Proof Score Development with Specification Calculus (CITP)

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Specification Verification

- Constructing specifications and verifying them in the upstream of system/software development are still one of the most important challenges in system/software development and engineering. It is because many critical defects are caused at the phases of domain, requirement, and design specifications.

- Proof scores in CafeOBJ are intended to meet this challenge.

Verification with Proof Scores (1)

- For verifying a system, a model of the system should be formalized and described as system specifications that are formal specifications of the behavior of the system. System specifications are formalized in equations and transition rules.
- In conjunction with the system specifications, functions and predicates that are necessary for expressing the system's supposed properties are formalized and described in equations as property specifications.

Verification with Proof Scores (2)

- Proof scores are developed to verify that the system's supposed properties are deduced from the system and property specifications.
- Proof scores are described in equations, and the deduction is done only by reduction (i.e. rewriting from left to wright) with the equations.

Transition Systems, Reachability, Invariants

- A **transition system** is defined as a three tuple (St, Tr, In). St is a set of states, $Tr \subseteq St \times St$ is a set of transitions on the states, and $In \subseteq St$ is a set of initial states.
- A sequence of states $s_1 s_2 \cdots s_n$ with $(s_i, s_{i+1}) \in Tr$ for each $i \in \{1, \cdots, n-1\}$ is defined to be a **transition sequence**.
- A state $s^r \in St$ is defined to be **reachable** if there exists a transition sequence $s_1 s_2 \cdots s_n$ with $s_n = s^r$ for $n \in \{1, 2, \cdots\}$ such that $s_1 \in In$.
- A state predicate p (i.e. a function from St to `Bool`) is defined to be an **invariant** (or an invariant property) if ($p(s^r)$ = `true`) for any reachable state s^r.

$\Sigma = (S, \leq, F)$, $T_\Sigma(X)$, (Σ, E)

- Let $\Sigma = (S, \leq, F)$ be a regular order-sorted signature with a set of sorts S, and let $X = \{X_s\}_{s \in S}$ be an S-sorted set of variables.
- Let $T_\Sigma(X)$ be S-sorted set of $\Sigma(X)$-terms, let $T_\Sigma(X)_s$ be a set of $\Sigma(X)$-terms of sort s, let E be a set of $\Sigma(X)$-equations, and let (Σ, E) be an equational specification with unique sort `State`.
- Let $\theta \in T_\Sigma(Y)^X$ be a substitution (i.e. a map) from X to $T_\Sigma(Y)$ for disjoint X and Y then θ extends to the morphism from $T_\Sigma(X)$ to $T_\Sigma(Y)$, and $t\,\theta$ is the term obtained by substituting $x \in X$ in t with $x\,\theta$.

Transition Rules

- Let $tr = (\forall X)(l \to r \texttt{ if } c)$ be a rewrite rule with $l, r \in T_\Sigma(X)_{\texttt{State}}$ and $c \in T_\Sigma(X)_{\texttt{Bool}}$, then tr is called a transition rule and defines the one step transition relation $\to_{tr} \in T_\Sigma(Y)_{\texttt{State}} \times T_\Sigma(Y)_{\texttt{State}}$ for Y being disjoint from X as follows.
- Note that $=_E$ is understood to be defined with $((\Sigma \cup Y), E)$ by considering $y \in Y$ as a fresh constant if Y is not empty.

$$(s \to_{tr} s') \stackrel{\text{def}}{=} (\exists \theta \in T_\Sigma(Y)^X)((s =_E l\,\theta) \texttt{ and } (s' =_E r\,\theta) \texttt{ and } (c\,\theta =_E \texttt{true}))$$

Transition Specification

- Let $TR = \{tr_1, \cdots, tr_m\}$ be a set of transition rules, let $\rightarrow_{TR} \stackrel{\text{def}}{=} \bigcup_{i=1}^{m} \rightarrow_{tr_i}$, and let $In \subseteq (T_\Sigma/=_E)_{\texttt{State}}$. In is assumed to be defined via a state predicate $init$ that is defined with E, i.e. $(s \in In)$ `iff` $(init(s) =_E \texttt{true})$.

