Certification of Constrained Rewriting

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joint work with Christian Sternagel

Computational Logic, University of Innsbruck

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Outline

- Task 5: Certification
- IsaFoR + CeTA
- Progress in Task 5

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- working plan: late start

	1st year	2nd year	3rd year
Thiemann	1M	2M	1M
Zankl		1M	1M
PD2		6M	12M
Sternagel			2M

Goal 1: Loops w.r.t. strategy

Consider leftmost-outermost rewriting: $\xrightarrow{\mathsf{lo}}$

$$\begin{aligned} x == y &\to \mathsf{eq}(\mathsf{chk}(x), \mathsf{chk}(y)) \\ \mathsf{eq}(x, x) &\to \mathsf{true} \\ \mathsf{chk}(x) &\to \mathsf{false} \\ \mathsf{eq}(\mathsf{false}, y) &\to \mathsf{false} \end{aligned}$$

Above TRS can be used to check equality of arbitrary terms in constant number of reductions

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$\Rightarrow$ (Non-)Termination analysis has to be strategy aware

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Considered strategies for rewrite step $s \rightarrow_p t$

- innermost (no redex strictly below position p)
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Loop under strategy

- loop $t \to \dots u \to_p v \to \dots C[t\mu]_q$ respects strategy iff reduction

$$\underbrace{C[C[C[\dots C[}_{n}u\,\underbrace{\mu]\dots\mu]\mu]\mu]}_{n} \to_{q\dots qp} \underbrace{C[C[C[\dots C[}_{n}v\,\underbrace{\mu]\dots\mu]\mu]\mu]}_{n}$$

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  respects strategy for all n and all steps $u \to_p v$ in the loop
- decidable for
  - context-sensitive rewriting (trivial)
  - innermost strategy (difficult)
  - outermost strategy (even more difficult)
  - forbidden patterns (most difficult)

Goal 2: Unraveling

- Unraveling is transformation $\mathcal{U}$ from conditional to standard rewriting
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- Variations in how many auxiliary variables are used as arguments of U
- Example:

$$
\begin{array}{ll}
\mathsf{div}(x, \mathsf{s}(y)) \to 0 & \mid x \leq y \to^* \mathsf{true} \\
\mathsf{div}(x, \mathsf{s}(y)) \to \mathsf{s}(\mathsf{div}(x - \mathsf{s}(y), \mathsf{s}(y))) & \mid x \leq y \to^* \mathsf{false}
\end{array}
$$

becomes

$$
\begin{array}{ll}
\mathsf{div}(x, \mathsf{s}(y)) \to \mathsf{U}_1(x \leq y, {\color{red}x}, {\color{red}y}) & \mathsf{U}_1(\mathsf{true}, {\color{red}x}, {\color{red}y}) \to 0 \\
\mathsf{div}(x, \mathsf{s}(y)) \to \mathsf{U}_2(x \leq y, x, y) & \mathsf{U}_2(\mathsf{false}, x, y) \to \mathsf{s}(\mathsf{div}(x - \mathsf{s}(y), \mathsf{s}(y)))
\end{array}
$$

red variables are subject to optimizations

Goal 2: Unraveling

- Completeness of unraveling is easy: $s \rightarrow_{\mathcal{R}} t$ implies $s \rightarrow^{+}_{\mathcal{U}(\mathcal{R})} t$
- Soundness is much harder and does not always hold: if $s \rightarrow^{*}_{\mathcal{U}(\mathcal{R})} t$ then $s \rightarrow^{*}_{\mathcal{R}} t$ for s and t not containing U-symbols sufficient conditions like
  - ultra left linearity (for optimized and non-optimized unraveling)
  - ultra weak left linearity

Goal 3: Certifying constrained rewriting

- Completely depends on results of Task 1:
  - what kinds of constraints?
  - definition of rewrite relation?
  - ...

