# Write-Once Memory Codes for Low-Complexity Decoding of Asymmetric Multiple Access Channel

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***Abstract*—Write-Once Memory (WOM) codes are designed for data storage, which allow re-writing on $n$ cells that can change their bit value from 0 to 1 but not vice versa. This paper focuses on applying "WOM codes" to cooperative wireless communications. Due to the characteristics of WOM codes, the Asymmetric Multiple Access Channel (AMAC) (also referred to as the MAC with degraded messages) is considered, which is a conventional Multiple Access Channel (MAC), where one user can observe the other user's message. We describe an AMAC system where WOM codes are used to deal with the interference between two users. For a specific AMAC model, WOM codes can achieve the AMAC capacity, and using WOM codes for the AMAC with no errors leads to a low-complexity decoder. Finally, we comment of how the AMAC model can be applied to the relay channel. While we consider primarily the AMAC with no errors, this study forms a foundation for future work on the AMAC with errors.**

## I. Introduction

The motivation of this work is reduction complexity in wireless communications that it is possible using coding techniques.

A Multiple Access Channel (MAC) is a system which has two separated users where each one simultaneously communicates data using a common channel [1]. The Asymmetric Multiple Access Channel (AMAC) with two encoders is a MAC where one of the users has access to the other user's message, and to its own private message [2]. In this scheme, one user can encode both messages whereas the other user encodes only its own message.

In addition to the AMAC topic, codes for Write-Once Memory (WOM) (e.g. flash memory) is also important for the understanding of the system proposed in this work. A WOM consists of a number of write-once bit locations (wits). Each wit initially represents a bit value of 0 that can be irreversibly overwritten with a bit value 1. Observing the state of the WOM medium, it is possible to reuse it using WOM codes by introducing redundancy into the recoded bit sequence, and to determine how to update the content of the memory with a new bit sequence [3].

Rivest and Shamir demonstrated how to reuse such a write-once memory multiple times using coding techniques [4]. A coding scheme which can write a variable of cardinality $v_1$ for the first write, a variable of cardinality $v_2$ for the second write and so on, to the $n$ binary cells defined, a total of $t$ times writting, which Rivest and Shamir called $\langle v \rangle^t/n$-WOM codes [4], where $v = v_1, \dots, v_t$.

WOM codes were designed for data storage applications, however, they have never previously been used for wireless communications. This paper is inspired in part by the application of WOM codes to reducing read latency in memories [5]. Hence, we focus on and show how to apply existing WOM codes to the AMAC (and, thereby, relay channel with a noiseless link between the sender and the relay).

The major advantage of applying WOM codes to the AMAC is reducing the complexity of decoding. Numerous schemes for joint iterative decoding of the two transmitted codewords at the receiver (e.g. LDPC codes for the MAC [6]) have been proposed. The proposed coding scheme aims to efficiently separate the two codewords at the receiver, without using successive interference cancellation. This low-complexity approach is appealing for power-constrained applications.

## II. System description

### A. 2-user AMAC

This section describes the AMAC. The key differences between the MAC and the AMAC are: (1) User 2 knows the message of User 1 and (2) for the capacity, the input distribution is optimized over $p(x_2|x_1)p(x_1)$ rather than $p(x_1) \cdot p(x_2)$, where $p(x_1), p(x_2)$ are the probability distribution for User 1's and User 2's codewords, respectively.

Some notations are introduced; $U^1$ and $U^2$ denote the message sequence sent by User 1 where $U^1 \in \{1, 2, 3, ...2^{nR_1}\}$ and User 2 where $U^2 \in \{1, 2, 3, ...2^{nR_2}\}$. User 1 sends codewords from a codebook $\mathcal{C}_1$ and User 2 sends codewords from a codebook $\mathcal{C}_2$; $X_1^1, X_2^1, ..., X_n^1 \in \mathcal{C}_1$ and $X_1^2, X_2^2, ..., X_n^2 \in \mathcal{C}_2$ denote user variables corresponding to a codeword. Let $f_1 : \{1, 2, ...2^{nR_1}\} \to \mathcal{C}_1$, $f_2 : \{1, 2, ..., 2^{nR_1}\} \times \{1, 2, ..., 2^{nR_2}\} \to \mathcal{C}_2$, which denote the encoding of the first and second users, respectively. The message $U^1$ is encoded into a codeword $X_1^1, X_2^1...X_n^1$ given by $f_1(U^1)$. Similarly, $U^2$, follows the rule with the addition that User 2 knows the message $U^1$, is encoded by $X_1^2, X_2^2, ..., X_n^2 = f_2(U^1, U^2)$.