- Then a transition specification (Σ, E, TR) defines a transition system $((T_\Sigma/=_E)_{\texttt{State}}, \rightarrow_{TR}, In)$.
- The state space of a transition system is formalized as a quotient set (i.e. a set of equivalence classes) of terms of a topmost sort `State`, and the transitions are specified with conditional transition rules (rewrite rules) over the quotient set.

Properties to be Verified

A property to be verified is either

- an invariant (i.e. a state predicate that is valid for all reachable states), or
- a (p leads-to q) property for two state predicates p and q ((p leads-to q) means that from any reachable state s with (p(s) = `true`) the system will get to a state t with (q(t) = `true`) no matter what transition sequence is taken).

- Given a transition system $TS = (St, Tr, In)$, let $p_1, p_2, \cdots, p_n$ $(n \in \{1, 2, \cdots\})$ be state predicates of TS, and $inv(s) \stackrel{\text{def}}{=} (p_1(s)\ \texttt{and}\ p_2(s)\ \texttt{and}\ \cdots\ \texttt{and}\ p_n(s))$ for $s \in St$.
- The following three conditions are sufficient for a state predicate p^t to be an invariant.

  (1) $(\forall s \in St)(inv(s)\ \texttt{implies}\ p^t(s))$
  (2) $(\forall s \in St)(init(s)\ \texttt{implies}\ inv(s))$
  (3) $(\forall (s, s') \in Tr)(inv(s)\ \texttt{implies}\ inv(s'))$

- A predicate that satisfies the conditions (2) and (3) like inv is called an **inductive invariant**. If p^t itself is an inductive invariant then taking $p_1 = p^t$ and $n = 1$ is enough. However, $p_1, p_2, \cdots, p_n$ $(n > 1)$ are almost always needed to be found for getting an inductive invariant, and to find them is a most difficult part of the invariant verification.

It is worthwhile to note that there are following two contrasting approaches for formalizing $p_1, p_2, \cdots, p_n$ for a transition system and its property p^t.

- Make $p_1, p_2, \cdots, p_n$ as minimal as possible to imply the target property p^t;
  - usually done by lemma finding in interactive theorem proving,
  - it is difficult to find lemmas without some comprehensive understanding of the system.
- Make $p_1, p_2, \cdots, p_n$ as comprehensive as possible to characterize the system;
  - usually done by specifying elemental properties of the system as much as possible in formal specification development,
  - it is difficult to identify the elemental properties without focusing on the property to be proved (i.e. p^t).

- For a sort Srt and a predicate p on Srt we get $((p(X:Srt) \rightarrow_E^* \texttt{true})\ \texttt{implies}\ (\forall t \in (T_\Sigma)_{Srt})(p(t) =_E \texttt{true}))$ and $(p(X:Srt) \rightarrow_E^* \texttt{true})$ is a sufficient condition to prove $(\forall t)p(t)$.
- However, usually p is not simple enough to obtain $(p(X:Srt) \rightarrow_E^* \texttt{true})$ directly, and we need to analyze the structure of terms in $(T_\Sigma)_{Srt}$ and E for (1) **generating** a set of terms $\{t_1, \cdots, t_m\} \subseteq T_\Sigma(Y)_{Srt}$ that covers all possible cases of $(T_\Sigma)_{Srt}$, and (2) **checking** $(p(t_i) \rightarrow_E^* \texttt{true})$ for each $i \in \{1, \cdots, m\}$.

- The generation & checking can be a theorem proving method for transition systems based on
  (1) generation of finite state patters that cover all possible infinite states, and
  (2) checking the validities of verification conditions for each of the finite state patterns.

- **Induction** is a similar kind of method for proving $(p(X : Srt) \rightarrow_E^* \texttt{true})$ by covering all the terms of a constrained sort Srt by making use of inductive structure of the terms of the constrained sort.

