

# An executable semantics of Object-Oriented models for simulation and theorem proving

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**Abstract.** This paper presents an executable semantics of OO models. We made it possible to conduct both simulation and theorem proving on the semantics by implementing its underlying heap memory structure within the expressive intersection of the functional language ML and the theorem prover HOL. This paper also presents a verification system ObjectLogic which supports simulation and theorem proving of OO models based on the executable semantics. As an application example, we show a verification of a UML model of a practical firewall system.

## 1 Introduction

As our society has become more dependent on the information systems, it has become more important to ensure the correctness and validity of those systems. Especially, there is a growing need for the verification on the analysis level of the development since the scale of systems is becoming large and the bugs found in the coding stage lead to a fatal loss for constructors. Verification on the analysis levels allows early detection of bugs and, as a result, reduces the total cost of development.

Among many verification methods, we focus on theorem proving which is recently gathering attention in industrial areas. The prominent feature of theorem proving is induction by which we can prove the correctness of system behavior exhaustively for arbitrary inputs. In order to apply theorem proving to the analysis models such as UML (Unified Modeling Language [1]), we need to implement a formal semantics of OO models in theorem provers.

We consider that the semantics should be executable. This is because an executable semantics allows us to conduct not only theorem proving but also simulation. Even though theorem proving is quite a powerful verification method, it is not efficient when it comes to the cost-effectiveness because it requires manual intervention of users through proofs. But, simulation can, to some extent, compensate the disadvantage of theorem proving. Simulation is efficient in that it allows us to identify the result of system execution at a glance. Although it cannot ensure the 100% correctness, we can immediately check if the result is apparently correct or not. This is especially useful in the early stage of model construction where many trivial bugs are included. By simulation, we can efficiently find trivial bugs in advance of high-cost theorem proving, and as a result, we can optimize the total cost of verification.

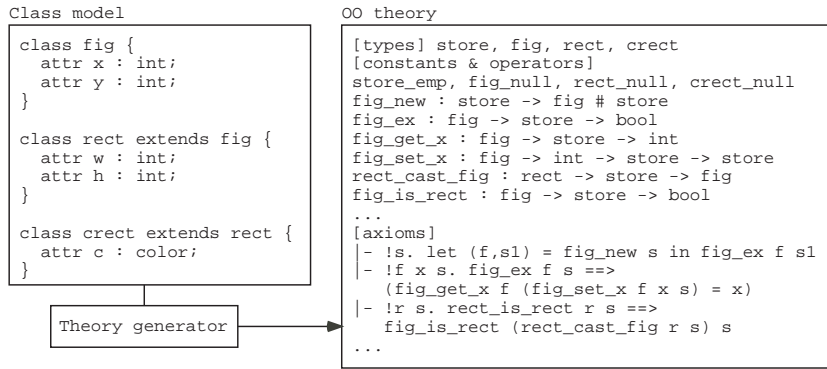
So far, we have implemented a semantics of OO models as a theory in the theorem prover HOL [2] and conducted verification of a simple library system [16]. But, only theorem proving was possible because the semantics was not executable. So this time, for the purpose of simulation, we made the semantics executable by implementing it in the functional language Moscow ML [3] (the meta-language of HOL). In HOL, the semantics is implemented based on a heap memory structure, i.e. The types and operations in the semantics are represented by those of the heap memory, and all the axioms are derived from their definition. We implemented the same data structure as actual operation in ML and made the semantics directly executable. The trick is that this memory structure is implemented within the intersection of the expressive power of HOL and ML. This makes it possible to implement the semantically equivalent memory structures both in HOL and ML. To have an equivalent semantics is important to make the result of simulation and theorem proving consistent with each other.

In this paper, we present the overview of the executable semantics and its implementation. We also present a verification system ObjectLogic which supports simulation and theorem proving of OO models based on the executable semantics. As an application, we show a verification of a firewall system where we proved that UML sequence diagrams satisfies constraints written in OCL (Object Constraint Language [4]).