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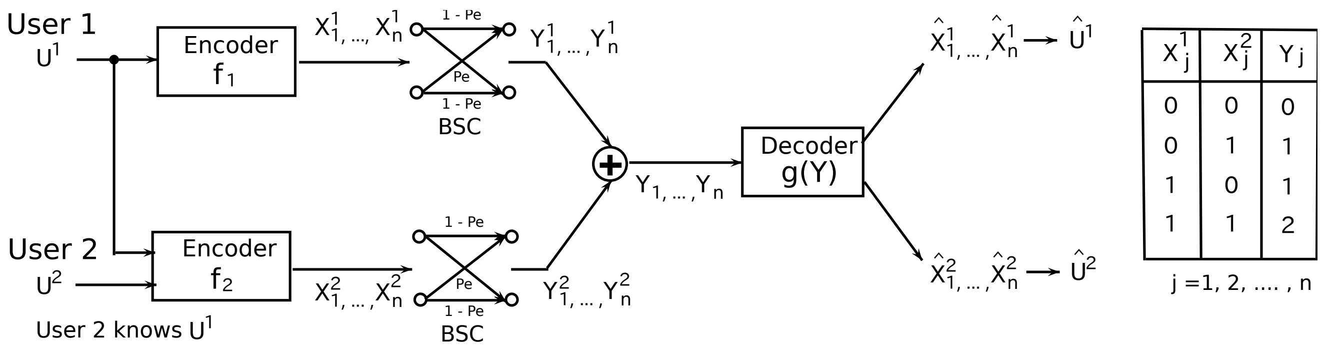


Fig. 1. Binary Symmetric AMAC — system diagram

The channel is characterized by its input sequence $X_1^1, X_2^1, ..., X_n^1$ and $X_1^2, X_2^2, ..., X_n^2$ (in the case of only two senders), and a set of conditional probability measures $p(y|x^1, x^2)$ on the output sequence $Y_1, Y_2, ..., Y_n$ given the inputs sequence $X_1^1, X_2^1, ..., X_n^1$, and $X_1^2, X_2^2, ..., X_n^2$.

*Definition 2.1:* A (two users) discrete memoryless multiple-access channel is denoted by $\{(\mathcal{X}^1, \mathcal{X}^2), \mathcal{Y}, p(y|x^1, x^2)\}$, where $X^1 \in \mathcal{X}^1$ and $X^2 \in \mathcal{X}^2$ are the input, $Y \in \mathcal{Y}$ is the output, and $p(\cdot|x^1, x^2)$ is a probability mass functions on $\mathcal{Y}$ indexed by the input data $x^1 \in \mathcal{X}^1$, $x^2 \in \mathcal{X}^2$. The channel is memoryless if

$$P(y_1, ..., y_n | x_1^1, x_2^1, ..., x_n^1, x_1^2, x_2^2, ..., x_n^2) = \prod_{j=1}^{n} p(y_j | x_j^1, x_j^2) \quad (1)$$

where $x_j^1$, $x_j^2$ and $y_j$ denote the *j*-th bit of the input and output sequence.

## B. Capacity of the AMAC

In a MAC, multiple users send data at the same time sharing the channel, however they interfere with each other. The capacity of the MAC is well-known [1] and given here for reference.

*Theorem 2.1:* The MAC capacity region is the closure of the convex hull of the set of points $(R_1, R_2)$ satisfying:

$$R_1 \leq I\left(X^1; Y | X^2\right), \quad (2)$$

$$R_2 \leq I\left(X^2; Y | X^1\right), \quad (3)$$

$$R_1 + R_2 \leq I\left(X^1, X^2; Y\right) \quad (4)$$

over all distributions $p(x^1) \cdot p(x^2)$ on $\mathcal{X}^1 \times \mathcal{X}^2$.