[Subsume] A term $t' \in T_\Sigma(Y)$ is defined to be an **instance** of a term $t \in T_\Sigma(X)$ iff there exits a substitution $\theta \in T_\Sigma(Y)^X$ such that $t' = t\,\theta$. A finite set of terms $C \subseteq T_\Sigma(X)$ is defined to **subsume** a (may be infinite) set of ground terms (i.e. terms without variables) $G \subseteq T_\Sigma$ iff for any $t' \in G$ there exits $t \in C$ such that t' is an instance of t.

[Generate&Check-S] Let $(T_\Sigma/=_E)_{\texttt{State}}, \rightarrow_{TR}, In)$ be a transition system defined by a transition specification (Σ, E, TR). Then, for a state predicate p_{st}, doing the following **Generate** and **Check** are sufficient for verifying

$$(\forall t \in (T_\Sigma)_{\texttt{State}})(p_{st}(t) =_E \texttt{true}).$$

Generate a finite set of state terms $C \subseteq T_\Sigma(X)_{\texttt{State}}$ that subsumes $(T_\Sigma)_{\texttt{State}}$.

Check $(p_{st}(s) \twoheadrightarrow_E^* \texttt{true})$ for each $s \in C$. $\square$

Built-in Search Predicate (1)

Let `q` be a predicate with arity "`State State`" for stating some relation of the current state and the next state, like ($inv(s)$ `implies` $inv(s')$). Let the function `valid-q` be defined using the CafeOBJ's built-in search predicate

`pred _=(*,1)=>+_if_suchThat_{_} : State %State %Bool Bool Info`

as follows.

```
-- predicate to be checked for a State
pred valid-q : State .
eq valid-q(S:State) =
   not(S =(*,1)=>+ SS:State if CC:Bool
       suchThat not((CC implies q(S,SS)) == true)
       {(ifm S SS CC q(S,SS))}) .
```

Built-in Search Predicate (2)

For a state term $s \in T_\Sigma(Y)_{\texttt{State}}$, the reduction of the Boolean term: `valid-q(s)` with $\twoheadrightarrow_E^* \cup \rightarrow_{TR}$ behaves as follows based on the definition of the behavior of the built-in search predicate.

1. Search for evey pair (tr_j, θ) of a transition rule $tr_j = (\forall X)(l_j \rightarrow r_j \texttt{ if } c_j)$ in Tr and a substitution $\theta \in T_\Sigma(Y)^X$ such that $\texttt{s} = l_j\,\theta$.
2. For each found (tr_j, θ), let $(\texttt{SS} = r_j\,\theta)$ and $(\texttt{CC} = c_j\,\theta)$ and print out `(ifm s SS CC q(s,SS))` and tr_j if $(\texttt{not((CC implies q(s,SS)) == true)} \twoheadrightarrow_E^* \texttt{true})$.
3. Returns `false` if any print out exits, and returns `true` otherwise.

[Cover] Let $C \subseteq T_\Sigma(Y)$ and $C' \subseteq T_\Sigma(X)$ be finite sets. C is defined to **cover** C' iff for any ground instance $t'_g \in T_\Sigma$ of any $t' \in C'$, there exits $t \in C$ such that t'_g is an instance of t and t is an instance of t'.

[Generate&Check-T1] Let $((T_\Sigma/=_E)_{\texttt{State}}, \rightarrow_{TR}, In)$ be a transition system, and let $C' \subseteq T_\Sigma(X)$ be the set of all the left-hand sides of the transition rules in TR. Then doing the following **Generate** and **Check** are sufficient for verifying

