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Study on Turbo Decoding Using Hard Decision Decoder -Principle of Hard-in Soft-out Decoding-

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Abstract

We propose a new construction and decoding method for concatenated codes. One of the target coding schemes is a serially-concatenated coding scheme with a turbo code as the inner code and a Reed-Solomon (or other block) code as the outer code. The proposed decoding strategies include following two points. (1) New repetitive diagram of whole serial concatenated code is investigated. (2) To reduce complexity, hard-input soft-output decoding for the outer code is applied. Using a double error-correcting Reed-Solomon code as outer code, the results are evaluated and discussed.

1. Introduction

Turbo decoding (or the turbo principle) for decoding of concatenated codes widely uses soft-in soft-out decoding for component code [1]. In this study, the use of hard decision decoders to help turbo decoding is discussed.

Let us assume a concatenated coding scheme. Either serial or parallel concatenation is applicable to the scheme. We use hard decision decoder(s) for at least one component code of the concatenated coding scheme, and soft-input soft-output for one or more component code(s). Let us consider feedback of likelihood information from hard decision decoder to soft decision decoder. We need soft information from hard decision decode. This is the soft-output from hard decision decoder, and the decoder may call hard-input soft-output (HISO) decoder.

The principle of the HISO decode is as follows. We may evaluate frame, symbol and bit error rate of the component code for any known channel.

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We do not know the decoding results is correct or not, but we know how many errors are changed in the estimated codeword. Therefore, the conditional error rate can be calculated, given the number of the corrected errors. Of course, the hard decision decoder may fail to decode correctly, or may detect an uncorrectable error. In such cases, symbol and bit error rate for the received word is evaluated in similar way. These conditional bit error rates are used to transform the hard decisions into a soft-output value. These values may be determined either analytically or experimentally.

In this method, we assume all symbols and bits in an estimated codeword have same soft value. Therefore the code length of the corresponding component decoder must determined carefully.

In Chapter 2, we introduce concept of hard-input soft-output decoding. We show a method for deriving soft-output values from hard decision decoding, and discuss a suitable coding scheme for turbo decoding using hard-input soft-output decoder. In Chapter 3, a new decoding algorithm based on hard-input soft-output decoding is discussed. In Chapter 4, an example code is evaluated by computer simulation.

2. Concatenated Coding Scheme Using Hard Decision Decoder

Turbo decoding is performed for concatenated coding schemes using soft-input soft-output decoding of the component codes. Classical concatenated coding schemes, i.e. serial concatenation of Reed-Solomon (RS) and convolutional codes, use hard decision decoding and soft decision decoding. There should be some merit in hard decision algebraic decoding. If there is hard-in and soft-out decoding, we may apply turbo decoding to concatenated coding scheme using algebraic codes.

2.1. Hard-in and Soft-out (HISO) Decoding

In this section, we describe the computing method of soft-output information from the hard decision decoding. The hard decision decoder knows how many symbols are corrected in the decoding, i.e. Hamming distance $d(R, W)$ between the received word R and the estimated codeword W. We can estimate the reliability of the decoding based on the Hamming distance $t = d(r, v)$. From the well known analysis of word-error rate [4], [6] and bit-error rate of error correcting codes, we may describe the conditional probabilities with decoded distance $d(R, W)$, and derive the soft-output information of the decoding.

The calculated soft value is corresponds to all symbols of the estimated codeword. In case of ordinary soft-in soft-out decoding, every bit has the individual soft value. This is the differences between HISO and SISO decoding. If there are many codewords in the over all concatenated, the soft values help the other soft-in decoder. We discuss the effective structure of the concatenated coding scheme in next section.

Let us assume a linear code C over $GF(q)$ whose minimum distance is d_{min}, and perform the $\lfloor (d_{min} - 1)/2 \rfloor (= t_{max})$ error correcting, bounded distance decoding. The probability of correct decoding $P_c(W \mid t)$ for $t = 0, 1, \cdots, t_{max}$ error correction $(d(R, W) = t)$ is given as

$$P_C(W \mid t) = \left(\frac{P}{q-1} \right)^h P^t (1-P)^{n-t} \quad . \tag{1}$$