This paper is organized as follows. Section 2 explains the overview of the executable semantics. Section 3 explains its implementation. Section 4 explains how to execute the semantics. Section 5 introduces ObjectLogic. Section 6 shows a verification of a firewall system. Section 7 cites related works. Section 8 gives conclusion and future work.

## 2 The OO semantics

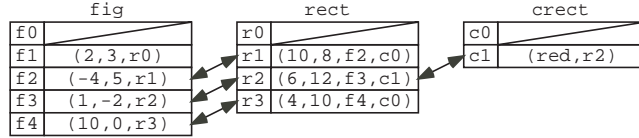
We implemented the OO semantics as a theory in HOL. The theory is not specific to particular models, but implements general OO concepts so that it can be used as a groundwork for various models. The OO concepts covered are classes, attributes, inheritance (tree), object subtyping and method dynamic dispatching. Besides covering general OO concepts, it has two characteristics. Firstly, it allows arbitrary types (without type variables) to be embedded into the types of object attributes (fields). Compared to the verification on the program level, verification on the analysis level requires high-availability of types since the analysis models are abstracted by various types such as set, stack and date. In fact, UML, the most major modeling language, does not limit available types to particular ones. Therefore, we enabled attributes to have arbitrary types. This feature is also beneficial in that we can utilize various types and pre-proved theorems in HOL libraries when constructing models and performing proofs. But it is not so easy to realize this feature in the first-order types of HOL. The problem lies in the structure of objects, i.e. an object is a data which holds multiple attributes of arbitrary types. So, we cannot represent objects by naively taking a product of type variables like  $\alpha * \beta * \gamma * \dots$  because we cannot predict how many variables we should put in the product. To cope with this problem, we take the approach of automatically constructing the semantics depending on the type information of the application. If types are given in advance, an object is easily represented by a product of those types like  $num * string * bool$ . The object referencing and inheritance are realized by putting



**Fig. 1.** The OO theory

those products in a heap memory structure. The implementation details are explained in the next section. Secondly, it is shallowly embedded, i.e. the concepts such as classes, attributes, and inheritance are represented directly as types and constants in HOL. This is because our verification target is each instance of OO models. Shallow embedding facilitates the proof of instance levels compared to deep embedding [5]. It also has the effect of making the theory simple because all the typing information is directly represented by the type system of HOL and there is no need to additionally include the typing constraints into the theory.

The OO theory is defined based on the class model of the target system. We implemented the theory generator which inputs a class model and outputs its OO theory. Fig.1 shows an example. The class model defines static structure of a system such as classes, attributes and inheritance. The OO theory defines types, operators and axioms representing basic OO concepts. The type playing central part in the theory is `store`. It represents the environment which holds all alive objects in the system and has the constant `store_emp` representing an empty store. The type `fig` represents the type of object references for the class `fig` and has the constant `fig_null` representing NULL reference. The operator `fig_new` is a function to create a new `fig` instance in the store. The operator `fig_ex` is a predicate to test if a `fig` object exists in the store. The first axiom is a property about these operators saying “The newly created `fig` object is alive in the store after the creation.” The operators `fig_get_x` and `fig_set_x` are functions to read and write the attribute `x` of the class `fig`. The second axiom says “If the `fig` object is alive in the store, the value of the attribute `x` of the object obtained just after updating it to `v` equals to `v`.” The operator `fig_cast_rect` is a function to cast a `fig` object downward from `fig` to `rect`. The operator `fig_is_rect` is a predicate to test if a `fig` object is an instance of the class `rect`. After an object is created, its apparent type can be changed by cast operators, but instance-of operators remember the actual type of the object. The third axiom illustrates this. It says “If a `rect` object is an instance of the `rect` class, it is still the instance of the `rect` class even if it is cast to



**Fig. 2.** Snapshot of the heap memory

the `fig` class.” The instance-of operators are used to implement the dynamic method dispatching.

