

# Overview of Coding Methods for Flash Memories

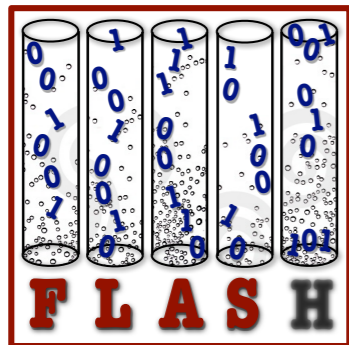
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フラッシュメモリ符号化に関するワークショップ

名古屋工業大学

<http://flashworkshop.org/>

**2010 April 3**

# Outline

- This talk is on coding for flash memories.
- Concentrate on codes for re-writing memories

1. Overview of flash memory: benefits and problems

2. Codes for rewriting memories

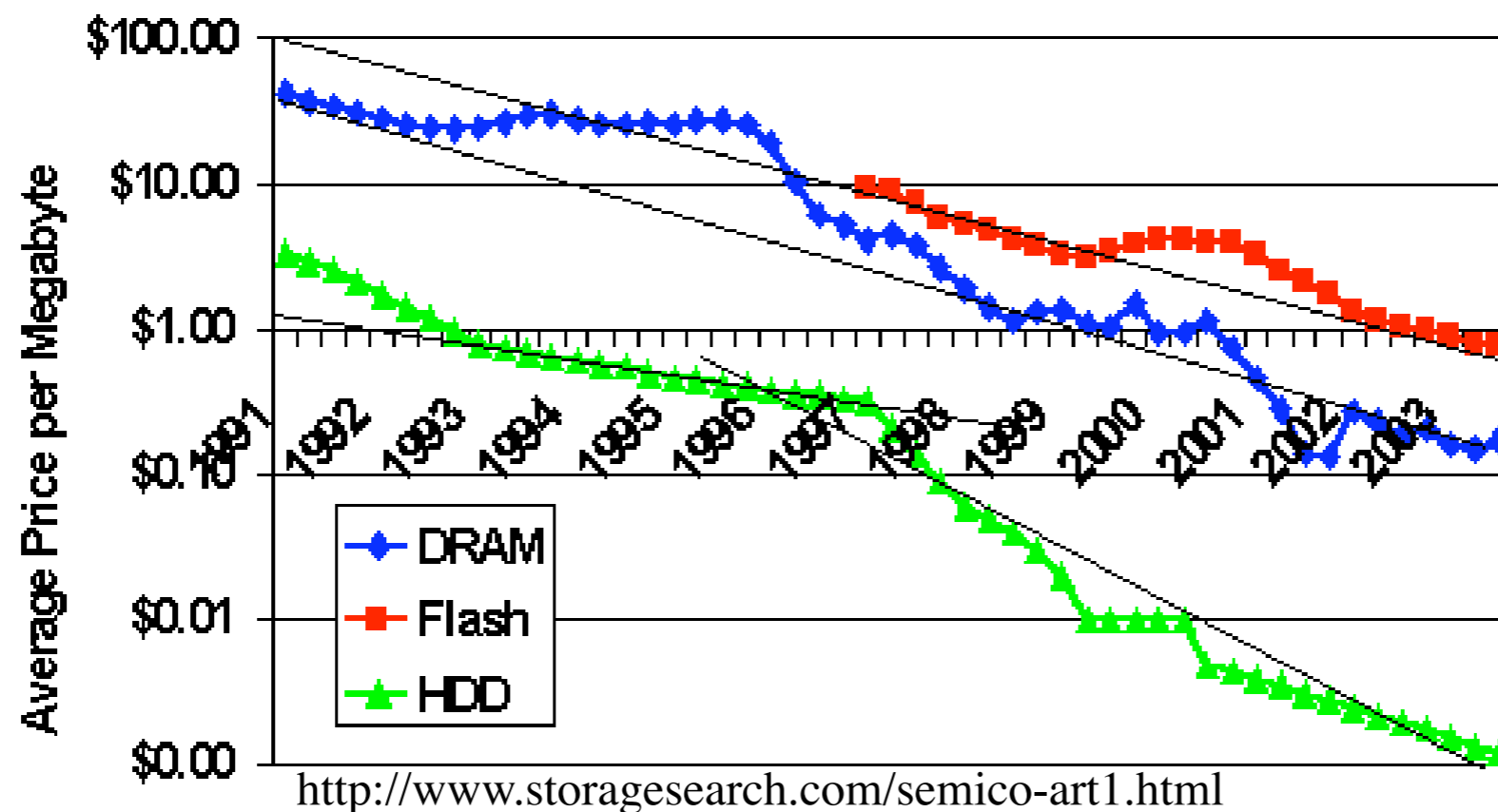
- Codes for binary  $q=2$  memories
- Codes for non-binary  $q>2$  memories

3. Traditional error-correction for flash

# Flash Memory

Flash is a semiconductor memory whose price has been rapidly decreasing. History:

- 1980 NOR Flash invented by Fujio Masuoka at Toshiba
- 1986 NAND Flash also invented by Masuoka
- 1988 Intel introduces commercial NOR Flash (PC BIOS, etc.)
- 1998 Early MP3 Player (32 MB, Korean SaeHan Information Systems)
- 2000 First USB Flash drive (8MB IBM)



# Flash vs. Hard Disks as a Storage Medium

Compared to hard disks, flash has advantages:

- Mechanically durable
- very fast random reading and writing (~100 times hard disks)
- fast sequential reads
- small memories are feasible (2 GB for ¥900)
- lower power

Flash disadvantages:

- performance decreases as SSD is used
- high cost per megabyte

But the cost of flash SSDs are almost reasonable



amazon.co.jp こんにちは、Brian Kurkoskiさん。おすすめ商品があります。本人でない場合は[こちら](#)。  
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すべてのカテゴリーを見る 検索 家電&カメラ

家電・カメラ Amazon ランキング メーカー・カテゴリー カメラ・デジタルカメラ テレビ・レコーダー

Intel Boxed X25-V Value SATA SSD 40GB  
SSDSA2MP040G2R5

インテリジェント (4件) この商品の詳細

¥ 11,440 通常 詳細

在庫あり

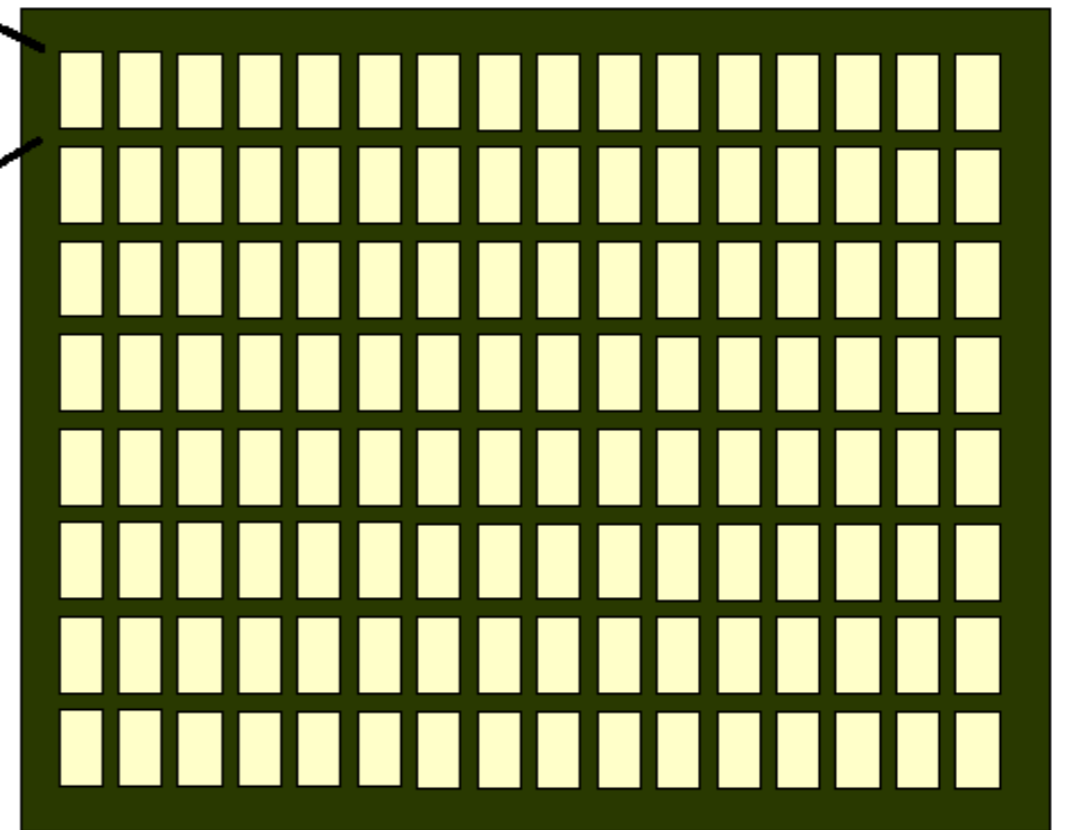
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2010/4/1 木曜日にお届けします！今から16時間と23分以内に「お急ぎ便」  
ション(有料)を選択して確定されたご注文が対象です。詳しくは[こちら](#)  
新品9点 ¥ 11,440より

# About Flash

- NOR Flash
  - Relatively low density
  - Small sizes blocks — “random” access is possible
  - Used to storing computer programs
- NAND Flash
  - Higher density
  - Has very large blocks
  - “Sequential” access
  - Widely used: cameras to SSDs

4KB  
Page

1 Block = 128 pages = 512KB



- NAND Flash is arranged:
  - one **page** consists of 512-4096 bytes
  - one **block** consists of 32-128 pages
  - one **plane** consists of 1024 blocks

<http://www.linux-mag.com/cache/7590/1.html>

# About Flash: Charge is easily added

- In flash memories, charge is stored on a “floating gate”, and read as a voltage.

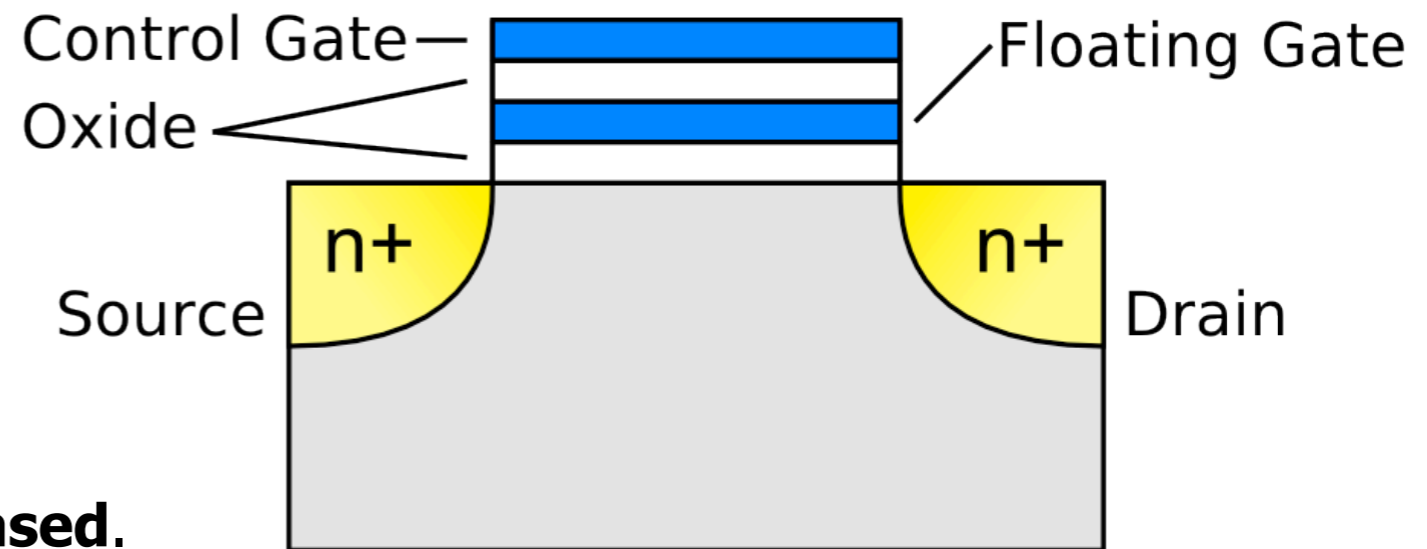
- Two very important things about flash:

1. Charge can easily be increased, but can only be decreased by an erasure operation. **Only**

**whole blocks of ~512 KB can be erased.**

2. Each block has a limited number of erase cycles it can handle. After 10,000 - 100,000 erasures, the block cannot be reliably be used. Also, erasures are slow.