The probability of undetectable error $P_e(W \mid t)$ is given as,

$$P_E(v \mid t) = \sum_{h=0}^{n} \left(\frac{P}{q-1} \right)^h (1-P)^{n-h} \sum_{l=1}^{n} A_l N(l, h; t), \tag{2}$$

where $N(h, l; t)$ is the number of received words R which satisfies following condition. Hamming weight of R is h. For the mis-estimated codeword W with Hamming weight l and R, $d(R, W)$ is t.

$$N(l, h; t) = \sum \binom{n-l}{k} (q-1)^k \binom{l}{i} (q-2)^i \binom{l-i}{j}, \tag{3}$$

where the integers i,j and k satisfies $i + j + k = s$, $l + k - j = h$. The symbol error probability with t error correcting condition is given by

$$p_{byte}(W \mid t) = \frac{1}{n} \sum_{l=d}^{n} l \{ A_l \sum_{h=t_{max}+1}^{n} N(h, l; t)(P/(q-1))^h (1-P)^{n-h} \} \tag{4}$$

There are received words which are uncorrectable, but the error is detected with the bounded distance decoding. Such a probability, probability of error detection P_{Fail} is given by

$$P_{Fail} = \sum_{h=t_{max}+1}^{n} \left(\frac{\binom{n}{h}}{(q-1)^h} - \sum_{l=0}^{n} \sum_{m=0}^{t} N(h, l; m) A_l \right) \cdot P^h (1-P)^{n-h} \tag{5}$$

Therefore the symbol error probability for the uncorrectable errors is:

$$p'_{byte} = \frac{1}{n} \sum_{h=t_{max}+1}^{n} h \{ \binom{n}{h} (q-1)^h - \sum_{l=0}^{n} \sum_{t=0}^{t_{max}} N(h, l; t) A_l \} P^h (1-P)^{n-h} \ . \tag{6}$$

From the symbol error probabilities, the bit error probabilities are given by

$$p_{bit} = \alpha \cdot p_{byte} \quad , \tag{7}$$

where p_{byte} is p'_{byte} or $p_{byte}(W \mid t)$. The soft-output information for all bits corresponds to estimated codeword W is

$$\lambda(u_k; t) = \ln \frac{1 - \frac{p_{bit}(t)}{P_E(t)}}{\frac{p_{bit}(t)}{P_E(t)}} \quad . \tag{8}$$

and that the received word R for undetectable error case is:

$$\lambda'(u_k) = \ln \frac{1 - \frac{p'_{bit}}{P_{Fail}}}{\frac{p'_{bit}}{P_{Fail}}} \quad . \tag{9}$$

2.2. Concatenated coding scheme based on hard-input soft-output component decoder

We assume algebraic decoding for hard-input soft-output component decoder. Various block codes are applicable to the component code. The code should use for improving the BER efficiency of turbo-like codes. Therefore, we choose the serial concatenation of Reed-Solomon codes and turbo code as an example. Figure

Figure 1: Structure of information block of outer turbo code

1 shows the structure of serial concatenation code. An (n, k) RS code over $GF(2^q)$ is applied to the outer code. S RS codewords are passed to the outer turbo code.

We may apply turbo decoding to the example concatenated code or classical two-step decoding, i.e. turbo decoding for the turbo code and hard-decision decoding for the RS code.

2.3. Decoding for double error correcting RS codes

The merit of hard-decision decoders is that the hardware and computational complexity is low. We may apply reasonable algebraic decoder to the proposed system. One good choice selection is RS codes, especially double-error correcting (distance five) RS codes. The double-error correcting RS code can be decoded by solving simple formula, which is much simpler than the famous decoding algorithms of multiple error correcting algorithms, i.e. Berlekamp-Massey [2], Euclid and other algorithms.

Here, briefly introduce the double-error correcting of RS code over $GF(q)$. Let us assume:
s_i: $i^{\{}$th syndrome
L_i: error locations
α^{L_i}: error location vectors
σ_i: coefficient of the error locator polynomial
E: error value,

respectively.

The non-repetitive algebraic decoding algorithm is described as follows.

In case of a single error:

1. If $s_1 = 0$ or $s_2 = 0$, go to the double error case.
2. Compute
3. Verify $s_3 = s_2 \cdot \sigma$ and $s_4 = s_3 \cdot \sigma$.