If User 2 knows $U^1$, this multiple access memoryless channel is regarded as the AMAC (Fig.1). The capacity region is given in [2].

*Theorem 2.2:* The AMAC capacity region is the closure of the convex hull of the set of points $(R_1, R_2)$ satifying:

$$\begin{aligned} R_2 &\leq I\left(X^2; Y | X^1\right) \\ &= H\left(X^2 | X^1\right) - H\left(X^2 | X^1, Y\right), \end{aligned} \quad (5)$$

$$\begin{aligned} R_1 + R_2 &\leq I\left(X^1, X^2; Y\right) \\ &= H\left(Y\right) - H\left(Y | X^1, X^2\right) \end{aligned} \quad (6)$$

TABLE I
JOINT INPUT DISTRIBUTION $p(x^1, x^2)$ ON $\mathcal{X}^1 \times \mathcal{X}^2$

| $p(x^1, x^2)$ | | $X^2$: $x^2 = 0$ | $X^2$: $x^2 = 1$ | $p(x^1)$ |
|---|---|---|---|---|
| $X^1$ | $x^1 = 0$ | $\alpha p_2$ | $p_1 - \alpha p_2$ | $p_1$ |
| | $x^1 = 1$ | $(1-\alpha)p_2$ | $1 - p_1 - (1-\alpha)p_2$ | $1 - p_1$ |
| | $p(x^2)$ | $p_2$ | $1 - p_2$ | |

over all joint distributions $p(x^1, x^2)$. Note that the AMAC capacity region is similar to the MAC capacity region, however one restriction (2) is removed.

## C. Binary Symmetric AMAC

This section describes the noise model used in this paper, which is called the *Binary Symmetric AMAC* and is pictured in Fig.1. It also describes a parameterized input distribution for the channel.

The AMAC capacity region will be characterized using the joint input distribution $p(x^1, x^2)$ on $\mathcal{X}^1 \times \mathcal{X}^2$, with three variables $p_1, p_2, \alpha$. Note that $p_1$ and $p_2$ denote the probablity on $\mathcal{X}^1 \times \mathcal{X}^2$, respectively, and that $p(X^1 = 0) = p_1$ and $p(X^2 = 0) = p_2$. Also, $\mathcal{X}^1 = \mathcal{X}^2 = \{0, 1\}$ and $\mathcal{Y} = \{0, 1, 2\}$.

*Definition 2.2:* Parameterized joint input distribution for the binary symmetric AMAC.

- User 2 knows the message of User 1, so $X^1$ and $X^2$ are jointly distributed according to $p(x^1, x^2)$ given by Table I.
- The joint distribution $p(x^1, x^2)$ depends on a parameter $\alpha$, which is bounded as:

$$\max(0, \frac{p_1 + p_2 - 1}{p_2}) \leq \alpha \leq \min(1, \frac{p_1}{p_2}).$$

If $\alpha = p_1$, then $X_1$ and $X_2$ are independent.

*Definition 2.3:* We define the binary symmetric AMAC as an AMAC with an independent BSC for each user:

- Input data $X^1$, $X^2$ corresponds to channel output $Y^1$, $Y^2$, respectively, passed through each Binary Symmetric Channel (BSC) with error probability $p_e$.

TABLE II
CONDITIONAL DISTRIBUTION $p(y|x^1,x^2)$ FOR BINARY SYMMETRIC AMAC

| $x^1, x^2$ | $Y=0$ | $Y=1$ | $Y=2$ |
|---|---|---|---|
| 0, 0 | $(1-p_e)^2$ | $2p_e(1-p_e)$ | $p_e^2$ |
| 0, 1 | $(1-p_e)p_e$ | $(1-p_e)^2$ | $(1-p_e)p_e$ |
| 1, 0 | $(1-p_e)p_e$ | $(1-p_e)^2$ | $(1-p_e)p_e$ |
| 1, 1 | $p_e^2$ | $2p_e(1-p_e)$ | $(1-p_e)^2$ |

- Conditional probability distributions $p(y^1|x^1)$, $p(y^2|x^2)$ are given by:

$$p(y^i=0|x^i=0)=p(y^i=1|x^i=1)=1-p_e,$$
$$p(y^i=0|x^i=1)=p(y^i=1|x^i=0)=p_e$$

  where $i=1,2$.
- The two BSC outputs are $y^1 \in \{0,1\}$ and $y^2 \in \{0,1\}$. Thus, the channel output $Y$ is the sum $Y=Y^1+Y^2$, which sets $y \in \{0,1,2\}$.
- The conditional probability distribution between each encoder and decoder is, $p(y|x^1,x^2) = \sum_{y_1,y_2} p(y|y^1,y^2)p(y^1|x^1)p(y^2|x^2)$ and is given by Table II.