- Thus, erasures should be avoided.



<http://elec424.rice.edu>

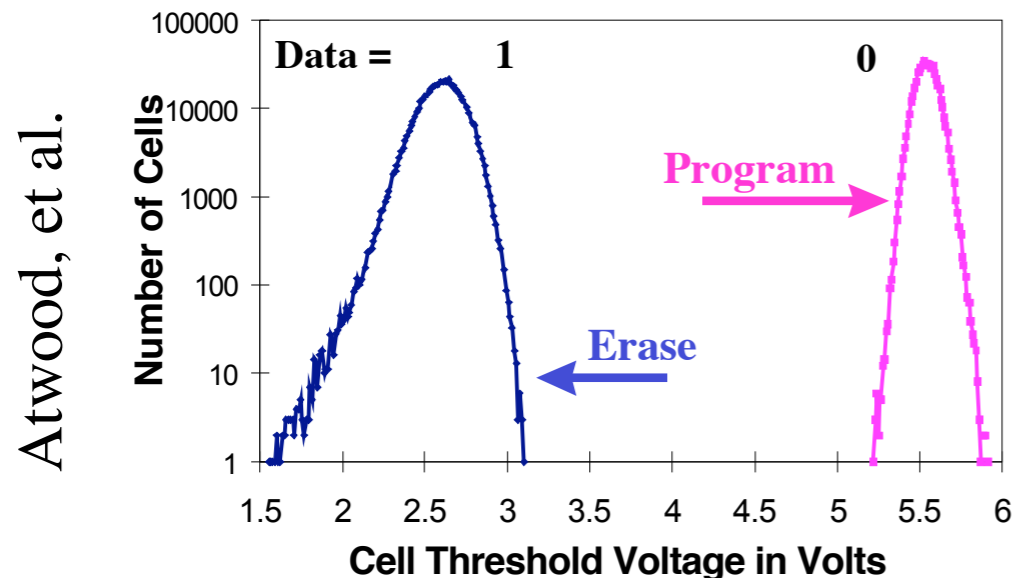
# About Flash: Single-Level and Multi-Level

- The transistor where charge is stored is called a “cell”
- SLC Single-Level cells store one bit
- MLC Multi-level cells store two or more bits
- Current flash chips use SLC and MLC with 4 levels (2 bits per cell)
- 8 levels (3 bits per cell) seem to be coming. Proposals as high as 256 levels.

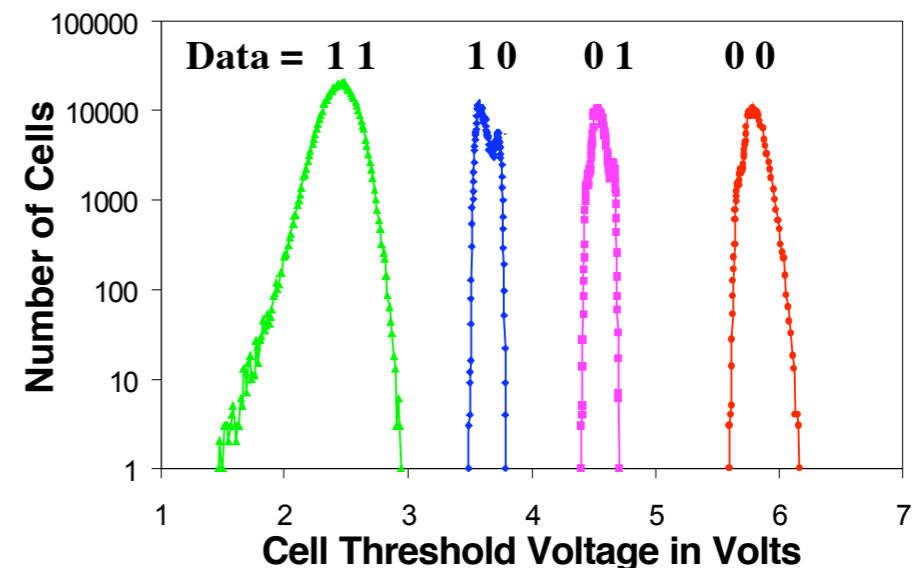
Voltage Easily Increases  
→

←  
Erasure is difficult

“Write Asymmetry”



**SLC**



**MLC (2 bits)**

# Current Approaches

Because erasures shorten the longevity (長寿命化) of flash memory:

- Current solution: flash translation layer (FTL) and wear leveling
- Computer science research: “log file systems,” garbage collection, etc.

## Big Question

“Can coding theory improve the longevity and performance of flash memories?”

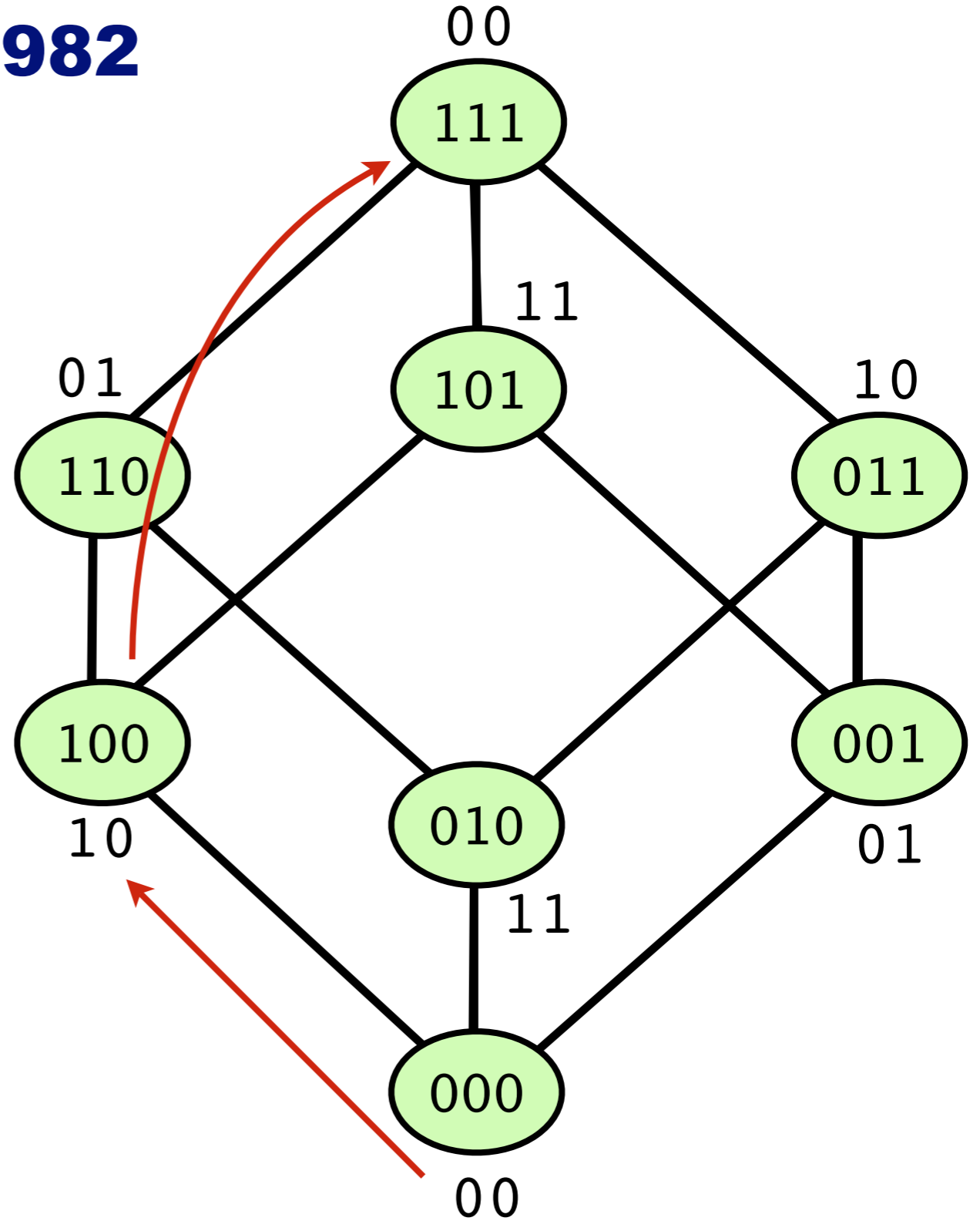
## A Few More Problems

- SLC -> MLC Errors are possible, ECC is needed
- Write-verify cycle: programming is imprecise, and must avoid overshoot
- Read disturb
- Write disturb