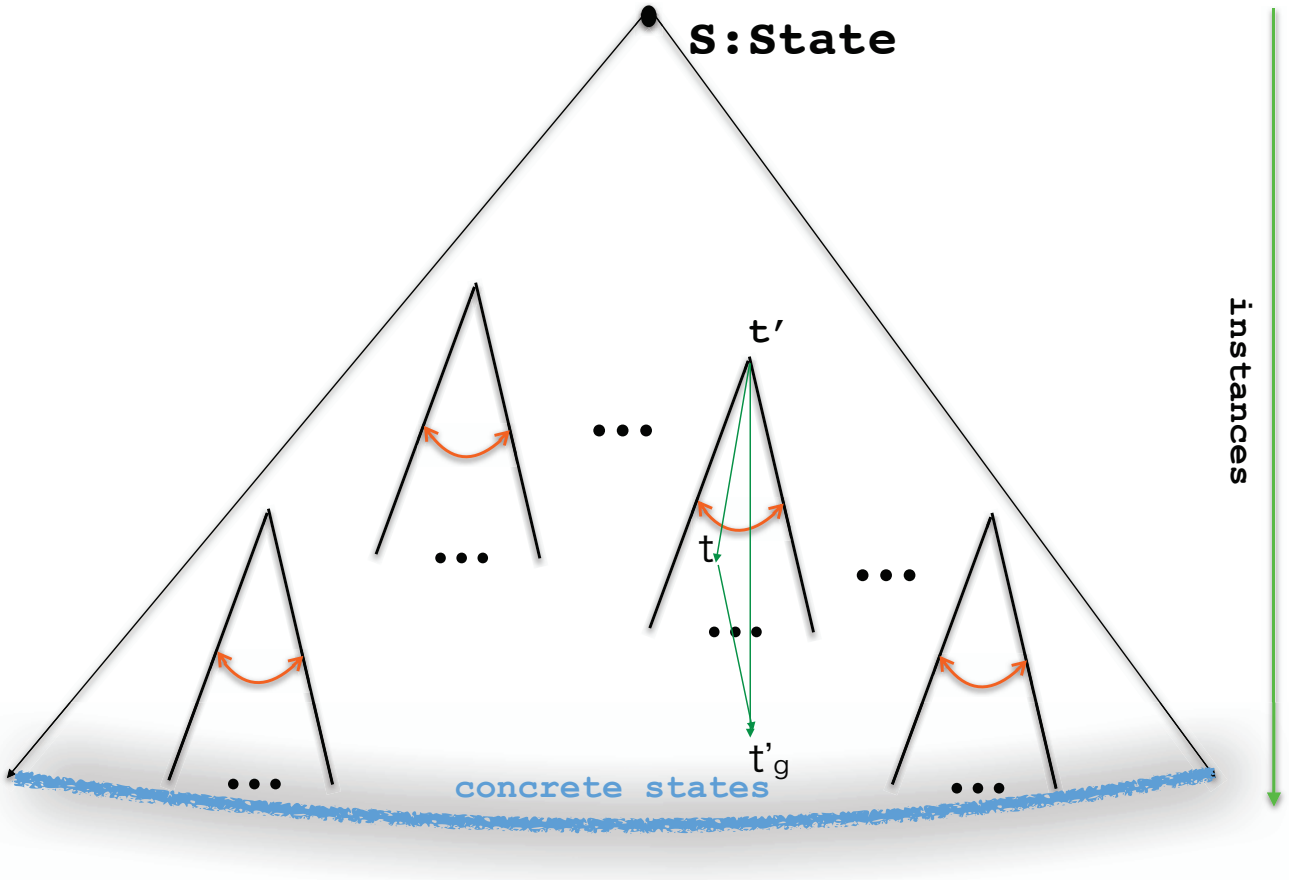
$$(\forall (s, s') \in ((T_\Sigma \times T_\Sigma) \cap \rightarrow_{TR}))(\texttt{q}_{\texttt{tr}}(s, s') =_E \texttt{true})$$

for a predicate "`pred` $\texttt{q}_{\texttt{tr}}$ `: State State`".

Generate a finite set of state terms $C \subseteq T_\Sigma(Y)_{\texttt{State}}$ that covers C'.