The capacity region for the binary symmetric AMAC is the closure of the convex hull of the set of point $(R_1, R_2)$ which satisfy (5) and (6).

### D. WOM codes and their capacity

Write once bits (or wits) are an array of bits with 2 possible values, which are 0 and 1. The initial state of every wit is 0, which can be irreversibly programmed to 1.

An $\langle v\rangle^t/n$ binary WOM code is coding scheme that uses $n$-bit (or wits) to represent one of $v=2^k$ values as codewords, where $k$ is the number of bits in the information word, so that it can be written a total of $t$ times [4].

In this paper, we treat the well-known $\langle 2^2\rangle^2/3$ WOM-code, which is designed for the storage of two bits twice using only three cells. The encoding and decoding rules for this WOM code are represented in Table III, which shows four values and two codebooks. The codebook $\mathcal{C}_1$ represents the first write and codebook $\mathcal{C}_2$ represents the second write, each one with four codewords of length $n=3$. For more details on WOM codes construction you may refer to Cohen et al. [7].

The rate of the $i^{\text{th}}$ write is given by:

$$R_i = \frac{\log_2 v_i}{n}$$

since there are $v_i = 2^{k_i}$ messages on write $i$, and there are $n$ bits, where $i=1,2,...,t$.

*Theorem 2.3:* For a binary WOM 2-write codes capacity region is given by:

$$R_1 \le h(p), \tag{7}$$
$$R_2 \le 1-p \tag{8}$$

for $0 \le p \le 0.5$, where $h(\cdot)$ represents the binary entropy function [8]. $\langle 2^2\rangle^2/3$ WOM codes have a rate pair $(R_1,R_2) = (2/3, 2/3)$ [4].

TABLE III
WOM CODE $\langle 2^2\rangle^2/3$

| Message | First Write $\mathcal{C}_1$ $X_1^1X_2^1X_3^1$ | Second Write $\mathcal{C}_2$ $X_1^2X_2^2X_3^2$ |
|---|---|---|
| 00 | 000 | 111 |
| 01 | 001 | 110 |
| 10 | 010 | 101 |
| 11 | 100 | 011 |

## III. AMAC WITH NO ERRORS

### A. AMAC using WOM codes

This section explains how WOM codes are used for the AMAC. For the binary symmetric AMAC with no errors $p_e=0$, the explanation is straightforward as follows: (1) The encoder for $U^1$ uses the WOM codebook of the first write $\mathcal{C}_1$ to achieve a rate $R_1$. (2) The encoder for $U^2$ refers the message $U^1$ and uses the WOM codebook of the second write $\mathcal{C}_2$ if $U^1 \ne U^2$ (and if $U^1 = U^2$, Encoder for $U^2$ outputs the same codeword as $U^1$) to achieve rate $R_2$. (3) Since the channel has no errors, the decoder receives a sequence $Y_1, Y_2, ..., Y_n$ which is the real-number addition of both codewords $X_1^1, X_2^1 ... X_n^1$ and $X_1^2, X_2^2, ..., X_n^2$.

A property of WOM codes is that the second write knows the current state of the memory. In the AMAC system, this corresponds to User 2 knowing User 1's codeword. Thus, the AMAC is more appropriate than the MAC to apply WOM codes to cooperative wireless communications.

When $Y=0$ and $Y=2$, the decoder can find easily the pair $(X^1,X^2)=(0,0)$ and $(X^1,X^2)=(1,1)$, respectively. However, if $Y=1$, it produces an uncertainty, i.e. the decoder cannot distinguish between $(X^1,X^2)=(1,0)$ or $(X^1,X^2)=(0,1)$. A property of binary WOM 2-write codes is that in any position where $X_j^1=1$, then $X_j^2$ also is 1. Thus, $(X^1,X^2)=(1,0)$ never occurs, and $Y=1$ is always decoded as $(X^1,X^2)=(0,1)$.