# Rivest and Shamir: “How to Reuse a ‘Write Once’ Memory” 1982

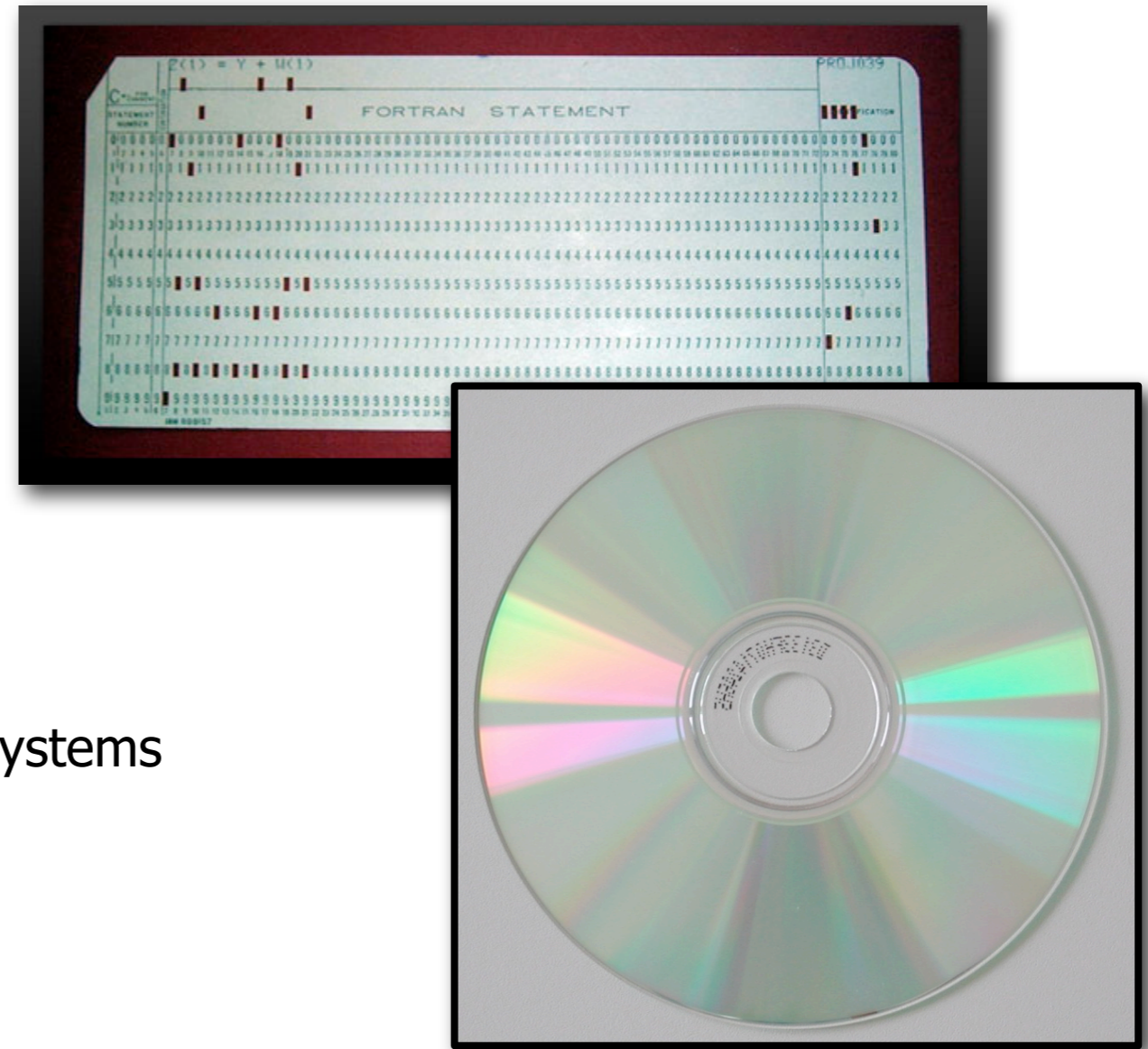
## Toy example:

- 3 storage “cells”
  - Initially (0,0,0) state
  - $0 \rightarrow 1$  is allowed
  - $1 \rightarrow 0$  is not allowed
- Store 2 bits of information.
- Can write data 2 times:
  - first write can be any two bits
  - second write can be any two bits
- Example:
  - store 01, then
  - store 00
- Code rate is  $2/3$
- Guaranteed minimum of two writes



# Literature Overview

- The problem of “asymmetrical writing” storage systems is not new!
  - write-once optical media: CD-R
  - PROM
  - punch cards
- Prior work mostly considered binary storage systems



Rivest and Shamir, Inf. and Contr

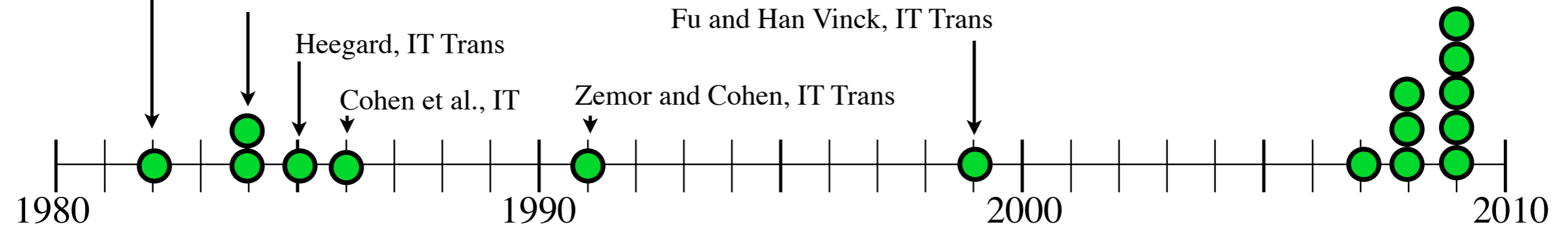
1. Wolf et al, Bell Labs TR
2. Fiat and Shamir, IT Trans

Heegard, IT Trans

Cohen et al., IT

Zemor and Cohen, IT Trans

Fu and Han Vinck, IT Trans



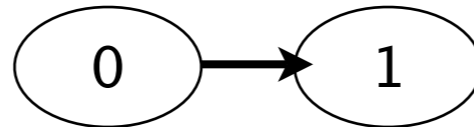
- This talk is about coding. However, there are also information theoretic results.

# Outline of Rewriting Codes

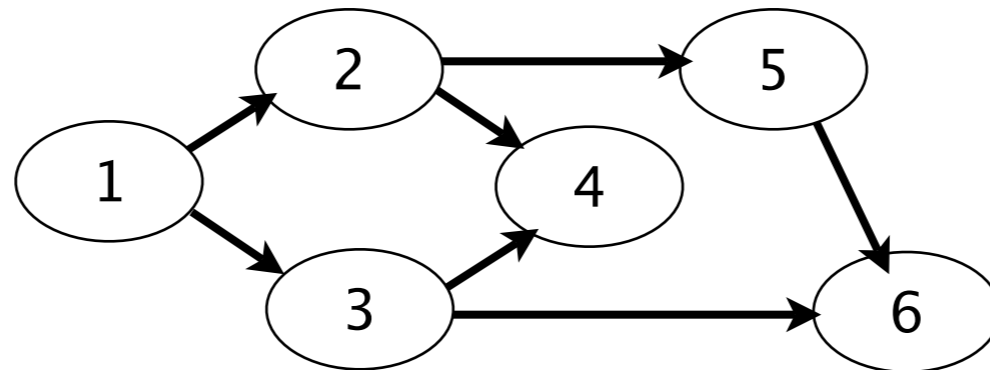
- Channel models, definitions and assumptions
- Codes for binary cells:
  - Rivest and Shamir bounds
  - linear code (based upon linear error-correcting codes) [Cohen et al, 1986]
- Codes for q-ary cells
  - JBB07 bounds
  - A low rate code

# Channel Models for One “Cell”

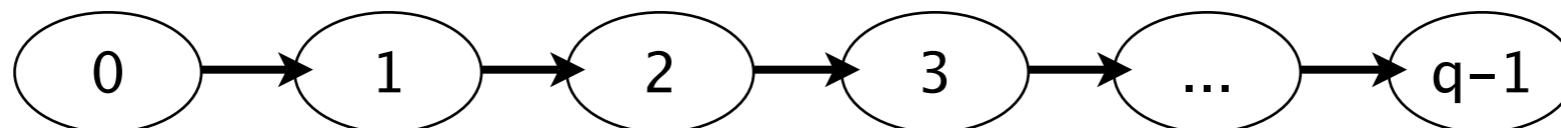
- “Write Once Memory” (Rivest and Shamir, 1982)



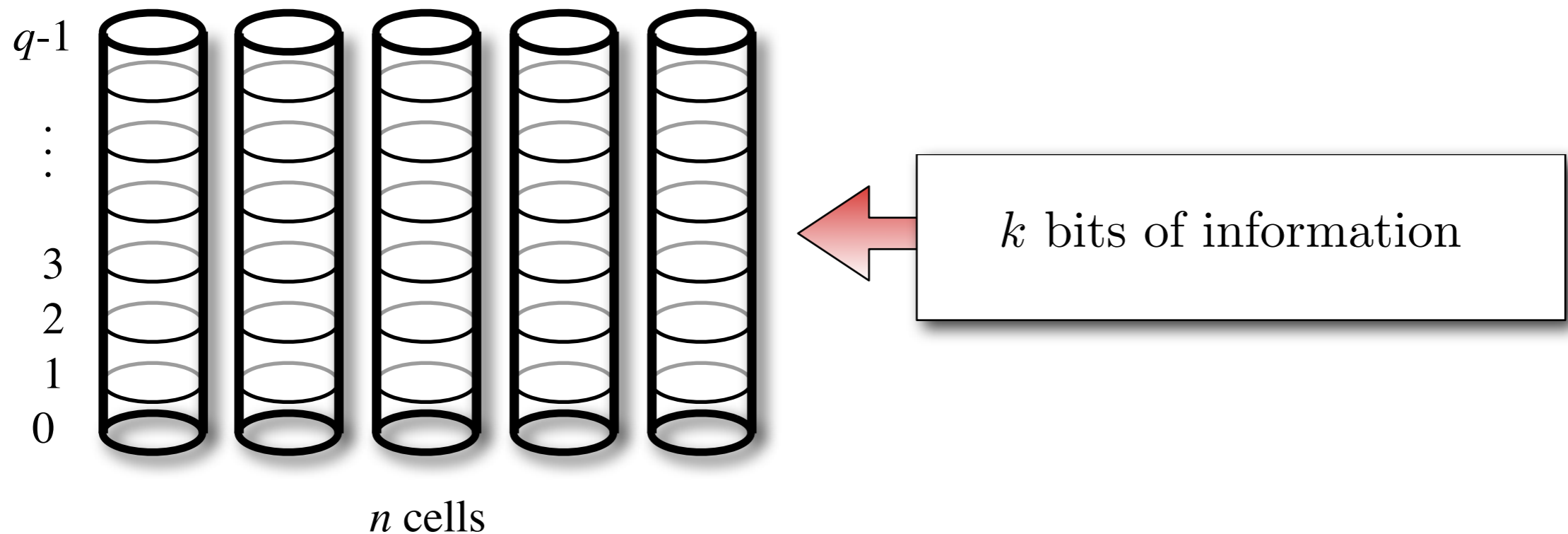
- Directed Acyclic Graph (DAG) Fiat and Shamir, 1984. Other capacity results.



- q-ary “Write Asymmetric Memory”, Jiang et al 2007



# Definitions and Assumptions



Floating code (or flash code)

- Encoding function:

$$\{\text{Information: One Variable}\} \times \{\text{Current Memory State}\} \longrightarrow \{\text{New Memory State}\} \cup \mathbf{E}$$

- Decoding function:

$$\{\text{Current Memory State}\} \longrightarrow \{k \text{ Information bits}\}$$

- Let  $t$  or  $T$  denote the *minimum* number of times information can be written, before erasure.

# Definitions and Assumptions: Point 1, Writes

There are *two* perspectives on the number of writes:

- Word Writing (Rivest-Shamir).

- $k$  bits are written simultaneously.
- A code allows *at least*  $T$  word writes, before the memory is “full”.

- Bit Writing (Jiang et al).

- Only 1 bit written at a time.
- A code allows *at least*  $t$  bit writes, before the memory is “full”.
- 1 word write performed by  $k$  bit writes,

$$T = \frac{t}{k}$$

**To make a fair comparison, choose *word writes*  $T$  as the metric.**

- Consistent with block-oriented nature of storage devices

1. More than  $T$  or  $t$  writes may be possible.
2. Full memories must be erased before next write.

# Definitions and Assumptions: Point 2, Rate

Fiat-Shamir (1984):

- for arbitrary DAG: NP Hard
- for a tree (including flash memory): polynomial time

Natural questions:

- For a given  $n, k, T, q$ , does a floating code exist? ←
- What is the relationship between  $n$  block length,  $k$  info bits,  $T$  writes and  $q$  cell levels?

Recent work has ignored the code rate (very low rates).