   If either test fails go to the two error case. item Compute
4. XOR value E at location L.
5. Exit

In case of double errors:

1. Compute

$$\sigma_1 = \frac{s_1 \cdot s_4 + s_2 \cdot s_3}{s_1 \cdot s_3 + (s_2)^2} \qquad \sigma_2 = \frac{(s_3)^2 + s_2 \cdot s_4}{s_1 \cdot s_3 + (s_2)^2} \quad (10)$$

   If the denominator or either numerator is zero, post an uncorrectable error flag and exit.
2. Compute $C = \sigma_2/(\sigma_1)^2$ and fetch Y_1 and Y_2 from the root of the two-error locator polynomial: $Y^2 + Y + C = 0$. If C does not correspond to a valid pair of roots, post an uncorrectable error flag and exit.
3. Compute $X_1 = \sigma_1 \cdot Y_1$ and $X_2 = \sigma_1 \cdot Y_2$
4. Compute $L_1 = \log(X_1)$ and $L_2 = \log(X_2)$
5. Compute

$$E_1 = \frac{\alpha^{L2} \cdot s_1 + s_2}{\alpha^{L1} + \alpha^{L2}} \qquad E_2 = s_1 + E_1 \quad (11)$$

6. XOR value E at location L_1 and L_2.
7. Exit.

3. Proposed Decoding algorithm

Let C1 be the RS component code of serial concatenation. Let C2A and C2B be the component convolutional code of the original turbo code. Standard turbo decoding without RS code is performed by soft-input soft-output decoders for C2A and C2B. For received sequence y, maximize $P(u_k|y)$ for information bit $u_k (1 \le k \le N)$, and determine $u_k = 1$ or -1. In the log-domain, the LLR value $L(\hat{u}_k)$:

$$L(\hat{u}_k) = \ln \frac{P(u_k = +1|y)}{P(u_k = -1|y)} \quad (12)$$

are computed. If $L(\hat{u_k}) \geq 0$, $u_k = +1$ otherwise $L(\hat{u_k}) < 0$,$u_k = -1$. The $L(\hat{u_k})$ for systematic code is given by

$$L(\hat{u_k}) = L_c \cdot y_k + L(u_k) + L_e(\hat{u_k}) \quad , \tag{13}$$

where $L_c \cdot y_k$ is the channel value given by y_k, $L(u_k)$ is the a priori information given by the u_k i.e. log ratio of a priori probability $P(u_k)$ and $L_e(\hat{u_k})$ is the extrinsic information for u_k.

The proposed decoding structure is shown in Figure 2. The key point of the structure is the usage of RS decoder (decoders 1 and 2 for C1). The double error correcting decoder of the RS code is very simple compared with C2A and B. So we use two RS decoders in the structure.

Figure 2: Proposed Turbo Decoding

step1 (decoder1 for C2A):

Soft-input soft-output decoding is performed on decoder1 for C2A (in case of first iteration a priori values are determined as $L(1)(u_k) = 0$), log likelihood value $L^{(1)}(\hat{u}_k)$ is computed.

step2 (decoder1 for C1):

Using the hard decision value of $L^{(1)}(\hat{u}_k)$ from decoder1 for C2A, and converting binary data to nonbinary symbols, hard-input soft-output decoding is performed by decoder1 for C1. If the RS code is defined over $GF(2^k)$, the conversion is simply to take the k-bit vector as the RS symbol. We may assume simple basis for RS symbols. Soft-output information $\lambda^{(1)}(\hat{u}_k)$ is computed as in Chapter 2, and combined with hard decision decoding results as $+1 \rightarrow 1$ or $-1 \rightarrow 0$ Therefore, update $L^{(1)}(\hat{u_k})$ as:

$$L^{(1)}(\hat{u}_k) = L^{(1)}(\hat{u}_k) \pm \lambda^{(1)}(\hat{u}_k) \tag{14}$$

and output $L_e^{(1)}(\hat{u_k})$.

step3 (decoder2 for C2B):

Soft-input soft-output decoding is performed by decoder2 for C2B. Input extrinsic value $L_e^{(1)}(\hat{u_k})$ as the a priori value $L^{(1)}(u_k)$, operate decoder2 for C2B and output $L^{(2)}(\hat{u}_k)$ log likelihood value $L^{(1)}(\hat{u_k})$ is computed.

step4 (decoder2 for C1):

Operate the hard-input soft-output decoding same as step 2, i.e.

$$L^{(2)}(\hat{u_k}) = L^{(2)}(\hat{u_k}) \pm \lambda^{(2)}(\hat{u_k}) \tag{15}$$

output $L_e^{(2)}(\hat{u_k})$ to decoder1 for C2A and return to Step 1, if the number of iterations is less than the maximum. Otherwise goto Step 5.

step5(output the decoding results):

Output hard quantized value $L_e^{(2)}(\hat{u_k})$.