The rate defined for WOM codes is the same as the rate for the AMAC, where $n$ represents the number of WOM code bits as well as the number of BSCs uses (AMAC) per user. The rates are also the same, in the AMAC case, user $i$ can transmit one of $2^{nR_i}$ messages, which is identical to the WOM code definition of rate.

### B. Capacity region with no errors

We here compare the capacity region of binary WOM 2-write codes and the AMAC with no errors $p_e=0$, in order to utilize binary WOM 2-write codes for wireless communications.

The AMAC capacity region can be calculated following (5) and (6) with joint input distribution $p(x_1,x_2)$. $I(X^1,X^2;Y)$ has maximum value 1.585, (for which $I\left(X^2;Y|X^1\right) \approx 0.918$), indicated by (a) in Fig. 2, when

$$p(X^1=0,X^2=0)=p(X^1=1,X^2=1)=1/3,$$
$$p(X^1=0,X^2=1)=p(X^1=1,X^2=0)=1/6, \qquad (a)$$

which corresponds to $Y$ having a uniform distribution.

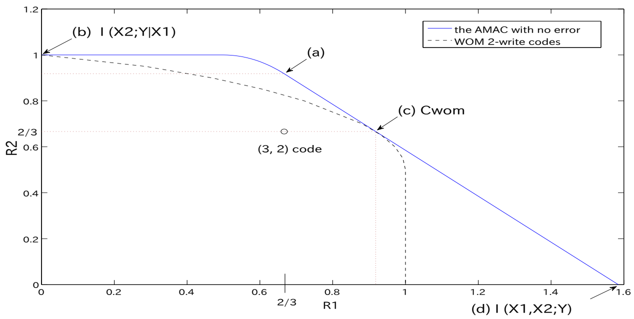


Fig. 2. The AMAC with no errors and WOM 2-write codes capacity region

On the other hand $I(X^2;Y|X^1)$ has maximum value 1, (for which $I\left(X^1,X^2;Y\right)=1$), also indicated by (b) in Fig. 2, when

$$p(X^1=0,X^2=0)=p(X^1=0,X^2=1)=0,$$
$$p(X^1=1,X^2=0)=p(X^1=1,X^2=1)=1/2. \qquad (b)$$

Other points in the achievable rate region were found by an exhaustive search over $p_1$, $p_2$ and $\alpha$ (See Table I), and the convex hull is shown in Fig.2.

### *C. Comparison of WOM codes and the AMAC*

For the AMAC, if the input distribution is chosen as:

$$p(X^1=x^1,X^2=x^2)=\begin{cases}0 & \text{if } x^1=1, x^2=0,\\ 1/3 & \text{otherwise.}\end{cases} \qquad (c)$$

Then, the rate pair $(R_1,R_2)\approx(0.918,0.667)$ is on the boundary of the AMAC capacity region. For the WOM code, this rate pair corresponds to the unrestricted-rate capacity $C_{WOM}$ of binary WOM 2-write codes, which is the maximum of the achievable sum-rates given by:

$$\begin{aligned} C_{WOM} &= \max_{(R_1,R_2\in R_{WOM})}(R_1+R_2)\\ &= \max_{p\in[0,\frac{1}{2}]}(h(p)+(1-p)), \end{aligned} \qquad (9)$$

where $R_{WOM}$ denotes the capacity region of WOM 2-write codes given in *Theorem 2.3*. This sum is maximized when $p=1/3$, implying

$$\begin{aligned} R_1 \approx 0.9183, R_2 &= \frac{2}{3},\\ C_{WOM} &= \log_2 3 \approx 1.585. \end{aligned} \qquad (10)$$

Using the WOM code, the corresponding input distribution for $\mathcal{C}_1$ is $p_1=2/3$ (i.e. the probability of a one is $1/3$). The input distribution for $\mathcal{C}_2$ can be shown to be $p_2=1/3$ (i.e. the probability of a one is $2/3$) [6].

Finally, we observe that the binary WOM 2-write capacity-achieving codes can achieve the AMAC capacity with no errors, and the WOM codes capacity region touches the AMAC one only at the point $(R_1,R_2)\approx(0.918,0.667)$, indicated by (c) in Fig. 2.