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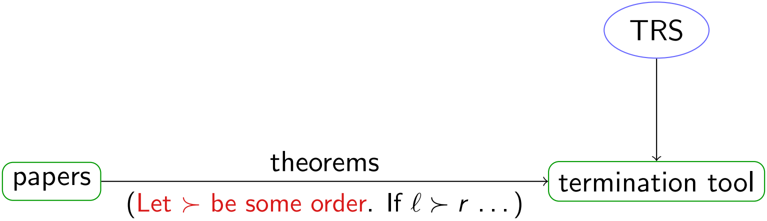
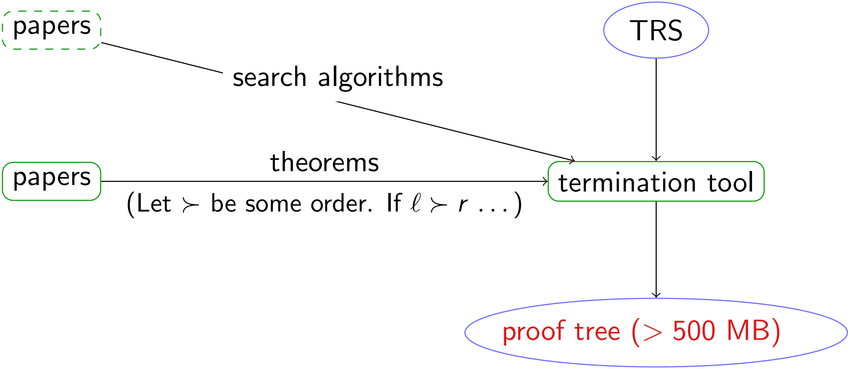