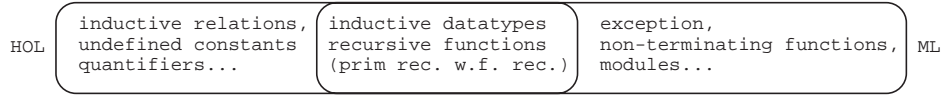
### 3 Implementation in HOL

In order to guarantee the soundness of the theory, we implemented the theory conservatively by definitional extension. This is a standard method to construct sound theories in HOL, where new theories are derived from existing sound theories by only allowing introduction of definition and derivation by sound inference rules. This can be comparable to axiomatic theory construction where axioms are directly introduced in the theory, which often makes the theory inconsistent. The theory is derived from a heap memory structure which is defined by primitive theories already existing in HOL. The types and constants in the theory are represented by those in the heap memory and the axioms are derived from their definitions.

Fig.2 shows a snapshot of the heap memory for the example model. The heap memory consists of three sub-heaps which are introduced corresponding to the three classes. Each sub-heap is represented by a list and the whole heap is represented by a tuple of them. Object references are represented by indices of the memory. For example, the references `f0`, `f1`, `f2`,... are represented by natural numbers 0,1,2, ... (`f0` represents a null reference `fig_null`). Object instances are represented by a tuple or multiple tuples stored in the sub-heaps. For example, the tuple in `f1` represents a `fig` instance whose attribute are `x=2` and `y=3`. Two tuples in `f2` and `r1` together represent a `rect` instance whose attribute are `x=-4`, `y=5`, `w=10`, and `h=8`. The two tuples are linked by storing the references `r1` and `f2` each other. Object subtyping is modeled by this linked-tuple structure. For example, the three references `f3`, `r2` and `c1` all point at the same `crect` instance. This means the `crect` instance can have three apparent types `fig`, `rect` and `crect`. The operators in the theory are implemented as the functions to manipulate the heap memory. Their definition is detailed in [16].

### 4 Executing the semantics

In HOL, the theory is derived from the heap memory structure. By defining the same data structure in ML, the theory becomes executable. It is known that HOL and ML have similar type systems and there exists an intersection of expressiveness between them. Fig.3 illustrates this. The common concepts are inductive datatypes and recursive functions (primitive recursion and well-founded recursion) [8][9]. The heap memory



**Fig. 3.** Expressiveness of HOL and ML

structure in HOL is defined within this intersection and the same data structure can be defined in ML in a straightforward way.

For example, the following HOL function `write` is the function on the sub-heap `l` to update the data in the address `n` by data `x`:

```
write (n:num) (x:'a) (l:'a list) =
  if 0 < n /\ n < LENGTH l then write1 n x l else l
(write1 0 x l = x::(TL l)) /\
(write1 (SUC n) x l) = (HD l)::(write1 n x (TL l))
```

This primitive recursive function can be also defined in ML as follows:

```
fun write (n:int) (x:'a) (l:'a list) =
  if 0 < n andalso n < length l then write1 n x l else l
fun write1 0 x l = x::(tl l)
  | write1 n x l = (hd l)::(write1 (n-1) x (tl l))
```

We can see that the definition is directly correspondent except that we are using the integer type `int` in ML instead of the natural number type `num`. This is because ML does not have a type of natural numbers. This does not make any difference in their semantics. As for exception, we cannot use ML exception because HOL does not have the concept. In Java, exceptions are raised in the cases such as NULL reference accessing and illegal down-casting. We handle such cases by returning appropriate values. This is summarized by the following axioms:

```
|- !s. fig_get_x fig_null s = 0
|- !x s. fig_set_x fig_null x s = s
|- !f s. fig_is_fig f s ==> (fig_cast_rect f s = rect_null)
```

The first axiom means “When an attribute of a NULL reference is referred, it returns the default value for the attribute type.” The alternative approach could be to return an undefined constant like `fig_unknown_x:int`, but as it is not supported in ML, we simply return the default value. The second axiom means “When an attribute of a NULL reference is updated, it actually does not cause the effect to the state.” The third axiom means “When illegal down-casting occurs, it returns the NULL reference of the destination class.”