Define code rate as:

$$R = \frac{k}{n} \text{ bits per cell}$$

( $R > 1$  is possible, for example:  $q = 16 \Rightarrow R \leq 4$ )

Previous papers:

$$R = \frac{n}{k}, \quad R = \frac{kT}{n}$$
$$R = \frac{k}{n \log_2 q}$$

Will concentrate on this question:

- What is the relationship between  $T$  and  $R$ , for various  $n$ ?

# Codes for Binary Memory, $q=2$

## Asymptotic bounds

Rivest & Shamir use  $w(\langle 2^k \rangle^T)$  to mean “the length of an optimal code”.

For  $T = 2$ , the rate as  $k \rightarrow \infty$  is:

$$\text{“capacity” (or achievable rate)} = \lim_{k \rightarrow \infty} \frac{k}{w(\langle 2^k \rangle^2)} = 0.7729$$

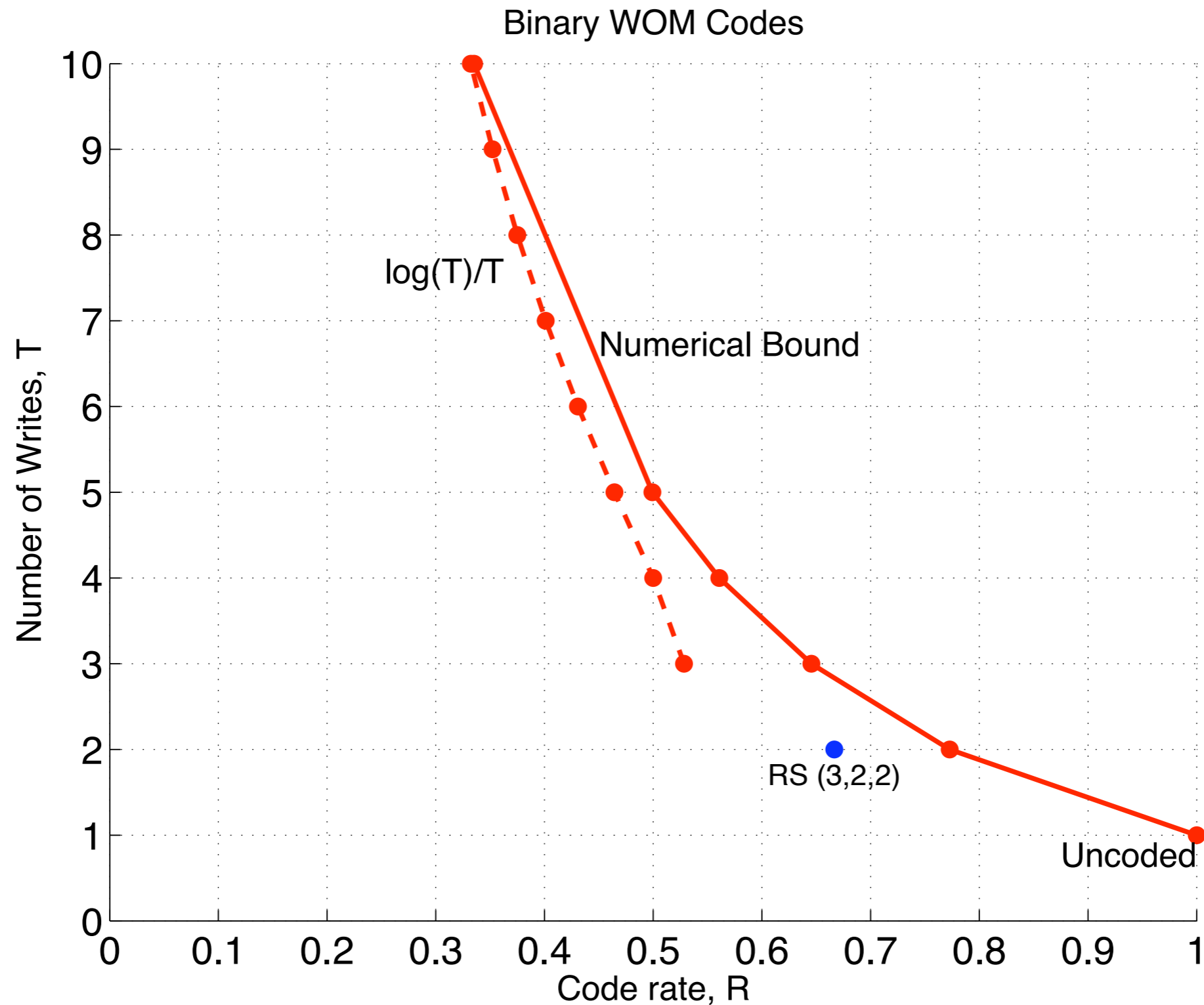
Interestingly, 0.7729 is the solution to the equation  $H(1 - p) = p$ .

For other values of  $T$ ,  
estimate are given

$T$	“Capacity” Estimate	$\log(T)/T$
3	0.6456	0.5283
4	0.5609	0.5
5	0.4993	0.4644
10	0.3352	0.3322
20	0.2142	0.2161
50	0.1116	0.1129
100	0.0658	0.0664
200	0.0380	0.0382

# Binary WOM Codes

## Code Rate vs. Number of Writes



# Simple Scheme to Write 1 Bit in n cells

As a “sanity check”, consider a simple scheme for encoding  $k = 1$  info bit.

Example  $n = 5$ . Stored sequence is:

$$\begin{array}{ccccccccc} (0, 0, 0, 0, 0) & \rightarrow & (1, 0, 0, 0, 0) & \rightarrow & (1, 1, 0, 0, 0) & \rightarrow & (1, 1, 1, 0, 0) & \rightarrow & (1, 1, 1, 1, 0) & \rightarrow & (1, 1, 1, 1, 1) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 0 & & 1 & & 0 & & 1 & & 0 & & 1 \end{array}$$

The information in each stage is the mod-2 sum of the stored sequence.

For any  $n \geq 1$ , this simple scheme has rate:

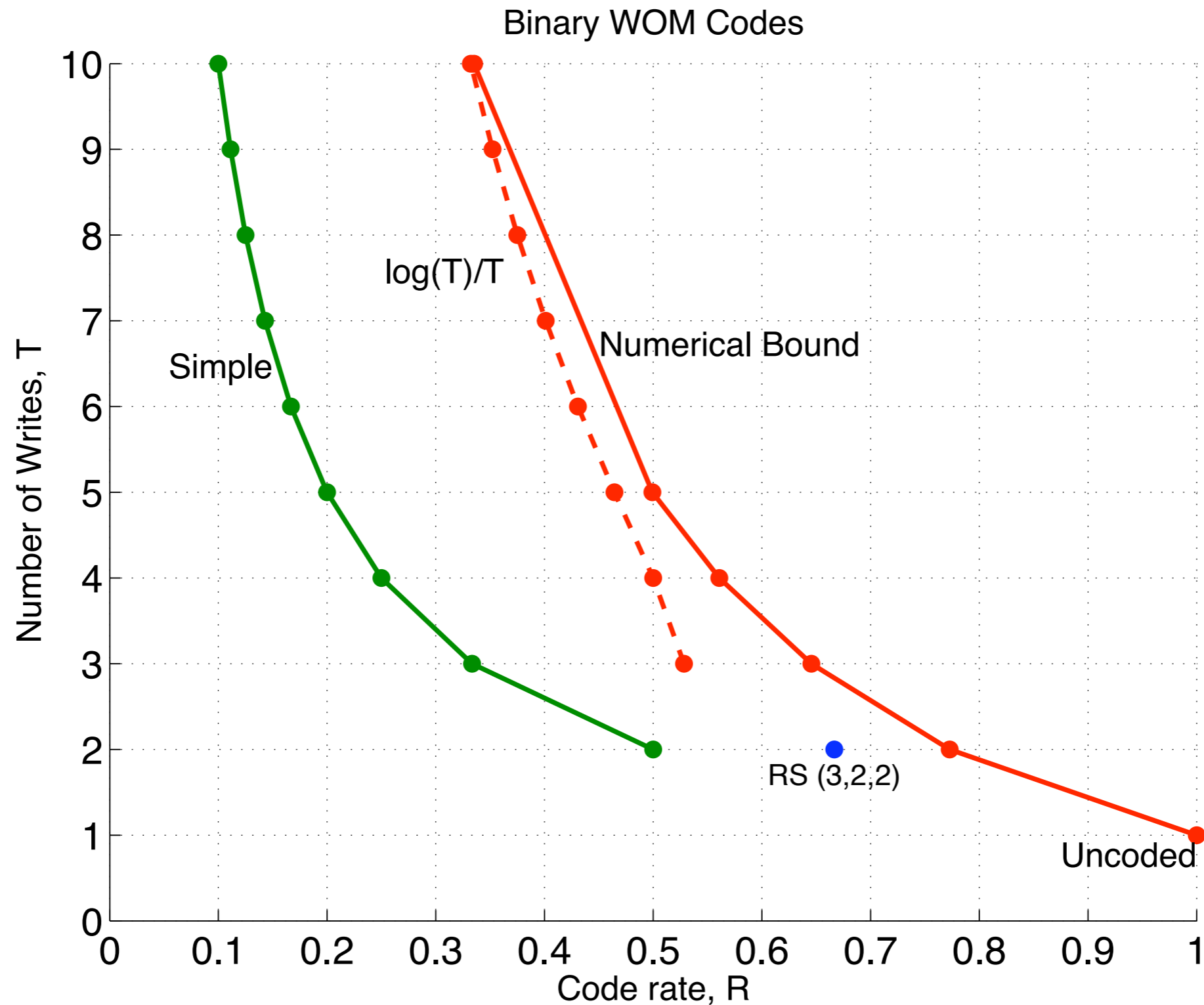
$$R = \frac{1}{T}$$

and allows for  $n$  writes:

$$T = n$$

# Binary WOM Codes

## Code Rate vs. Number of Writes



# A Binary Index-Type Scheme

The following codes was given by MahdaviFar, et al [MSVWY] at ISIT 2009 to illustrate a more complicated code.

Has poor rate, but explains an index-type scheme.

