Ideas on SAT solvers for enhancing efficiency of SMT solvers

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What Nagoya group did related to SAT solving

- Introduction of special clauses into CNFs
  - Elementary symmetric clauses (ES-clauses): $\{l_1, l_2, \ldots, l_n\}_k$ having the meaning that

    "Exactly k literals are true"
  - Ordered clauses: $\{l_1, l_2, \ldots, l_n\}_{\mathsf{OD}}$ having the meaning that

    "l_{i+1} implies l_i for each i

Comparison from experiments

		$\{l_1, l_2, \ldots, l_n\}_k$	$\{l_1, l_2, \ldots, l_n\}_{\mathrm{OD}}$
CNF size		${}_nC_k$	n
var. sel. priority	high	low speed	low speed
	low	high speed	high speed

- Speed depend on the sizes of representation if variable selection is the same
- Separate huristic tuning with low priority is effective

Variable selection heuristic: VSIDS

- Firstly introduced in Chaff [Matthew el.al. 2001]
- Priority value for each variables (or literals)
  - Static analysis: number of clauses having the variable (or literal)
  - Dynamic analysis: periodical update based on conflict clauses

Variable selection heuristic

- nanosat (es-k, od): VSIDS: number of occurrences for each literal
  Static analysis:
    counts literals in ES-clauses as one
    almost ignoring literals in OD-clauses
      very low co-efficient for binary search
  Dynamic analysis: natural handling
- minisat: modified VSIDS: originally no syntactic analysis and care about variable (not literal)
  Static analysis: positively count for ES-clauses Dynamic analysis: nagatively count for falsifying inference

Experiment 1 (1/2)
– SAT-competition problems –

- **SAT Competition 2011**
  [FI]: all 300 problems in application or crafted classes
  [10]: 34 problems in which more than 10% clauses are reduced by extraction of ES_1-clause
- **Environment**
  - **FreeBSD 8.1R amd64**
  - **Xeon W5590 (3.33GHz) x 2**
  - **48GB main memory**

Experiment 1(2/2)

- Subsets of SAT-competition problems

System	ES_1-cnf	[FI] 300 problems		[10] 34 problems	
		Solved	Time	Solved	Time
es1-sat	x	76+54	226302	8+0	31979
nanosat	x	49+24	286779	10+0	30777
minisat		76+56	223279	8+0	31567

SAT problems + UNSAT problems
Including conv. time and timeout 1200 sec
Unit: sec in Physical time

Experiment 2(1/3)
– Puzzle problems –

- Puzzle problems
  (14 problems)

	ES_1-cnf	Solved	Time
es1-sat	x	14	90
nanosat	x	14	50
minisat		13	1465

no UNSAT problems
direct generation of ES_1-CNF
time and timeout 1200 sec
Unit: sec in Physical time

Experiment 2(2/3)
– Detail –

problem (red. cl.)	size	nanosat		es1-sat	minisat
		ES_1-cnf	cnf	ES_1-cnf	cnf
Sudoku (99.9%)	49^2	1.6	14	4.6	3.7
	64^2	1.9	42	684	13
Magic sqr (70%)	4^2	0.68	0.12	0.37	0.34
	5^2	12	146	270	608
Pict logic (30%)	40×50 **7**	0.63	2.4	0.06	0.41
	50×60	2.1	17	1.3	1.3
Knight Patrol (30%)	7^2	4.9	1.8	0.03	1.3
	8^2	21	26	0.62	24
	9^2	24	86401	5.8	9.9
Hanoi (30%)	4	0.15	0.72	0.03	0.04

direct generatation of ES_1-CNF

Experiment 2(3/3)
– statics –

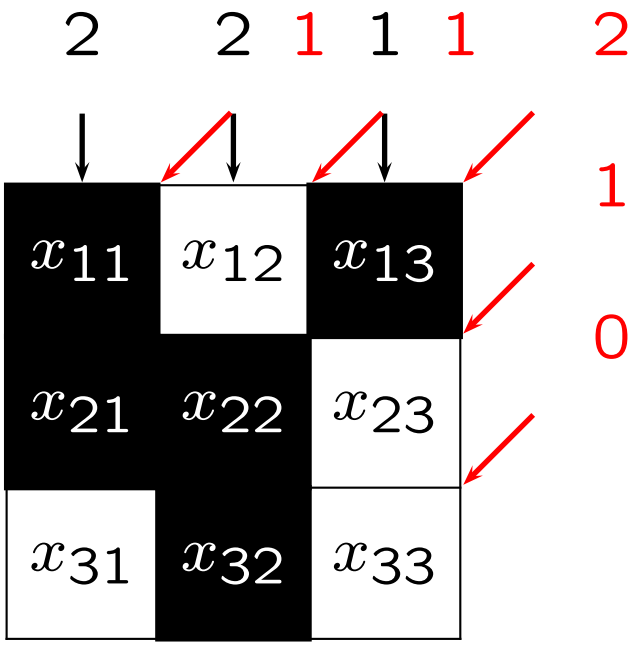
	Sudoku			
	(size=64^2)		(size=81^2)	
	es1-sat	minisat	es1-sat	minisat
Variables	262K		531K	
Clauses	16K	33M	26K	85M
Parsing time	0.13	4.9	0.3	12.6
Restarts	16	13	24	25
Conflicts	97K	35K	2.2M	4.3M
Decision	6.2M	4.3M	17M	63M
BCP	36M	18M	346M	802M
Memory	278MB	1955MB	2.8GB	30GB
CPU time	25.6	13.4	4779	14980

direct generatation of ES_1-CNF

Experiment 3(1/2)
– CNFs with ES_k-clauses –

- **Discrete computer tomography:**
  - **Instance: 4 sequences of numbers of black cells**
  - **Solution: The picture**
  - **ES_k-coding**

$$\{x_{11}, x_{21}, x_{31}\}_2$$
$$\{x_{12}, x_{22}, x_{32}\}_2$$
$$\{x_{13}, x_{23}, x_{33}\}_1$$
$$\vdots$$



Experiment 3(2/2)

size	nanosat	Clauses	Solved	Time
5^2	esk-cnf	28	100/100	0.04
	cnf	212	100/100	0.07
10^2	esk-cnf	58	100/100	0.33
	cnf	8818	100/100	2.50
15^2	esk-cnf	88	42/100	2193
	cnf	320044	53/100	2186

direct generatation of ES_k-CNF, incl. timeout 30 sec

Experiment 4 (1/2)
– Magic square –

- **Coding tips:**
  - **Ordered clause represent a number** $1 \leq x \leq 9$ **by variables** $x_2, \ldots x_9$**, e.g.,**

x	x_2	x_3	x_4	$\cdots$	x_9
3	1	1	0	$\cdots$	0

  - **Summation and euqality constraints by using ES-clause, e.g.**
    $x + y + z = 15$ **is represented as**

$$\{x_2, \ldots, x_9, y_2, \ldots, y_9, z_2, \ldots, z_9\}_{12}$$

Experiment 4 (2/2)
– Magic square –

solver	VSIDS	Form.	Size of magic square			
			4^2	5^2	6^2	7^2
nanosatk		ES_k	1.0	7.9	T.O.	T.O.
nanosatk +OD			1.0	6.2	T.O.	T.O.
	natural	ES_{k+OD}	0.38	5.6	18	T.O.
	ignore		0.13	5.4	10	T.O.
	bin.		0.007	0.29	3.0	T.O.
Sugar		B.lin	0.5	0.8	1.1	4.6

Unit: sec., timeout: 1200 sec