TRS
papers
theorems
(Let $\succ$ be some order. If $\ell \succ r$ ...)
termination tool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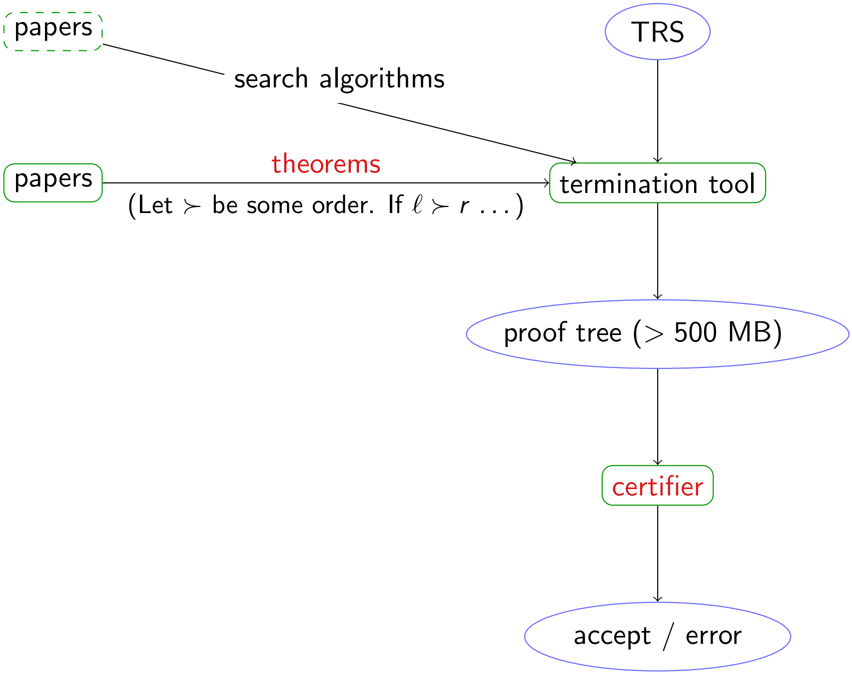
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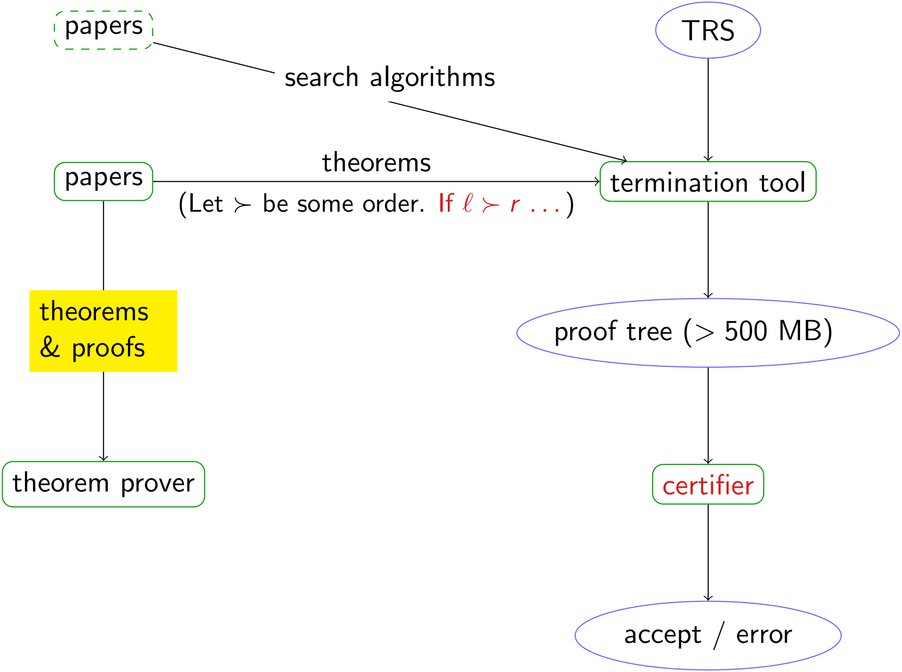
certifier