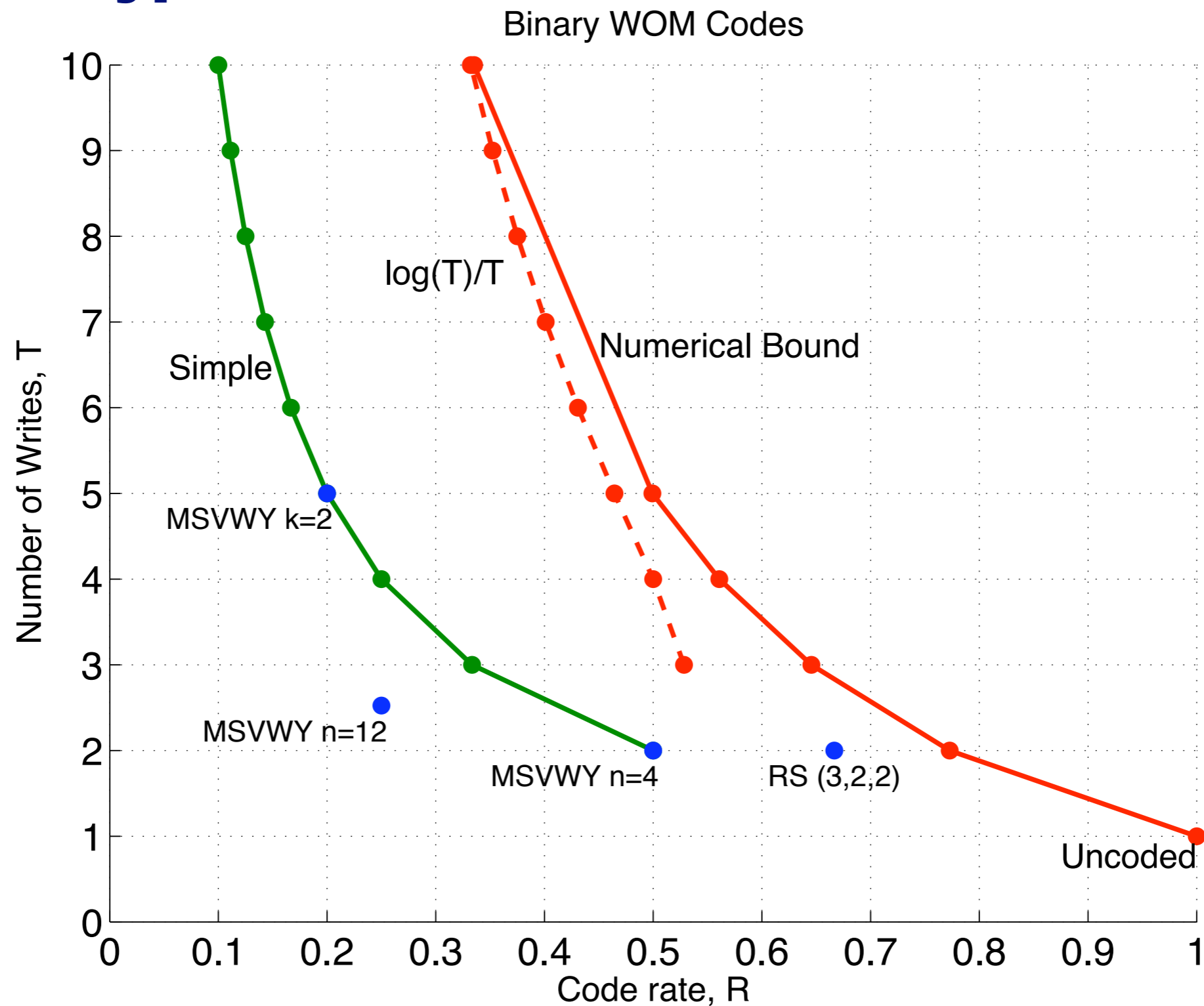
Encoding: partition  $n$  cells into blocks of size  $\log_2 k$ . When an information bit changes, record its index in the next available block.

Example:  $n = 12$ ,  $R = \frac{1}{4}$ ,  $k = 3$ . Results in  $T = \frac{1}{R \log_2 k}$  word writes.

index 1 2 3 ↓ ↓ ↓ info = 0 0 0	0	0	0	0	0	0	0	0	0	0	0	0
info = 0 1 0	1	0	0	0	0	0	0	0	0	0	0	0
info = 0 1 1	1	0	1	1	0	0	0	0	0	0	0	0
info = 0 0 1	1	0	1	1	1	0	0	0	0	0	0	0
info = 1 0 1	1	0	1	1	1	0	0	1	0	0	0	0

# Binary WOM Codes

## Index-Type Scheme



# A Linear Scheme

Cohen, Godlewski and Merks, “Linear Binary Code for Write-Once Memories,” IT Trans., 1986.

Use coset coding to encode information. Pick a linear code. Encoding:

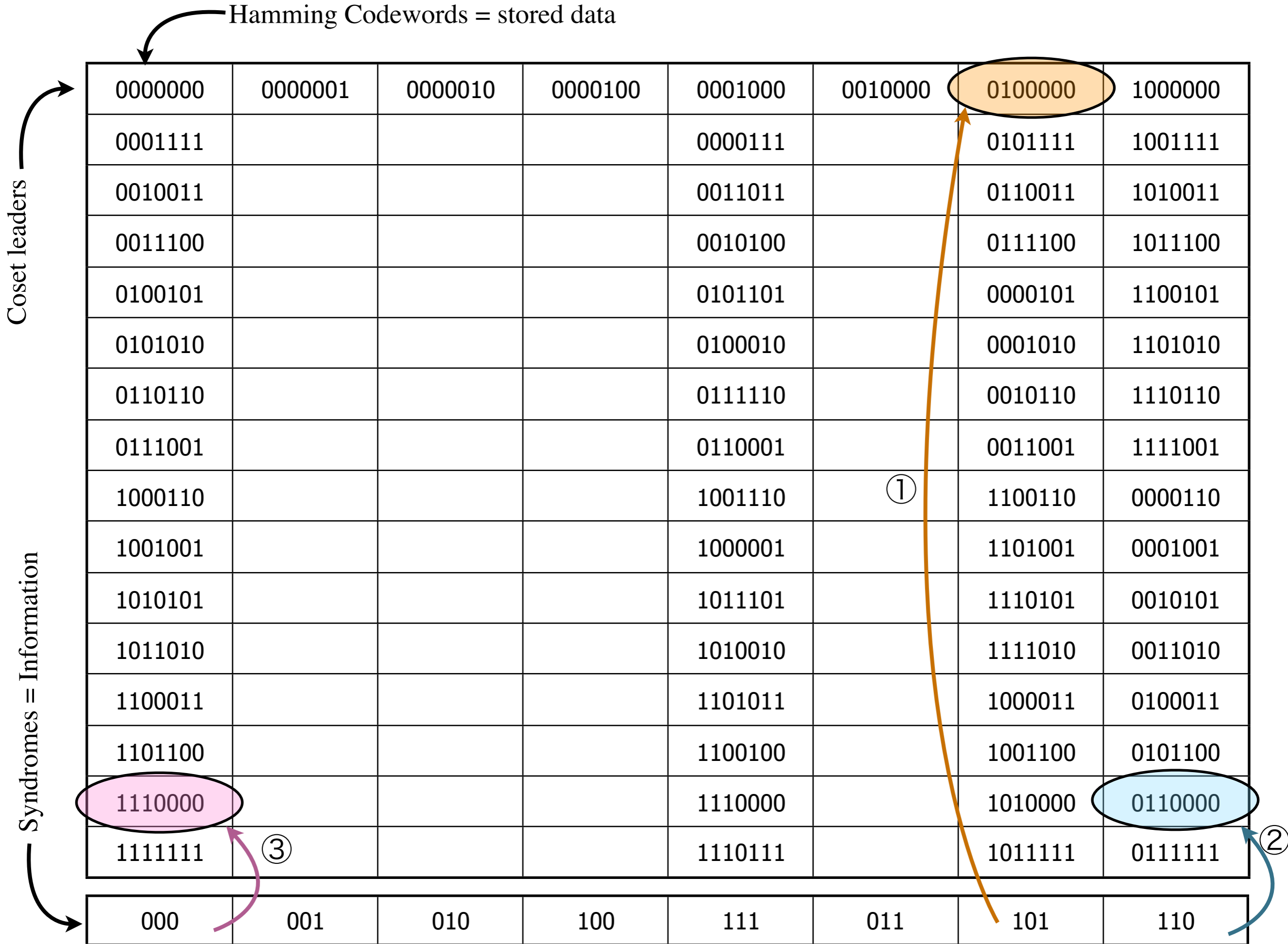
1. Information is encoded as the syndrome of a sequence
2. From the coset of that syndrome, select the coset codeword with the minimum weight.
3. Write that coset codeword to memory.

Decoding:

1. Compute the syndrome of the recorded sequence.

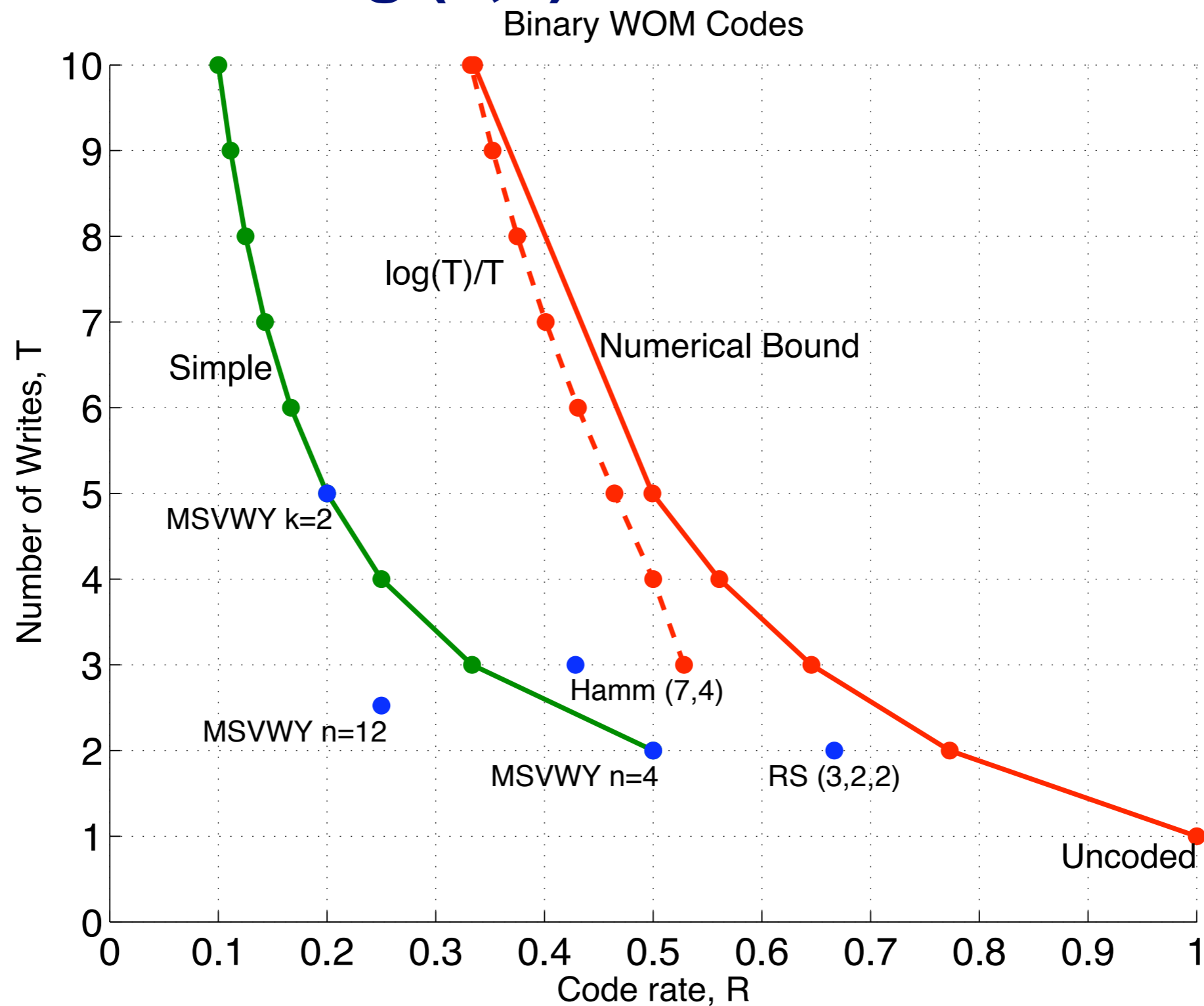
Example: Use Hamming (7,4) code to encode information with  $T = 3$  writes:

$$\begin{array}{ccccccc} (0, 0, 0) & \rightarrow & (1, 0, 1) & \rightarrow & (1, 1, 0) & \rightarrow & (0, 0, 0) \\ & & \textcircled{1} & & \textcircled{2} & & \textcircled{3} \end{array}$$