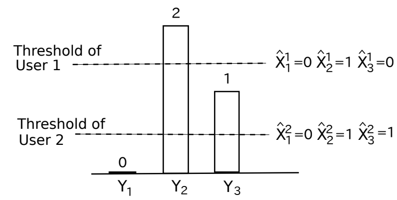


Fig. 3. Low Complexity Decoding using Threshold

TABLE IV
(A)ENCODING RULES WHEN $U^1=U^2$ (B)ENCODING RULES WHEN $U^1\neq U^2$

(a)

| $U^1$ | $\mathcal{C}_1$ | $U^2$ | $\mathcal{C}_2$ |
|---|---|---|---|
| 0 0 | 0 0 0 | 0 0 | 0 0 0 |
| 0 1 | 0 0 1 | 0 1 | 0 0 1 |
| 1 0 | 0 1 0 | 1 0 | 0 1 0 |
| 1 1 | 1 0 0 | 1 1 | 1 0 0 |

(b)

| $U^1$ | $\mathcal{C}_1$ | $U^2$ | $\mathcal{C}_2$ |
|---|---|---|---|
| 0 0 | 0 0 0 | 0 0 | 1 1 1 |
| 0 1 | 0 0 1 | 0 1 | 1 1 0 |
| 1 0 | 0 1 0 | 1 0 | 1 0 1 |
| 1 1 | 1 0 0 | 1 1 | 0 1 1 |

## IV. Low-Complexity Decoding

This section describes the low complexity decoding which is achieved owing to the key property of WOM codes where a bit value 1 can not be changed into a bit value 0. Thus, a bit value 0 is not assigned into a codeword $X^2$ where in the same position a codeword $X^1$ has a bit value 1. In order to explain the ideas, we use (3,2) WOM codes as an example and consider the special case $U^1=U^2$, where two different codebooks are required for User 2.

Table IV shows how to encode each user's data into codeword in the case of $U^1=U^2$ and $U^1\neq U^2$, in more detail. User 1's 2-bit data is encoded into a 3-bit codeword. (1) If User 2's 2-bit data is the same as User 1's, its data is encoded into a 3-bit codeword using Table IV-(a). (2) If User 2's 2-bit data is different from User 1's, its 2-bit data is encoded into a 3-bit codeword using Table IV-(b).

Once the decoder receives the $Y$ sequence it is needed to separate it into two estimated sequences. We set a threshold for each user. If each element of the $Y_1Y_2...Y_n$ sequence is above a threshold, the estimated bit of $\hat{X}^1_1\hat{X}^1_2...\hat{X}^1_n$ should be 1; otherwise, beneath the threshold each bit of $\hat{X}^1_1\hat{X}^1_2...\hat{X}^1_n$ should be 0. In summary, for User 1:

$$\hat{X}^1_j=\begin{cases}0 & \text{if } Y_j=0,1\\ 1 & \text{if } Y_j=2,\end{cases}$$

and for User 2:

$$\hat{X}^2_j=\begin{cases}0 & \text{if } Y_j=0\\ 1 & \text{if } Y_j=1,2\end{cases}$$

where $j=1,2,...,n$.

Let us show a concrete example. A pair of messages is transmitted over an AMAC. Suppose that User 1 transmits a message $U^1=[10]$, and User 2 transmits a message $U^2=[11]$ at the same time. To encode each message, Table IV-(b) is used because $U^1\neq U^2$. User 1 generates a codeword $X^1_1X^1_2X^1_3=[010]$ that corresponds to the first WOM code $\mathcal{C}_1$, and User 2 generates $X^2_1X^2_2X^2_3=[011]$ as the second WOM code $\mathcal{C}_2$. A