Check $(\texttt{valid-q}_{\texttt{tr}}\texttt{(t)} \twoheadrightarrow_E^* \cup \rightarrow_{TR} \texttt{true})$ for each $\texttt{t} \in C$. □

Covering



[Generate&Check-T2] Let $TR = \{tr_1, \cdots, tr_m\}$ be a set of transition rules, and let $tr_i = (\forall X)(l_i \rightarrow r_i \texttt{ if } c_i)$ for $i \in \{1, \cdots, m\}$. Then doing the following **Generate** and **Check** for all of $i \in \{1, \cdots, m\}$ is sufficient for verifying

$$(\forall (s, s') \in ((T_\Sigma \times T_\Sigma) \cap \rightarrow_{TR}))(\texttt{q}_\texttt{tr}(s, s') =_E \texttt{true})$$

for a predicate "`pred` $\texttt{q}_\texttt{tr}$ `:  State State`".

Generate a finite set of state terms $C_i \subseteq T_\Sigma(Y)_{\texttt{State}}$ that covers $\{l_i\}$.

Check $(\texttt{valid-q}_\texttt{tr}\texttt{(t)} \twoheadrightarrow_E^* \cup \rightarrow_{tr_i} \texttt{true})$ for each $\texttt{t} \in C$. $\square$

The conditions (1) and (2) for invariant properties can be verified by using Generate&Check-S with $\mathtt{p_{st-1}}(s)$ and $\mathtt{p_{st-2}}(s)$ defined as follows respectively.

(1) $\mathtt{p_{st-1}}(s) = (inv(s)\ \mathtt{implies}\ p^t(s))$
(2) $\mathtt{p_{st-2}}(s) = (init(s)\ \mathtt{implies}\ inv(s))$

Note that, if $inv \stackrel{\text{def}}{=} (p_1\ \mathtt{and} \cdots \mathtt{and}\ p_n)$ and $p^t = (p_{i_1}\ \mathtt{and} \cdots \mathtt{and}\ p_{i_m})$ for $\{i_1, \cdots, i_m\} \subseteq \{1, \cdots, n\}$, then condition (1) is directly obtained.

The condition (3) for invariant properties can be verified by using Generate&Check-T1 or T2 with $\mathtt{q_{tr\text{-}3}}(s, s')$ defined as follows.

(3) $\mathtt{q_{tr\text{-}3}}(s, s') = (inv(s)\ \mathtt{implies}\ inv(s'))$

Specification Calculus (SpecCalc)

Let S_i be a specification and p_i be a predicate in S_i, then "$S_i \vdash p_i$" is called a goal and means that "p_i is provable in S_i". A specification is denoted by a module in CafeOBJ .

Proof Goal: $\{S_g \vdash p_g\}$

Proof Rules: $$\frac{S_{i_1} \vdash p_{i_1}, S_{i_2} \vdash p_{i_2}, \cdots, S_{i_{k(i)}} \vdash p_{i_{k(i)}}}{S_{i_0} \vdash p_{i_0}} \ (i \in \{1, 2, \cdots, m\})$$

Calculation Rules:

- "$S_{i_0} \vdash p_{i_0}$" is replaced with "$S_{i_1} \vdash p_{i_1}, S_{i_2} \vdash p_{i_2}, \cdots S_{i_{k(i)}} \vdash p_{i_{k(i)}}$".
- "$S_j \vdash p_j$" is erased if "$p_j \rightarrow^*_{E_{S_j}}$ `true`". That is, if "`red in` S_j `:` p_j `.`" returns `true`.
- `The proof is over if the proof goal becomes empty.`

CafeOBJ **code**

```
:def <RuleName> = :csp{eq <t1l> = <t1r> .
                       eq
                          ...
                       eq <tml> = <tmr> .}

  such that ((<t1l> = <t1r>) or (<t2l> = <t2r>) or
                          ... or (<tml> = <tmr>)) = true
```