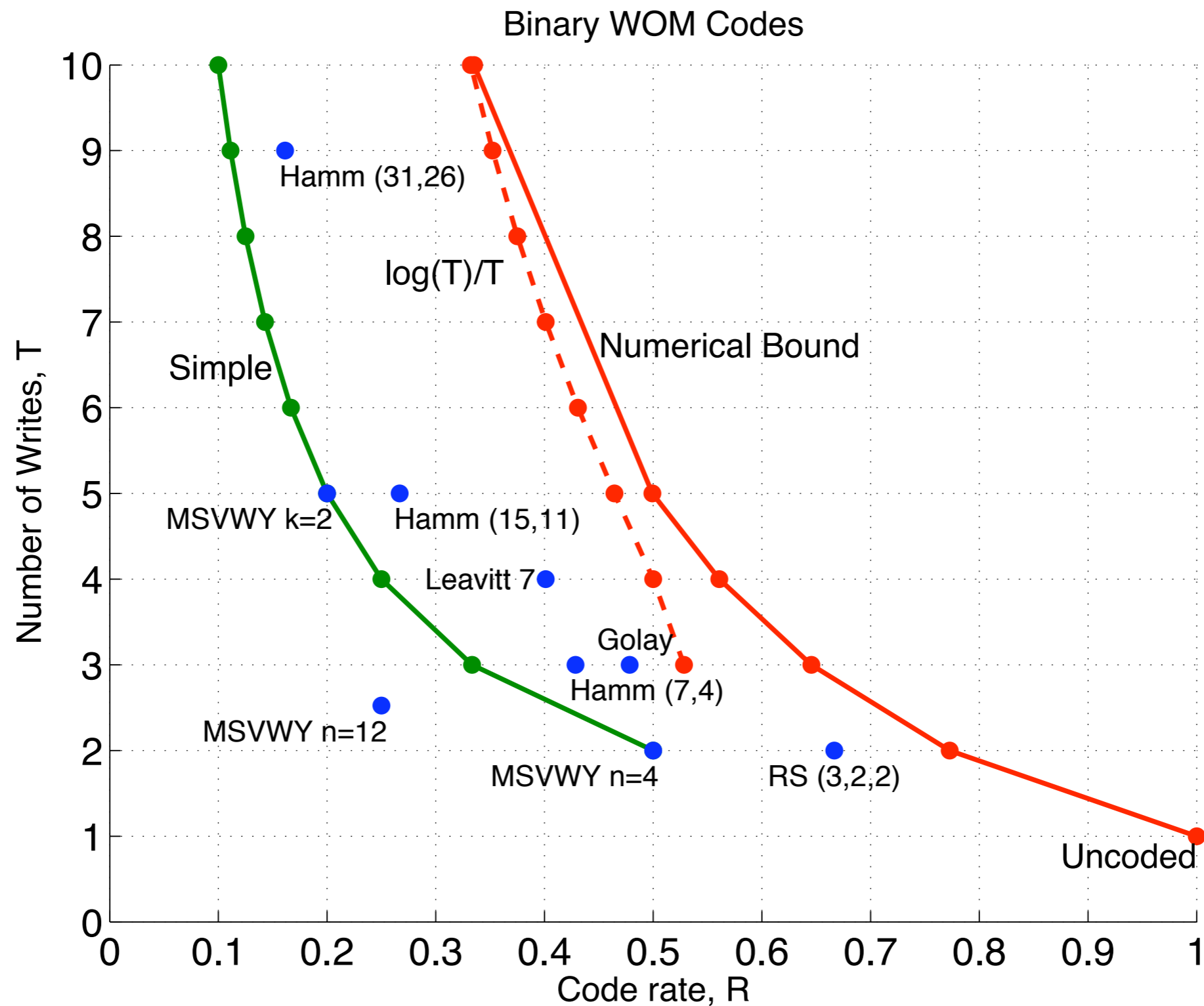
# Binary WOM Codes

## Linear Hamming (7,4) Code



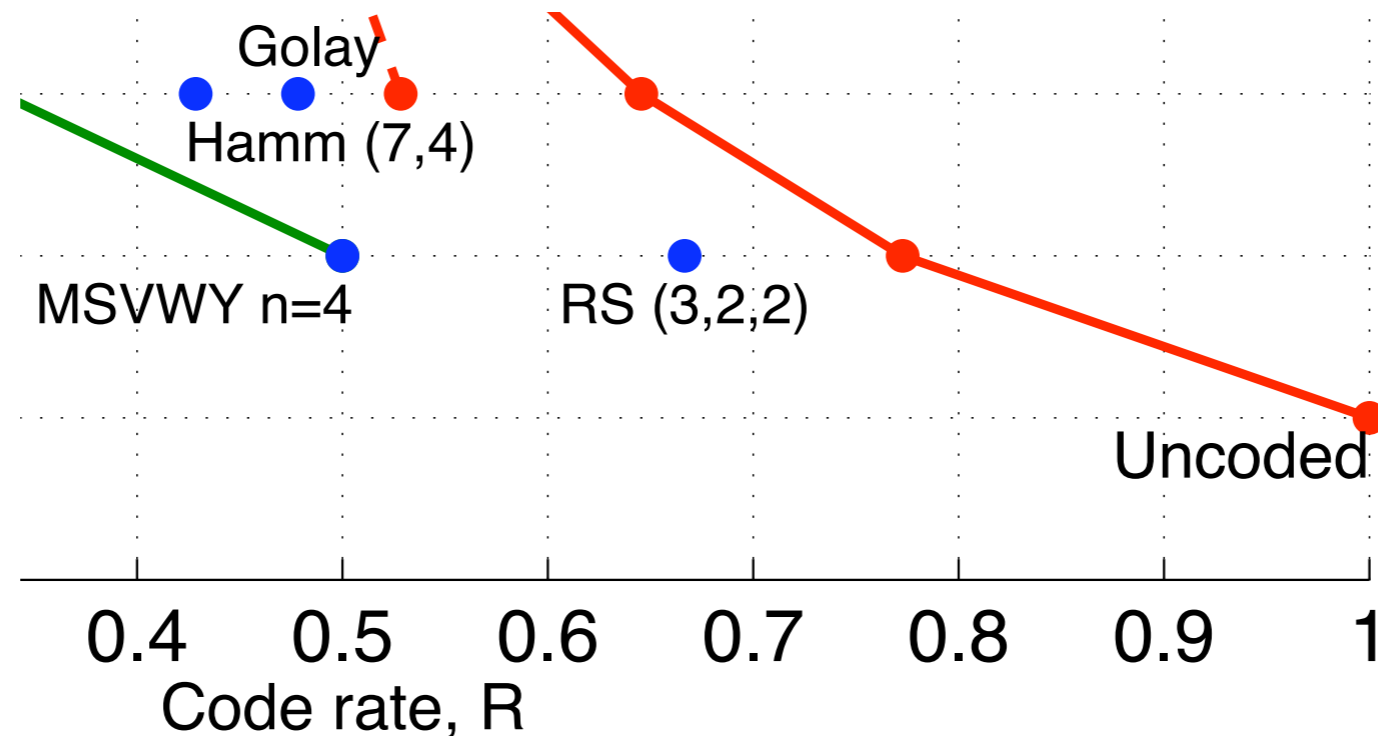
# Binary WOM Codes

## More Linear Codes



# Summary of Binary Codes

- Simple or “naive” coding:  $R = \frac{1}{T}$ .
- Rivest and Shamir showed that  $R = \frac{\log T}{T}$  is possible.
- Clearly, there is a tradeoff in number of writes and rate. But Rivest and Shamir showed you can do better than naive.
- For  $T = 2$ , the “toy example”  $n=3, k=2$  code has rate  $2/3$ .
- Optimal rate at  $T = 2$  is 0.77. This is fairly low rate. Practical?



# Codes for Multilevel Flash: $q > 2$

By increasing  $q$ , can we get better codes?

This is recent work, since 2007.

Trivial upper bound:

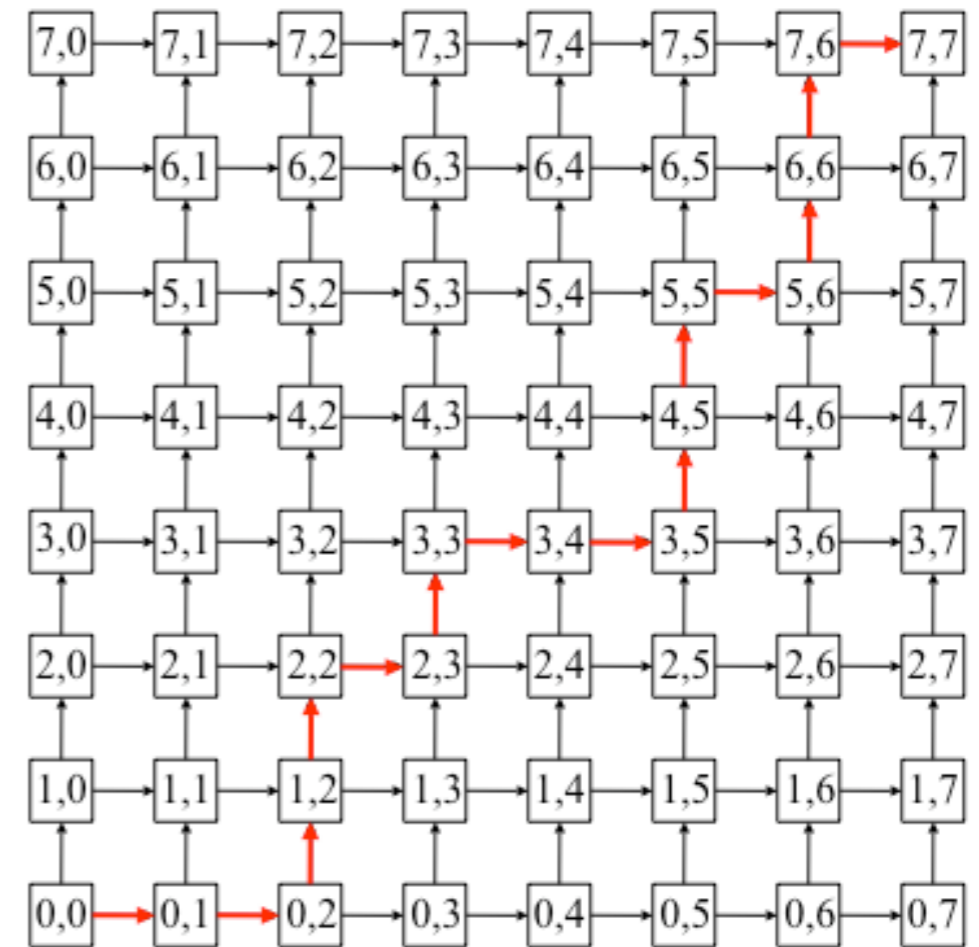
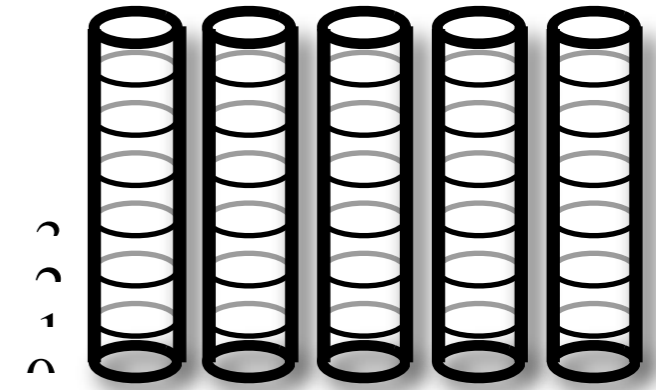
$$t \leq n(q - 1) \quad (\text{bit writes})$$

$$T \leq \frac{(q - 1)}{R} \quad (\text{word writes})$$

Tighter upper bound (approximate) Jiang, Bohossian, Bruck, ISIT 2007 (JBB07):

$$t \leq n(q - 1) - \frac{1}{2}(q - 1) \min(n, k - 1)$$

$$T \leq \frac{(q - 1)}{R} - \frac{1}{2}(q - 1) \min\left(\frac{1}{R}, 1\right)$$

