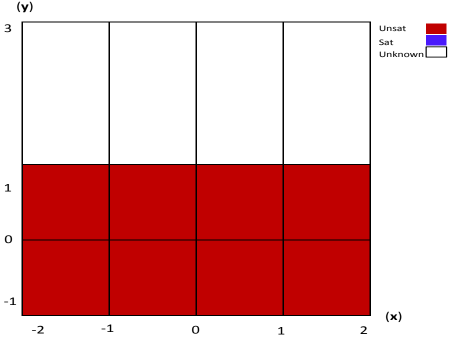

Unsat example by IA

Problem P2 Unsatifiable

$$(x \in [-2,-1]) \vee (x \in [-1,0]) \vee (x \in [0,1]) \vee (x \in [1,2])$$

$$(y \in [-1,0]) \vee (y \in [0,1]) \vee (y \in [1,2]) \vee (y \in [2,3])$$

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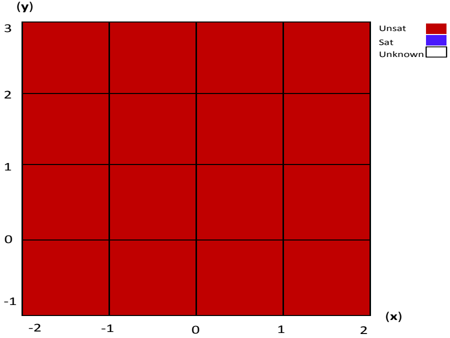

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Preliminary experiments: QF_NRA of SMT-LIB and problem P

Initial interval decomposition $x \geq 0$

$[0, 0.5] \vee ... \vee [2.0, 2.5]$

IA Number of variables

- AF1, AF2: ≥ 15
- CAI1: < 15
- CAI2: $<= 10$

Testing 2-random ticks

Problem	No. Variables	No. Constraints	Interval Arithmetic	Result	Time (s)
P	13	10	AF1	unknown	0.031
P	13	10	AF2	unknown	0.109
P	13	10	CAI1	UNSAT	0.046
P	13	10	CAI2	UNSAT	0.796
matrix-1-all-01	19	22	AF2	unknown	0.093
matrix-1-all-2	14	9	CAI1	SAT	8.328
matrix-1-all-3	19	21	AF1	SAT	175.968
matrix-1-all-4	16	20	AF1	SAT	20.328
matrix-1-all-11	19	17	AF1	SAT	17.687
matrix-1-all-14	14	16	CAI1	SAT	66.484
matrix-1-all-15	10	14	CAI1	unknown	26.656
matrix-1-all-18	6	10	CAI2	SAT	14.156
matrix-1-all-20	16	16	AF2	SAT	1.062
matrix-1-all-21	13	17	AF1	SAT	2753.72
matrix-1-all-24	11	12	CAI1	unknown	50.828
matrix-1-all-33	13	6	CAI1	SAT	68.756
matrix-1-all-34	20	14	AF2	SAT	3349.89
matrix-1-all-36	18	19	AF2	SAT	54.015
matrix-1-all-37	19	46	AF2	unknown	3730.66
matrix-1-all-39	19	23	AF2	unknown	85.781
matrix-1-all-43	16	9	AF2	unknown	0.343
matrix-2-all-6	17	10	AF2	unknown	15.75

Experiments: problem P

$$\exists x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \in [0,3]\ x_{11} \in [-3,2]\ x_{12} \in [-1,3].$$
$$x_1x_3 - x_1x_7 > 0\ \wedge\ x_1x_2 - x_1x_6 > 0\ \wedge x_1x_3 - x_3 > 0\ \wedge$$
$$x_1x_2 - x_4 > 0\ \wedge\ x_7 - x_3 > 0\ \wedge\ x_6 - x_2 > 0\ \wedge$$
$$x_8 + x_6x_9 - x_{10} > 0\ \wedge\ x_3x_9 - x_7x_9 > 0\ \wedge\ x_2x_9 - x_6x_9 > 0\ \wedge$$
$$x_{11}^3 - 2x_{11}^2 - 2x_{11}^2x_{12}^2 - 2x_{12}x_{11} - 2x_{12}x_{12} + x_{12} - 6.5 > 0$$

Experiments: problem P

$\exists x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \in [0,3]\ x_{11} \in [-3,2]\ x_{12} \in [-1,3].$
$x_1x_3 - x_1x_7 > 0 \ \wedge\ x_1x_2 - x_1x_6 > 0 \ \wedge x_1x_3 - x_3 > 0 \ \wedge$
$x_1x_2 - x_4 > 0 \ \wedge\ x_7 - x_3 > 0 \ \wedge\ x_6 - x_2 > 0 \ \wedge$
$x_8 + x_6x_9 - x_{10} > 0 \ \wedge\ x_3x_9 - x_7x_9 > 0 \ \wedge\ x_2x_9 - x_6x_9 > 0 \ \wedge$
$x_{11}^3 - 2x_{11}^2 - 2x_{11}^2x_{12}^2 - 2x_{12}x_{11} - 2x_{12}x_{12} + x_{12} - 6.5 > 0$

Initial interval decomposition

$(x_0 \in [0,1]) \vee (x_0 \in [1,2]) \vee (x_0 \in [2,3])$

$(x_1 \in [0,1]) \vee (x_1 \in [1,2]) \vee (x_1 \in [2,3])$

...