In this way, we can define a semantically equivalent heap memory structures both in HOL and ML. To have an equivalent semantics is important to make the result of simulation and theorem proving consistent with each other.

We extended the theory generator so that it outputs the ML structure which implements the heap memory structure. Its signature provides the types and operators corresponding to those in the OO theory. Fig.4 shows the execution of those operators. The internal structure of objects and stores are hidden by the opaque signature restriction.

```

- val (r,s1) = rect_new store_emp;
> val r = <rect> : rect
> val s1 = <store> : store
- val s2 = rect_set_x r 10 s1;
> val s2 = <store> : store
- val f = rect_cast_fig r s2;
> val f = <fig> : fig
- fig_get_x f s2;
> val it = 10 : int

```

Fig. 4. Execution of the theory

### 5 ObjectLogic

ObjectLogic is a verification system for OO models. It supports both simulation and theorem proving based on our executable semantics. It enables us to define models in a high-level language called OML (ObjectLogic Meta-Language). It is a sequential OO language whose syntax is closed to Java and is able to import arbitrary types and functions from HOL. In OML, we can specify assertions such as method contracts (pre- and post-conditions) and class invariants. From the assertions, ObjectLogic produces target propositions (proof obligation) in the OO theory. It also provides tactics which tries to prove the goal automatically by applying the axioms in the theory.

Fig.5 shows how it works. Firstly, ObjectLogic constructs the simulator and the theory from the classes using the theory generator (1). Then, it translates methods into functions both on the simulator and the theory (2). Finally, it translates assertions into HOL propositions (3). We can conduct simulation using the ML executables and theorem proving by proving the HOL propositions.

ObjectLogic can be used for verification of UML models. As shown in Fig.5, all we have to do is to translate UML class diagrams, sequence diagrams and OCL constraints into classes, methods and assertions in OML (Currently, this translation is done by hand and future version of ObjectLogic will support automatic translation). We can prove that the internal behavior of the sequence diagrams satisfies the method contracts and class invariants defined as OCL constraints in the class diagram.