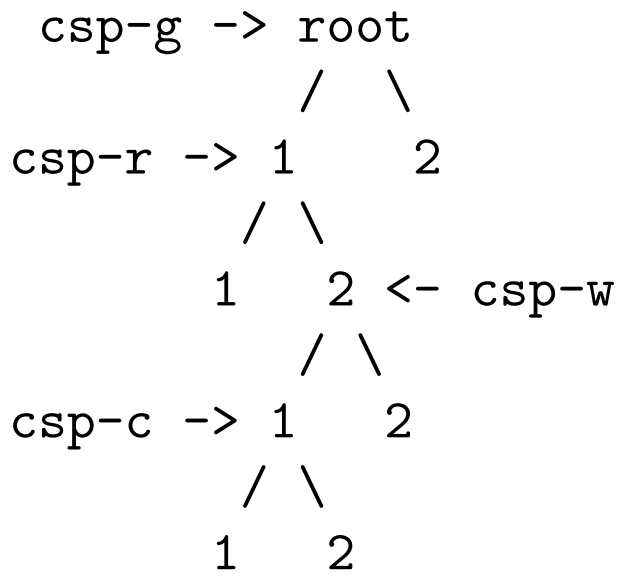
Proof Rule <RuleName>

$$\frac{\begin{array}{c} S \cup \{\texttt{eq <t1l> = <t1r> .}\} \vdash p, \\ S \cup \{\texttt{eq <t2l> = <t2r> .}\} \vdash p, \\ \cdots, \\ S \cup \{\texttt{eq <tml> = <tmr> .}\} \vdash p \end{array}}{S \vdash p}$$

Proof Score INITcheck

```
select INITcheck .
:goal{eq initCheck = true .}
:def csp-q = :csp{eq aq = empQ .
                  eq (aq =aq empQ) = false .}
:def csp-r = :csp{eq as-r = empS .
                  eq (as-r =as empS) = false .}
:def csp-w = :csp{eq as-w = empS .
                  eq (as-w =as empS) = false .}
:def csp-c = :csp{eq as-c = empS .
                  eq (as-c =as empS) = false .}
:apply(csp-q rd- csp-r rd- csp-w rd- csp-c rd-)
```

```
INITcheck(X.STATE)> :show proof
root*
[csp-q] 1*
[csp-r] 1-1*
[csp-r] 1-2*
[csp-w] 1-2-1*
[csp-c] 1-2-1-1*
[csp-c] 1-2-1-2*
[csp-w] 1-2-2*
[csp-q] 2*
```



Proof Score IINVcheck-ty

```
select IINVcheck-ty .
:goal{eq iinvCheck-ty = true .}
:def csp-c = :csp{eq as-c = empS .
                  eq as-c = (a-c-1 as-c-1) .}
:def csp-qc = :csp{eq a-q-1 = a-c-1 .
                   eq (a-q-1 =a a-c-1) = false .}
:apply(csp-c rd- csp-qc rd-)
```

```
IINVcheck-ty(X.STATE)> :show proof
root*
[csp-c] 1*
[csp-c] 2*
[csp-qc] 2-1*
[csp-qc] 2-2*

csp-c -> root
          /  \
         1    2 <- csp-gc
             / \
            1   2
```

Proof Score IINVcheck-exc

```
select IINVcheck-exc .
:goal{eq iinvCheck-exc = true .}
:def csp-qc = :csp{eq a-q-1 = a-c-1 .
                   eq (a-q-1 =a a-c-1) = false .}
:def csp-asc = :csp{eq as-c-1 = empS .
                    eq (as-c-1 =as empS) = false .}
:apply(csp-qc rd- csp-asc rd-)
```

```
IINVcheck-exc(X.STATE)> :show proof
root*
[csp-qc] 1*
[csp-asc] 1-1*
[csp-asc] 1-2*
[csp-qc] 2*

csp-gc ->   root
            /  \
csp-asc -> 1    2
          / \
         1   2
```