$(x_{10} \in [0,1]) \vee (x_{10} \in [1,2]) \vee (x_{10} \in [2,3])$

$(x_{11} \in [-1,0]) \vee (x_{11} \in [0,1]) \vee (x_{11} \in [1,2]) \vee (x_{11} \in [2,3])$
$(x_{12} \in [-1,0]) \vee (x_{12} \in [0,1]) \vee (x_{12} \in [1,2]) \vee (x_{12} \in [2,3])$

Experiments: problem P

$$\exists x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \in [0,3]\ x_{11} \in [-3,2]\ x_{12} \in [-1,3].$$
$$x_1x_3 - x_1x_7 > 0 \ \wedge\ x_1x_2 - x_1x_6 > 0 \ \wedge x_1x_3 - x_3 > 0 \wedge$$
$$x_1x_2 - x_4 > 0 \ \wedge\ x_7 - x_3 > 0 \ \wedge\ x_6 - x_2 > 0 \wedge$$
$$x_8 + x_6x_9 - x_{10} > 0 \ \wedge\ x_3x_9 - x_7x_9 > 0 \ \wedge\ x_2x_9 - x_6x_9 > 0 \wedge$$
$$x_{11}^3 - 2x_{11}^2 - 2x_{11}^2x_{12}^2 - 2x_{12}x_{11} - 2x_{12}x_{12} + x_{12} - 6.5 > 0$$

Initial interval decomposition

$$(x_0 \in [0,1]) \vee (x_0 \in [1,2]) \vee (x_0 \in [2,3])$$
$$(x_1 \in [0,1]) \vee (x_1 \in [1,2]) \vee (x_1 \in [2,3])$$

...

$$(x_{10} \in [0,1]) \vee (x_{10} \in [1,2]) \vee (x_{10} \in [2,3])$$
$$(x_{11} \in [-1,0]) \vee (x_{11} \in [0,1]) \vee (x_{11} \in [1,2]) \vee (x_{11} \in [2,3])$$
$$(x_{12} \in [-1,0]) \vee (x_{12} \in [0,1]) \vee (x_{12} \in [1,2]) \vee (x_{12} \in [2,3])$$
$$x_{11}^3 - 2x_{11}^2 - 2x_{11}^2x_{12}^2 - 2x_{12}x_{11} - 2x_{12}x_{12} + x_{12} - 6.5 > 0$$

UNSAT

Future works: How to handle polynomial equality (idea)

$$\exists x_1 \in [l_1, h_1] \cdots x_n \in [l_n, h_n]. \bigwedge_{j}^{m} f_j(x_1, \cdots, x_n) > 0 \wedge g(x_1, \cdots, x_n) = 0$$

Applying Intermediate value theorem

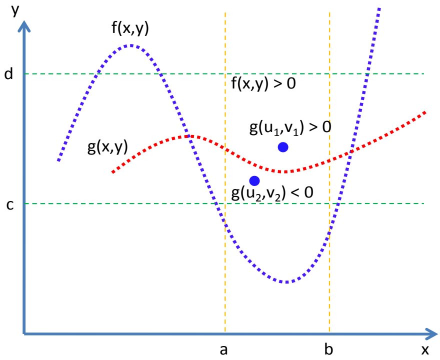
- By interval arithmetic

  $\forall x_1 \in [l_1, h_1] \cdots x_n \in [l_n, h_n].$

  $\bigwedge_j^m f_j(x_1, \cdots, x_n) > 0.$

- By testing
  - $g(a_1, a_2, ..., a_n) > 0$
  - $g(b_1, b_2, ..., b_n) < 0$

$\exists x \in [l_1, h_1]\, y \in [l_2, h_2].\, f(x,y) > 0 \wedge g(x,y) = 0$



Future works

1. Test data generation strategies
   - Reducing number of test cases for generation
2. Dynamic interval decomposition — not yet implemented
3. Heuristic strategies for learnt clauses — IA, Testing
   - Based on bounds of constraints getting from IA
   - The number of test cases are taken
4. Scalability and practical experiments
   - Number of variables: > 20

Thank you :)

Question?