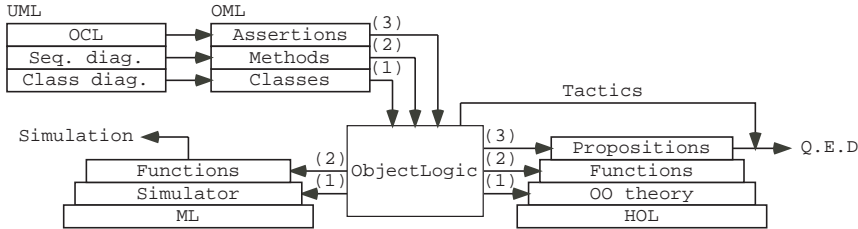


Fig. 5. ObjectLogic

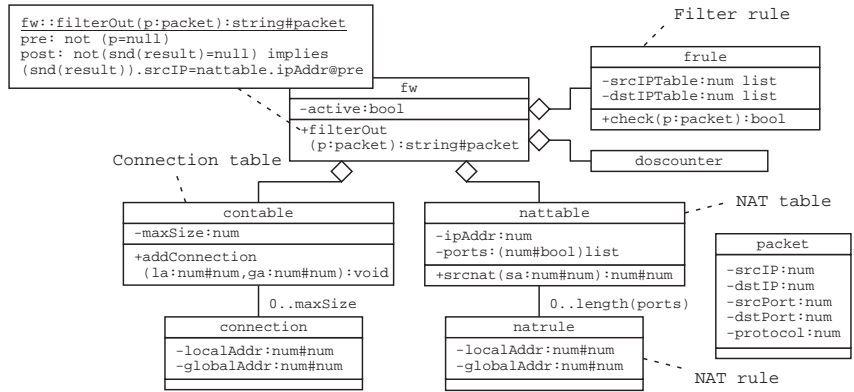


Fig. 6. UML class diagram of the firewall system

## 6 Verification of a firewall system

We applied ObjectLogic to verification of a practical firewall system. The specification of the firewall system is our original one based on a real product of a company close to Cisco<sup>®</sup> PIX Firewall. We conducted verification as follows. Firstly, we translated the specification into UML models (structural part into class diagrams and behavioral part into sequence diagrams) and the requirements into constraints in OCL. Then, we translated the UML models and the OCL constraints into OML and input it to ObjectLogic obtaining the ML executables and the HOL propositions. Finally, we conducted simulation using the executables and proved propositions in HOL. The OML code is about 1200 lines containing 8 classes, 35 attributes and 102 methods.

The firewall system is a stateful packet filter with NAT (Network Address Translation) which is the mechanism to translate between a public IP address of the firewall and multiple private IP addresses of local hosts. It has dual effects of sharing a single IP address among multiple hosts and hiding private addresses of the local hosts. Fig.6 shows the class diagram of the firewall system. The model is abstracted with respect to datatypes using HOL types. For example, IP addresses are represented by natural numbers and the filter rules are represented by lists of permissible address numbers. The key function of the system is the packet filtering function which is defined as the method `filterOut()` of the class `fw`. It inputs a packet and outputs a string message and the filtered packet. The internal behavior of this method is defined as sequence diagrams, which we omit to show due to space limitation.

For this firewall, we conducted simulation and theorem proving to verify the properties such as “The outbound packets which do not meet the filter rules are always dropped unless they belong to existing connections” and “The source IP address of the outbound packet is always updated by the public IP address of the firewall”. Both of them are crucial for the security of the firewall. The first property ensures that a local host never connects to illegal hosts in the outside network. The second property (NAT property) ensures that the private IP addresses of the local network never leak to the

```

- val (fw,s) = new_fw store_emp; (* Creat a FW *)
(* Set configuration values *)
- val (_,s) = fw_setIpAddr pfm 200 s;
- val (_,s) = fw_setPorts pfm [1200,1210,1220] s;
- val (_,s) = fw_setFilterRules pfm SRCADDR [10,20,30] s;
...
- val (p,s) = new_packet 20 1070 250 80 TCP s; (* Create a packet *)
> val p = <packet> : packet
  val s = <store> : store
- val (msg,p,s) = fw_filterOut fw p s; (* Apply outbound filtering *)
> val msg = "pass: new connection" : string
  val p = <packet> : packet
  val s = <store> : store
- packet_getInfo p s; (* Display packet information *)
> val it = ((200, 1200), (250, 80), 0) : (int * int) * (int * int) * int
...

```

**Fig.7.** Simulation of the firewall system

outside network. The class diagram includes the OCL constraint representing the NAT property which is defined as the contract of the method `filterOut()`.

Fig.7 shows simulation of the firewall. We firstly created a firewall object and set the configuration values such as the public IP address, the port numbers and the filter rules. Then, we applied the filtering function to an outbound packet and identified that the firewall correctly passed the packet and updated the source IP address by the NAT rule. By simulation, we were able to find some trivial bugs. For example, we found the lack of method invocation to add a connection by seeing the result where the connection table remained unchanged. We also found that `then-` and `else-`parts of `if` statement were reversed by seeing the result where an apparently correct packet was dropped. We were able to find these kinds of easy bugs efficiently by simulation and avoid the worst case to find them by high-cost theorem proving.

The NAT property is proved in HOL as the following theorem:

```

|- !(this:fw) (p:packet) (s:store).
  let (msg,p',s') = fw_filterOut this p s in
  packet_ex p s /\ invariants fw s ==>
  packet_ex p' s' ==>
  (packet_getSrcAddr p' s' = fw_getIpAddr this s)

```

The function `fw_filterOut` represents the filtering method of the firewall. It means “If the output packet `p'` is not NULL (the packet is passed), the source IP address of `p'` is equal to the public IP address of the firewall.” By this theorem, we can ensure that private addresses never leak outside. The proof took 8 hours and we proved 21 lemmas in the course. The proof code length is about 450 lines (about one tactic per line). The entire proof is done in the OO theory level using the tactics in ObjectLogic without digging down to the heap memory level. To be able to conduct proof in the OO level, which is close to human intuition, is the major advantage of ObjectLogic.