accept / error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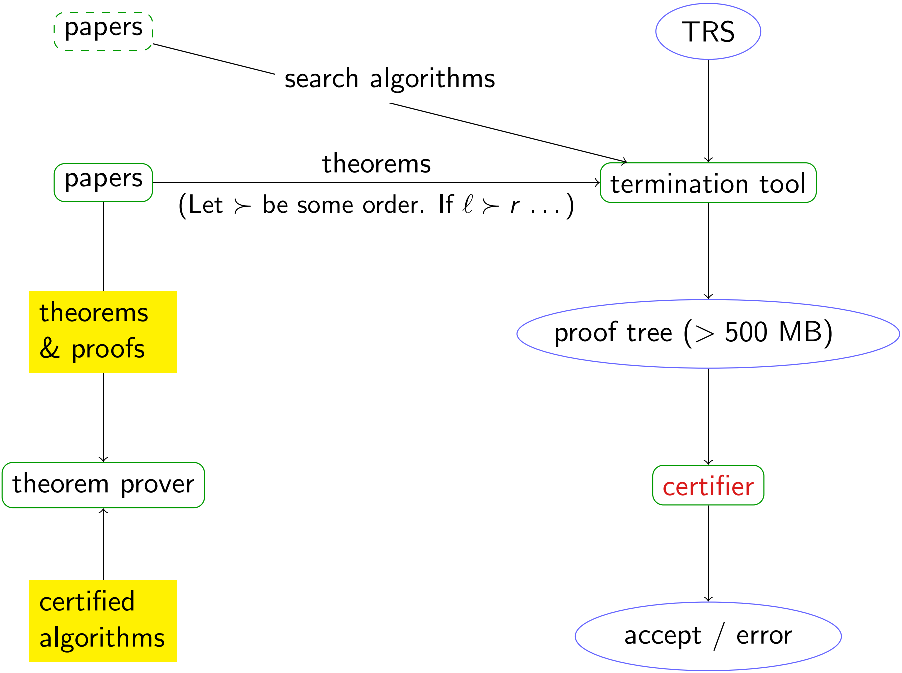
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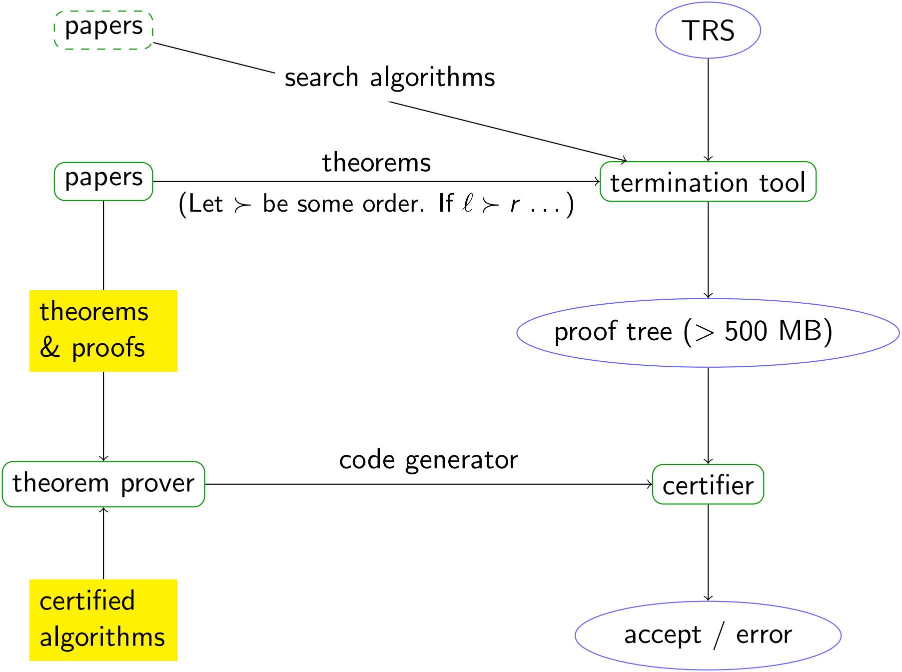
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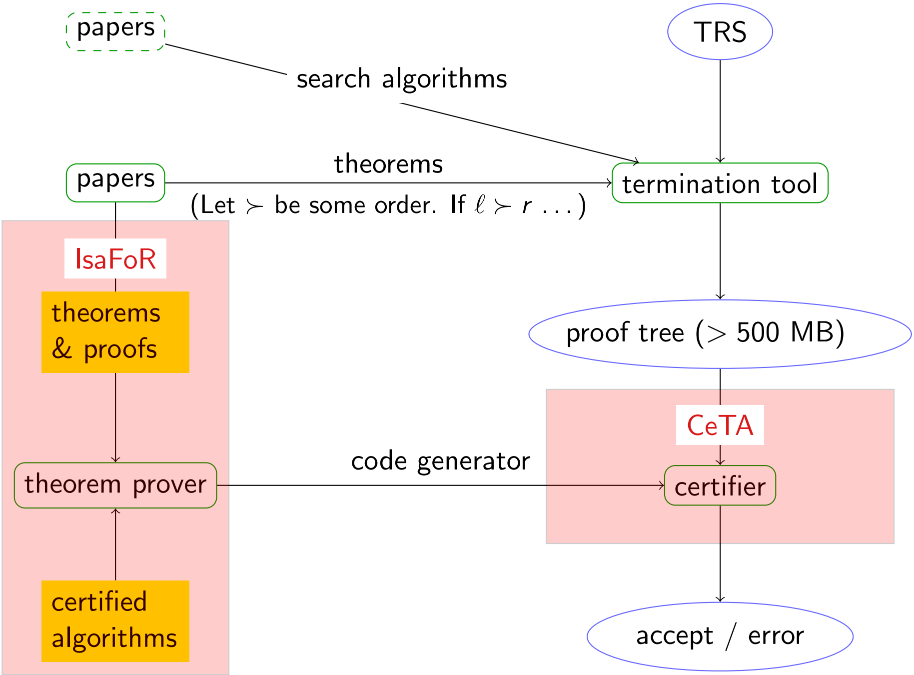
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Isabelle/HOL