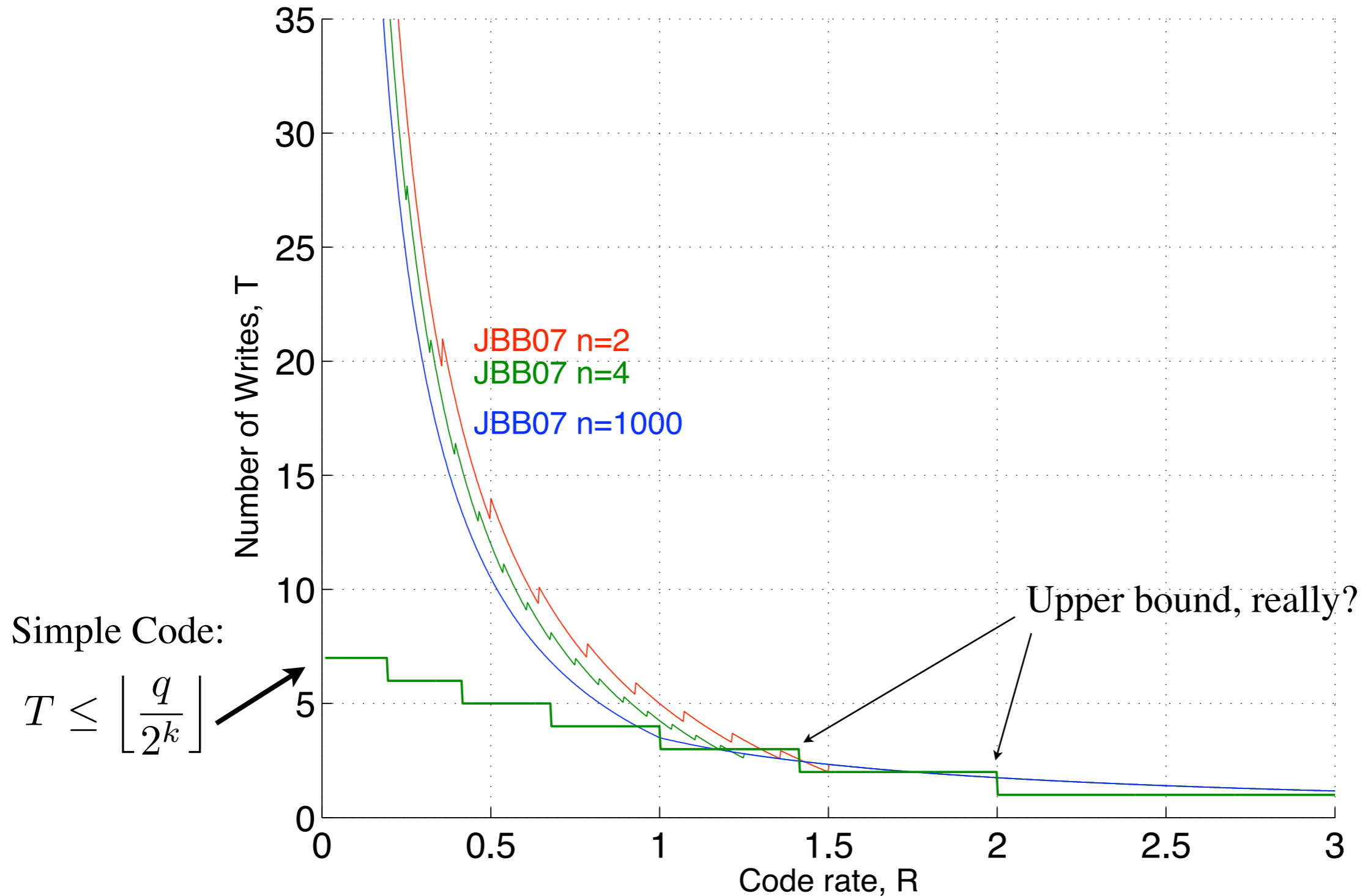


Trivial bound:  $n=2, q=8$

Image: Eitan Yaakobi

# Rewriting Codes for $q=8$ : Bounds

$q=8$



# Codes for $q > 2$

- Jiang, Bohossian and Bruck [ISIT 2007] also proposed a re-writing code for  $k=2$  bits
  - It complicated and hard to understand.
  - It is a low rate code
  - It achieves:

$$t = (n - 1)(q - 1) + \left\lfloor \frac{q - 1}{2} \right\rfloor$$

- Yaakobi, Vardy, Siegel and Wolf [Allerton 2008] proposed “multidimensional codes”
  - Achieves the same re-writing rate.
  - Easier to understand the construction

- Jiang, et al. [ISIT 2009] “Trajectory Code” :

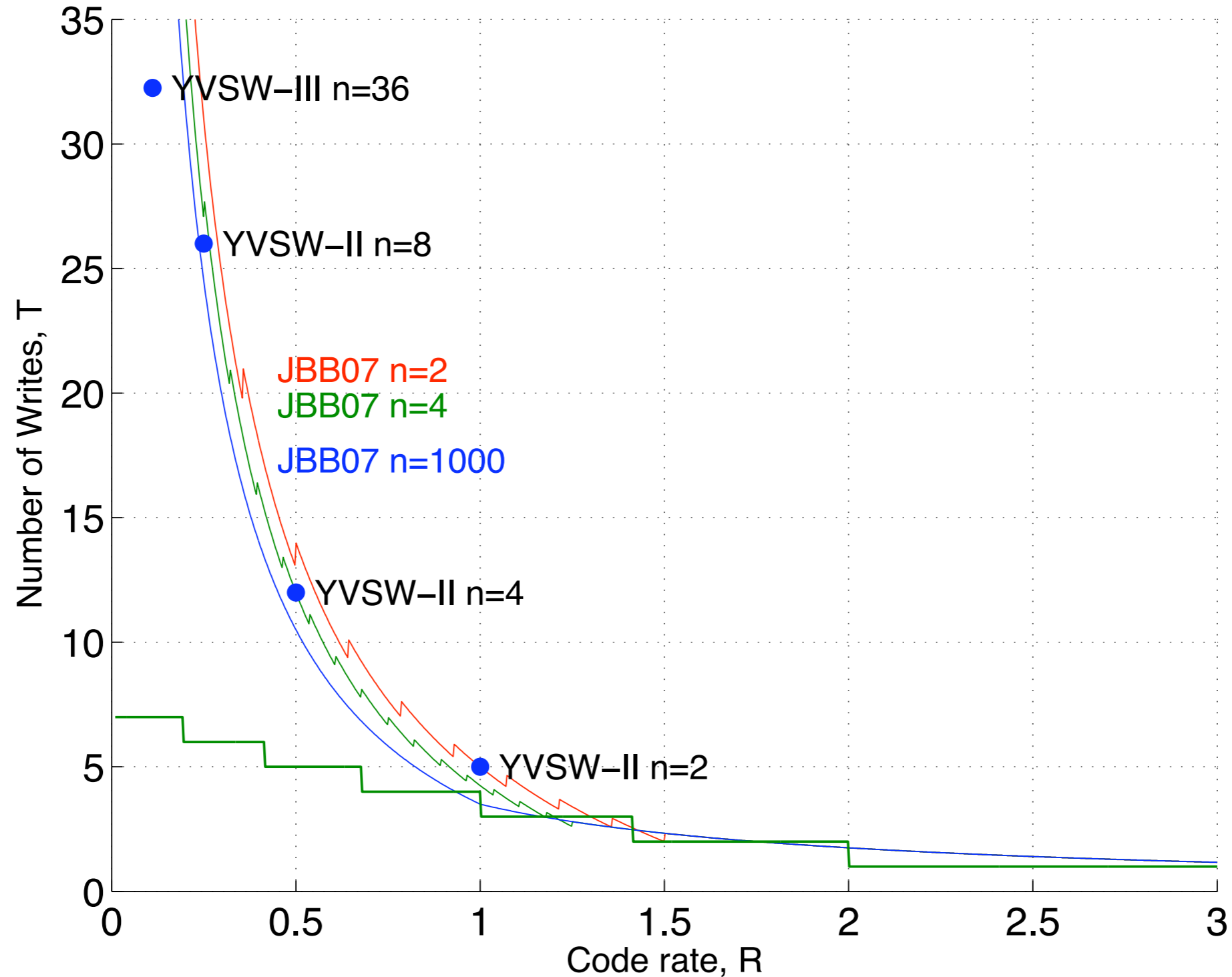
- MahdaviFar, et al. [ISIT 2009]

$$2^k \leq 2^{\sqrt{n}} \Rightarrow R \leq \frac{1}{k}$$

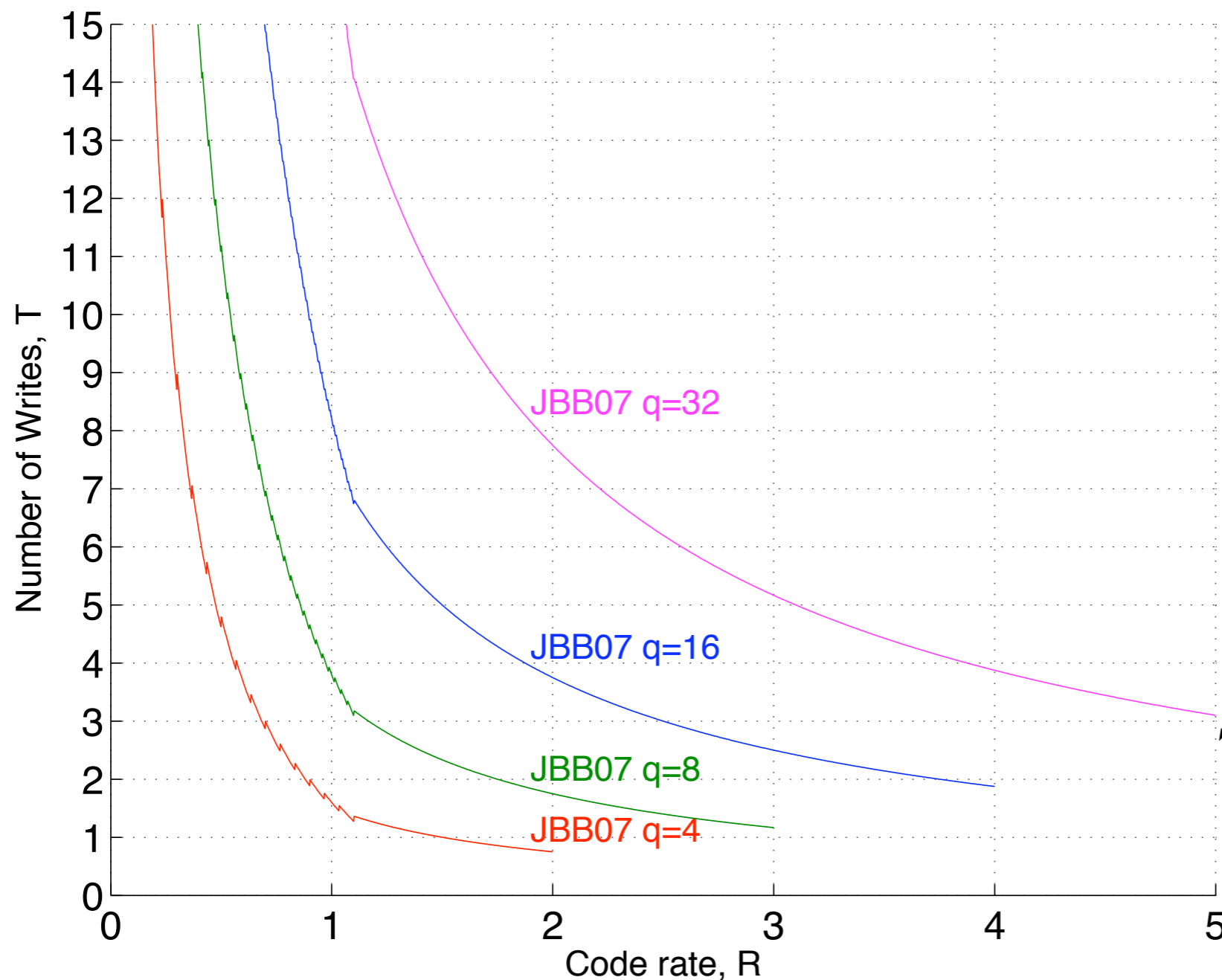
- Most constructions appear to be low rate!