4. Computer Simulation

To obtain good BER performance, we select the finite field and RS code for an example scheme.

In this study we choose double error correcting (32,27) RS code over GF(32) ($d_{min} = 5$)as an example scheme.

The $s = 10$ codewords of the RS code, 310 symbols = 1550 bits are inputted to the encoder of rate 1/3 turbo code based on 8 state convolutional encoder and prime interleaver[8]. The overall concatenated scheme is (4650,1350) RS-Turbo serial concatenation, whose over all rate is 0.290.

Figure 3 shows the comparison of BER and word error rate(WER) performances propose scheme to simple rate 1/3 turbo coding for AWGN channel. The following coding are compared. In comparison, number of iteration of all turbo decoding is four and decoding of the component codes uses log domain BCJR algorithm[7]. To improve the performance, soft value of hard-input soft-output decoding results are scaled and transferred to the BCJR decoder of the component code. In the figure, following coding schemes are evaluated.

1. Proposed coding and decoding: RS-turbo serial concatenated coding and turbo decoding using HISO decoder, is indicated "Turbo-RS-Proposed" in the figure.
2. Proposed coding and conventional decoding: The conventional decoding perform inner turbo decoding for turbo code firstly, and outer decoding

(hard decision decoding for RS code) secondly, ”Turbo-RS-Conventional”.

3. Conventional turbo coding ”Turbo-RS-Conventional”.

From the results, the propose coding scheme has better bit error rate and word error rate performances. This means the hard-input soft-output decoding helps over all turbo decoding. The soft-output information is effectively transferred in the repetitive decoding. Especially proposed scheme improve the floor level. This is given by the decoding algorithm and the good side effect of RS component code.

Figure 3: Simulation Result of concatenated coding using HISC

5. Conclusion

In this study, hard-input soft-output decoding is proposed. The decoding is applied to serial concatenated coding scheme using RS code and turbo code. New turbo like decoding algorithm using hard-input soft-output decoding is proposed and evaluated.

This scheme is easily extend to scheme based on hard-and-erasure-input soft-output decoding. We are evaluating such cases. Including the scaling for hard-input soft-output decoding results, the other approaches improving the performance will be discussed in the next study.

References

[1] C. Berrou, A. Glavieux, and P. Thitimajshima C “Near Shannon limit error-correcting coding and decoding F Turbo code(1) ,”IEEE C Int.Conf.Commun ICC’93 C vol2/3 C pp.1064-1071 C May. 1993.

[2] E. R. Berlekamp, “Algebraic Coding Theory, ”McGarw-Hill, 1968.

[3] Neal Glover and Trent Dudley C“PRACTICAL ERROR CORRECTION DESIGN FOR ENGINEERS C”Data System Tecnology C1988.

[4] Hideki Imai, “Code Theory,”IEICE, 1990.

[5] G. David Forney, Jr, “Concatenated Code C”MIT Press C Cambrige C Mass 1966.

[6] Richard E. Blahut C“THEORY AND PRACTICE OF ERROR CONROL CODES, ”ADDISION-WESLEY PUBLISHING COMPANY.

[7] L. R. Bahl, J.Cocke, F.Jelinek, and J.Raviv, “Optimal decoding of liner codes for minimizing symbol error rate,”IEEE, Transaction on Fundamentals, vol.J85-A , pp.1168-1181, Nov, 2002.

[8] H. Sida, A. Shibutani, H. Imai,“An embedded interleaver for turbo codes based on prine-field,”IEICE Transaction on Fundamentals, vol.J85-A, pp.1188-1181, Nov, 2002.

[9] S. Maehara, Brian M. Kurkoski, K. Yamaguchi, K. Kobayashi, “Evaluation of turbo decoding for a concatenated code using hard decision Reed-Solomon decoding,”The 28th Symposium on Infomation Theory and its Applications(SITA2005) Onna, Okinawa, Japan,pp.351-354, Nov, 2005