Precoders for Message-Passing Detection of Partial-Response Channels

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Abstract — Parallel message-passing detectors for partial-response channels have the property that a bit is estimated using channel symbols in a window of size $W$ centered upon that bit. Distinct input sequences that produce the same output sequence result in undesirable failure of window decoders, but precoding can eliminate this input-to-output mapping ambiguity. For a class of partial-response channels, we show necessary and sufficient conditions on a precoder to unambiguously map input sequences to output sequences.

I. INTRODUCTION

It is well-known that the BCFJR algorithm is an instance of the sum-product algorithm [1], and it can be described by applying a serial schedule to a message-passing graph. When a parallel schedule is applied to such a graph, a new algorithm results, and it has been considered for detection on partial-response channels [2]. This algorithm estimates a bit using channel symbols in a window of size $W$ centered upon that bit. It can distinguish some, but not all of the possible transmitted sequences, even in the absence of noise.

We will represent a sequence of symbols $a_{i-1}, a_i, a_{i+1}, \ldots$ using delay operator notation $a(D)$; also, we will write $a_i^W$ to mean $a_i, \ldots, a_{i+W}$. We consider the system shown in Fig. 1. The precoder input is the binary sequence $x(D)$. The partial-response channel has input $a(D)$ and output $y(D)$. In this paper, we are considering partial-response channel polynomials of the form $\gamma(D) = (1 - D)^m (1 + D)^n$, for integers $m,n \geq 0$, and $m$ not both zero. The window detector matched to the channel produces as its estimate $\hat{a}(D)$, and the output of the postcoder with transfer function $f(D)$ is $\hat{x}(D)$.

Definition Let $a(D)$ and $a'(D)$ be two distinct partial-response channel input sequences (or precodings), and let $y(D) = h(D) a(D)$ and $y'(D) = h(D) a'(D)$. If $y_i = y'_i$ for $i = 1, \ldots, W$, then $y^W$ is an ambiguous output sequence.

II. PRECODERS FOR MESSAGE-PASSING DETECTORS

Ambiguous output sequences will lead to failure of the window algorithm because the detector must choose between the distinct input sequences $a_i^W$ and $a'_i^W$ with equal probability; choosing the wrong input sequence generally results in a large number of bit errors. In systems with additive white Gaussian noise operating at high signal-to-noise ratio, ambiguous output sequences are the dominant source of bit errors, particularly when the window size $W$ is small. Thus, we are motivated to eliminate errors due to ambiguous output sequences.

The solution is to introduce a precoder, which maps binary user data $x(D)$ to the channel preimage $a(D)$. Lemma 1 gives necessary and sufficient conditions for a precoder to eliminate the problem of ambiguous output sequences.

**Lemma 1** Consider a binary input partial-response channel with transfer function $h(D) = (1 - D)^m (1 + D)^n$ (integers $m,n \geq 0$), preceded by a binary input, binary output precoder with transfer function $f(D)$. An ambiguous output sequence of length $W$ greater than or equal to some constant $W^{*}_{m,n}$ corresponds to a unique precoder input sequence if and only if:

\[
(1 + D) | f(D) \text{ mod } 2 \quad \text{for} \quad (m = 0 \text{ and } n > 0) \quad \text{or} \quad (m > 0 \text{ and } n = 0)
\]

\[
(1 + D^2) | f(D) \text{ mod } 2 \quad \text{for} \quad (m > 0 \text{ and } n > 0).
\]

When $W \geq W^{*}_{m,n}$, ambiguous output sequences correspond exactly to null error sequences, which are bit-infinite sequences $\varepsilon_a(D) = a(D) - a'(D)$ such that $\| \varepsilon_a(D) h(D) \| = 0$ [3]. Let $x'(D)$ be the preimage of $a'(D)$. In the proof of Lemma 1, we found restrictions on the coefficients of $f(D)$ such that $x^W = x'^W$, when $a_i^W = a'_i^W$ is a substring of a null error sequence.

The following values of $W^{*}_{m,n}$ were found by computer search: $W^{*}_{1,1} = 1$ (decoder), $W^{*}_{1,2} = 1$ (PR4), $W^{*}_{1,3} = W^{*}_{2,3} = 4$ (EPFR4, E$^2$PR4).

We note that precoders have been used in the past to avoid the effects of quasi-catastrophic error propagation in partial-response systems [4]. This use of precoders is similar to, but distinct from the use discussed in this paper.

**REFERENCES**


