Notes on Indexing Cosets of Lattice Codes

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Let $\Lambda_{\rm c}$ and $\Lambda_{\rm s}$ be two lattices in the n-dimensional Euclidean space. If $\Lambda_{\rm s} \subseteq \Lambda_{\rm c}$, then $\Lambda_{\rm c}/\Lambda_{\rm s}$ is a quotient group over the set of cosets. If the coset leaders $\mathcal{C}$ are those set elements inside the zero-centered Voronoi region of $\Lambda_{\rm s}$, then $\mathcal{C}$ is a finite lattice code with a sphere-like distribution, which is highly suitable for coding on the AWGN channel. In addition, the group property makes the code suitable for physical layer network coding, compute-and-forward relaying and so forth.

Nested lattice codes are a common choice for constructing $\mathcal{C}$. If $\Lambda_{\rm s} = M\Lambda_{\rm c}$ where $M \geqslant 2$, then all of the above properties are obtained. In addition, the indexing operation is simple — indexing means mapping the information (a finite subset of $\mathbb{Z}^n$) to $\mathcal{C}$. However, $\Lambda_{\rm s} = M\Lambda_{\rm c}$ may not always be a good choice. For example, $\Lambda_{\rm c}$ may not simultaneously be good for coding and shaping, or there may be complexity constraints associated with performing the shaping operation. If $\Lambda_{\rm s}$ is not a scaled version of $\Lambda_{\rm c}$, but $\Lambda_{\rm s} \subseteq \Lambda_{\rm c}$ still holds, then code $\mathcal{C}$ exists, but the indexing operation is not necessarily simple.

The subject of this talk is an indexing scheme for the cosets of $\Lambda_{\rm c}/\Lambda_{\rm s}$. Given a generator matrix for $\Lambda_{\rm s}$ and a check matrix for $\Lambda_{\rm c}$, a condition is given on the existence of a simple indexing scheme. This serves as a guide for practical schemes, where for example, the shaping lattice $\Lambda_{\rm s}$ may be fixed as a well-known lattice with a low-complexity shaping algorithm (for example E_8), and the coding lattice $\Lambda_{\rm c}$ has some freedom in the design parameters (for example, the non-zero coefficients for a low-density lattice code).

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