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- HOL: functional programming + theorem proving

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(∗ *programming* ∗)
datatype 'a list = *Nil* | *Cons 'a ('a list)*
fun insert where
 insert x Nil = *Cons x Nil*
 insert x (Cons y ys) = *if* $x \leq y$ *then Cons x (Cons y ys) else* ...

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$(*$ *specifying* $*)$
$fun\ sorted\ \ where$
$sorted\ \ Nil = True$
$sorted\ (Cons\ x\ Nil) = True$
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$(*\ \mathit{proving}\ *)$
$\mathit{lemma} : \mathit{sorted}\ \mathit{xs} \Longrightarrow \mathit{sorted}\ (\mathit{insert}\ x\ \mathit{xs})$
$\mathit{proof}\,(\mathit{induction}\ \mathit{xs} \ldots)$

IsaFoR: Isabelle/HOL Formalization of Rewriting

- library on rewriting, deep embedding (TRS is data object)

$$\begin{array}{ll} \mathit{datatype}\ ('f,'v)\mathit{term} & = \mathit{Var}\ 'v \mid \mathit{Fun}\ 'f\ (('f,'v)\mathit{term}\ \mathit{list}) \\ \mathit{type\_synonym}\ ('f,'v)\mathit{trs} & = (('f,\ 'v)\mathit{term} \times ('f,'v)\mathit{term})\ \mathit{set} \end{array}$$

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  - substitution constraint *nfs* to allow free variables in right-hand sides:

$$C[\ell\sigma] \overset{\mathit{nfs}}{\rightarrow}{}^{\mathcal{Q}}_{\mathcal{R}} C[r\sigma] \text{ for } \ell \to r \in \mathcal{R} \text{ if}$$

  - all arguments of $\ell\sigma$ are in normal form w.r.t. $\to_{\mathcal{Q}}$ (innermost)
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- recently added results on confluence, complexity, completion, and equational reasoning

Supported termination techniques

- Dependency Pairs
- Dependency Pair Framework
- (Innermost) Dependency Graph Processor
- (Innermost) DP Transformations Narrowing, Rewriting, and (Forward)-Instantiation
- Flat Context Closure
- Innermost Switch
- Innermost- and Outermost-Loops
- Match- and roof-bounds
- Reduction Pair Processors (poly, matrix, LPO, RPO, KBO, ...)
- Root-Labeling
- Semantic Labeling
- Size-Change Termination
- String Reversal
- Subterm Criterion
- Uncurrying
- (Innermost) Usable Rules

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- both IsaFoR and CeTA are easily available (repository or distribution)

  `http://cl-informatik.uibk.ac.at/software/ceta/`

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- Innermost Loops: original proof works in three phases

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  2. $x\mu^n \neq y\mu^n$ for all n

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  3. complex algorithm to decide identity problems
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  3. True (the variable in both terms is toggling)

Goal 1: Loops under strategies for $t \to^+ C[t\mu]$

- Innermost Loops: original proof works in three phases
  1. setting up matching problems:

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  2. simplified to identity problems using matching algorithm:

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- everything formalized, deviation from original proof in 3. step: improved complex algorithm, Kruskal no longer required

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- main result formalized without any formalization of 3. step: for extended identity problems from outermost loops k in $(\star)$ can be fixed to 0
  $\Longrightarrow$ get non-extended identity problem

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  - complete formalization of 1. step
  - figure out whether trick for outermost loops (fix $k = 0$) is also possible for extended identity problems from forbidden patterns;
    if not, formalize complex algorithm for extended identity problems

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  - algorithm to compute $\to_{\mathcal{R}}$ (for quasi-decreasing $\mathcal{R}$)
  - soundness results for unravelings

Goal 3: Constraint rewriting

- wait for outcome of Task 1
- then there will be several open tasks

Task 5: Certification, Summary

- 1. goal: certifying loops
  - large parts already done (innermost, outermost)
  - generalization to forbidden patters currently open
- 2. goal: unraveling
  - only completeness results in IsaFoR at the moment
- 3. goal: constrained rewriting
  - not clear what to do at the moment

- open questions
  - which tools provide proofs to certify (TTT2?, VMTL?, Nagoya?)
  - which soundness result for unraveling (Nagoya?, Vienna?, combined?)