In this way, ObjectLogic enables both simulation and theorem proving in the equivalent semantics in ML and HOL. By proving crucial properties of the firewall system, we made sure that ObjectLogic can be applicable to practical systems of proper scale. In order to make it applicable to general large systems, we need to make the proof



more efficient. The key is how efficient we can make the inference of loop statements. We consider it effective to introduce high-level loop statements for manipulating object collections and their inference rules because they frequently appear in application domains: calculate the interest for all the accounts in a bank, calculate the total price of all the items in the cart in online shopping sites. It is also effective to implement a verification condition generator for OML. To prove verification conditions is much more efficient than to prove propositions directly in the OO theory because we can focus on the proof of datatypes apart from the axioms in the OO theory.

## 7 Related work

The semantic equivalence between simulation and theorem proving is a notable feature of our executable semantics, which is realized by defining it in the expressive intersection of ML and HOL. This is similar to ACL2 which combines a theorem prover and a programming language based on the same language, an applicative subset of Common Lisp. ACL2 is often used as a semantics for both simulation and theorem proving. The work by G. Al Sammane [14] presents a tool TheoSim which combines simulation and theorem proving of VHDL designs. The work by M. Wilding et al. [15] defines a formal model of a microprocessor to integrate simulation and formal analysis. Even though ACL2 is successful in hardware verification, we consider it has a limitation in software verification because the representation is low level and the types are limited to numerals. On the other hand, our tool ObjectLogic is suited for software verification because it supports objects and allows arbitrary types. We consider that, in the firewall verification, the high-abstractness of the semantics saved a lot of modeling and proving effort which would have been taken in the case of using ACL2.

There are a lot of work to implement OO semantics in theorem provers especially for Java and UML. The work by G. Klein et al. [6] implements semantics of Java for both source language level and bytecode level in Isabelle/HOL. The work by G. Barthe et al. [7] implements an executable semantics of JavaCard platform (virtual machine and bytecode verifier) in Coq. Both of them adopt a deep embedding because their verification target is on the platform level such as type safety, soundness of Hoare calculi and correctness of the bytecode verifier. We adopted a shallow embedding because our verification target is on the instance level such as method contracts and class invariants. A shallow embedding makes the proof on the instance level easier and the theory itself simpler than deep embedding. The work by J. Berg et al. [10] and C. Marché et al. [11] implements Java semantics for reasoning Java programs annotated with JML specifications as memory models in WHY and Isabelle/HOL, respectively. We defined a similar memory model, but it differs from them in that it allows arbitrary types for object attributes, which is effective in the verification on the analysis level. As for UML, the work by [12] implements a semantics of UML sequence diagrams in PVS. The work by A. D. Brucker et al. [13] implements a semantics of the specification language OCL as a conservative shallow embedding in Isabelle/HOL. Compare to these work, our semantics is not specific to particular languages but implements basic typical OO concepts. We are aiming at constructing a general-purpose semantics which can be used as a groundwork for various languages.

## 8 Conclusion and future work

In this paper, we presented an executable semantics of OO models for the foundation of both simulation and theorem proving. The semantics is implemented in two languages: HOL for theorem proving and ML for simulation. We preserved the semantics equivalence between them by implementing the underlying heap memory structure within the expressive intersection of HOL and ML. We also presented a verification system ObjectLogic which supports simulation and theorem proving based on the executable semantics. As an application, we showed a verification of a UML model of a practical firewall server system. Future work is to reinforce the verification capability of ObjectLogic by implementing a test suite generator and a verification condition generator.

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