# Rewriting Codes for $q=8$ : YVSW Code

$q=8$



# Number of Writes increases in $q$ !



DAG is directed acyclic graph, the memory model.

“The significant improvement in memory capability is linear with the DAG depth. For a fixed number of states a ‘deep and narrow’ DAG cell is always preferable to a ‘shallow and wide’ DAG cell.”

-Fiat and Shamir, 1984

**Tight bound, really?**

# Summary of $q > 2$ Codes and Open Problems

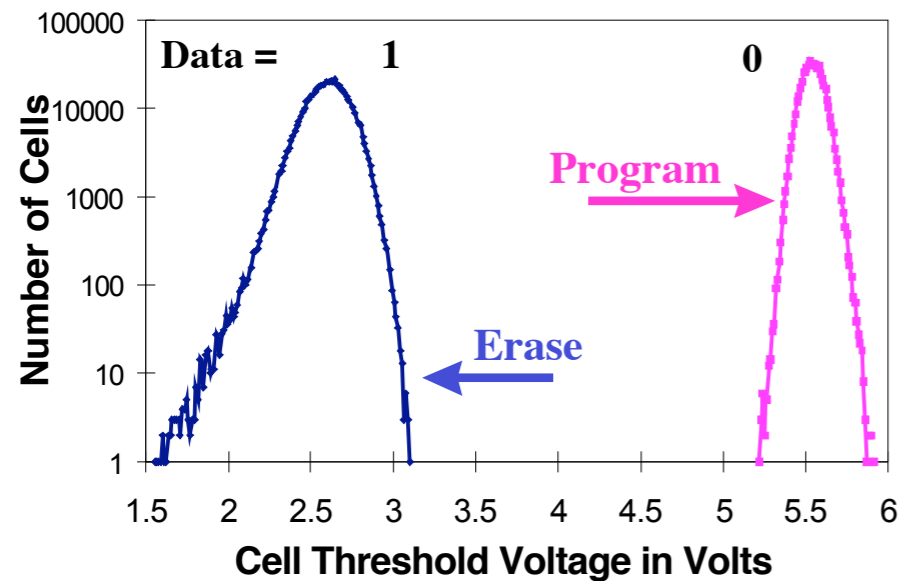
- In traditional coding theory,  $d_{\min}$  increases for increasing block length
  - But for rewriting codes, does  $T$  increase for increasing block length? (no?)
  - However, seems like  $T$  does increase for increasing levels  $q$
- High rate coding:
  - system designers use high rate codes, but there are few/no high rate codes
  - perhaps I'm too excited about high rate codes
  - Tighter bounds at high rate?
- Average vs. Minimum number of writes
  - $t$  and  $T$  was defined as the minimum number of writes
  - Average number of writes is always greater
  - Does average number of writes have better properties (improves with block length)?
- I did not mention other rewriting codes developed by Jiang, et al:
  - Buffer coding
  - Rank modulation

# Error-Correction for Flash Memories

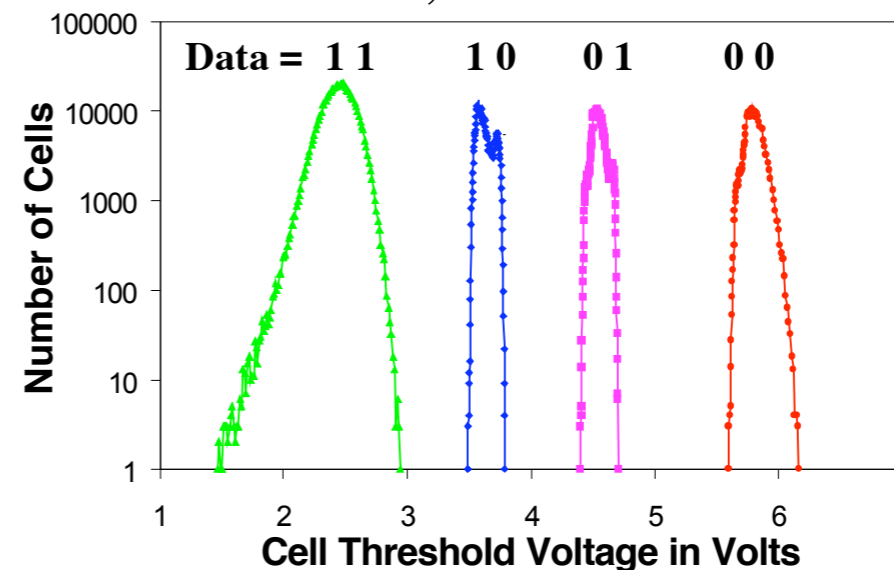
- Flash memories, particularly NAND flash are noisy.

Use Gray mapping

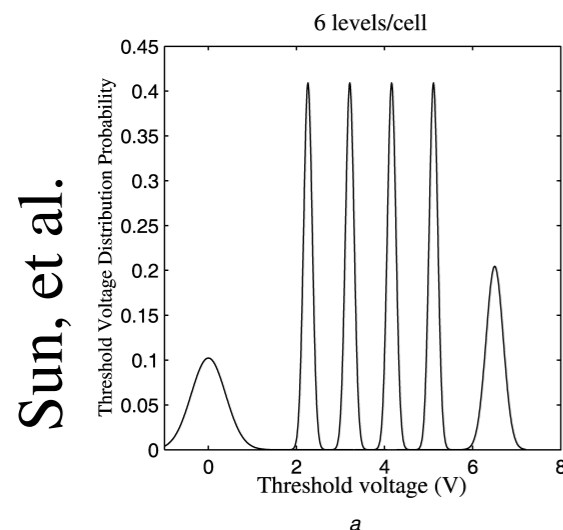
Atwood, et al.



**SLC**



**MLC (2 bits)**



Model:

- level 0:  $\sim N(0, 4\sigma^2)$
- level 1 to  $q - 2$  :  $\sim N(0, \sigma^2)$
- level  $q - 1$  :  $\sim N(0, 2\sigma^2)$

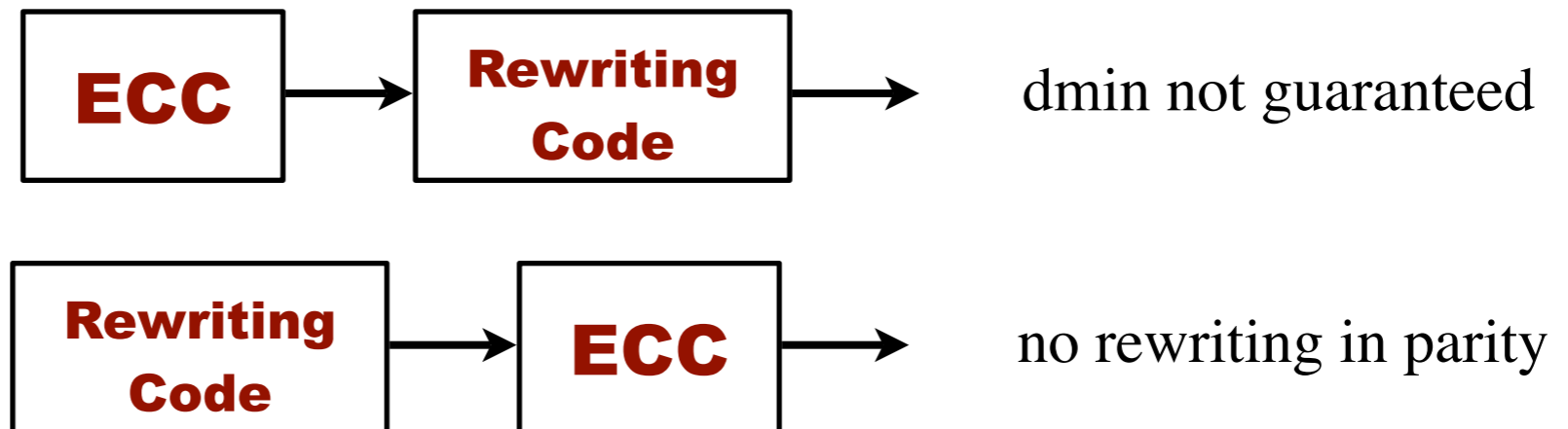
# Error-Correction for Flash Memories

- Most MLC flash uses error correction
  - Early chips: proposal to use Hamming codes to correct single bit errors
- MLC errors appear random:
  - Reed-Solomon codes correct burst errors well
  - Reed-Solomon codes, widely used in hard drives, DVDs, CD, etc, are not needed
  - However, Reed-Solomon has more efficient decoder [Chen et al., 2008]
- BCH codes can correct random errors well ( $R > 0.98$ )
  - Liu, Rho and Sung (2006): BCH (4148,4096) to correct 4 bit errors with 52 parity bits
  - Micheloni, et al. (2006): VLSI using BCH (32767,32692) to correct 5 errors
- LDPC Codes
  - Maeda and Kaneko (2009): Use non-binary LDPC codes of field size  $q$ 
    - $q=8, 16$ .  $R=1/2, 5/8$ . Found slight improvement in BER by using average column weight of 2.5

# More Open Problems

- Rewriting codes plus ECC

- Only a few papers on this topic. But, a serious problem (think RLL in hard drives)



- Intersymbol interference (ISI)

- Errors often appear independent , so BCH codes are used
- However, densities increase → errors become correlated, ISI occurs
- Need ISI models!

- Asymmetric Noise

- read disturb and retention problem: charge leaks from the cell → voltage decrease
- Errors are asymmetric

# Conclusion

- Flash memories are rapidly increasing in density, and should become widespread in the future.
- Flash memories have a limited number of write cycles. Avoid erasures by using coding
  - Binary codes are suitable for SLC, but SLC is being replaced by MLC
  - There appear to be few codes of sufficiently high rate for MLC
- Flash memories also have errors like a traditional communication system
  - Hamming codes, BCH codes, Reed-Solomon, LDPC codes appear to be effective