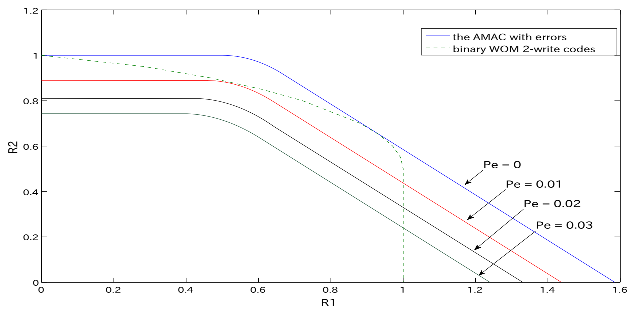


Fig. 4. binary WOM 2-write codes and the AMAC capacity region with errors

TABLE V
MAXIMUM VALUES OF MUTUAL INFORMATION OF THE AMAC ACHIEVABLE RATE REGION WITH ERRORS

| $p_e$ | 0 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\max I(X^1, X^2; Y)$ | 1.585 | 1.437 | 1.330 | 1.240 | 1.159 | 1.087 | 1.021 | 0.961 | 0.905 | 0.853 | 0.804 |
| $\max I(X^2; Y \vert X^1)$ | 1.000 | 0.890 | 0.810 | 0.743 | 0.684 | 0.631 | 0.583 | 0.539 | 0.498 | 0.460 | 0.426 |

bit by bit real-number addition for both codewords are taken place (recall there are no errors), and the decoder receives the sequence $Y_1Y_2Y_3 = [021]$; as shown in Figure 3, we obtain the estimated value $\hat{X_1^1}\hat{X_2^1}\hat{X_3^1} = [010]$ and $\hat{X_1^2}\hat{X_2^2}\hat{X_3^2} = [011]$ by each threshold, which were the transmitted codewords.

Using this scheme, the decoder can recover the transmitted data without errors.

## V. Preliminary studies for practical applications

Here we outline the preliminary studies on practical applications to channels with errors.

### A. AMAC Capacity Region with errors

We here find the binary symmetric AMAC capacity region with errors $p_e \geq 0$ (Fig.4). It is clear that for $p_e = 0$ (i.e. with no errors), the AMAC capacity region is greater than the WOM codes capacity region. We expect this is true for $p_e > 0$ as well. Hence, the AMAC region decreases, with increasing error probability $p_e$ (Fig. 4). Note that Table V shows each maximum value of (5) and (6) with errors which depend on various joint input distributions.

### B. Relay channel using WOM codes

This section describes a relay channel using WOM codes. This system consists of three nodes: a source node (S), a relay node (R), and a destination node (D) which is regarded as Block Markov coding [1], where codes are used in successive time slots.

We assume that the source can observe both message $U^1$ and $U^2$, whereas the relay can observe only message $U^1$. The relay node helps the transmission of the source. The transmission protocol of the proposed system is described as follows: (1) During the first time slot, at the source, message $U^1$ and $U^2$ are encoded into a codeword $X_1^2X_2^2...X_n^2 \in \mathcal{C}^2$ (note that $U^1$ is an empty message at this time) and broadcasted to both the relay and the destination nodes. At the relay, the message $U^2$ is re-constructed by performing channel decoding using side information $U^1$. (2) In the second time slot, at the relay, the re-constructed message is encoded using $\mathcal{C}^1$ into a codeword $X_1^1X_2^1...X_n^1$ and transmitted to the destination node; at the same time, a new codeword $X_1^2X_2^2...X_n^2 \in \mathcal{C}^2$ is generated from the current message $U^1$ and $U^2$ at the source node and broadcasted to both the relay and the destination nodes. Finally, the destination node can obtain both codewords $X_1^1X_2^1...X_n^1$ and $X_1^2X_2^2...X_n^2$ at the second time slot. The source (S) corresponds to User 2, and the relay (R) corresponds to User 1 in the AMAC because the source knows message $U^1$ and $U^2$ (i.e. User 2 knows User 1's message); therefore, the proposed WOM codes for the AMAC can be applied to the relay channel with block Markov coding.

## VI. Conclusion

We showed how to apply binary WOM 2-write codes to the AMAC and demonstrated that binary WOM 2-write codes can be used for the AMAC with no errors. In addition, binary WOM 2-write codes touch the boundary of the capacity region of the AMAC with no errors at the point $(R_1, R_2) = (0.918, 0.667)$ where WOM codes attain its capacity. Hence, binary WOM 2-write codes can be used for cooperative wireless communication systems with no errors, and we described low-complexity decoding with codewords of length 3. This scheme can be applied to the case of longer codes.

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